COMPUTER STUDY OF HARMONIC ORBIT CORRECTION IN THE AGS BOOSTER

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A. U. LUCCIO
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ACCELERATOR DEVELOPMENT DEPARTMENT
BROOKHAVEN NATIONAL LABORATORY
UPTON, NEW YORK 11973

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A.U. LUCCIO

I. Introduction

As a contribution to the more general problem of model based control of the AGS Booster, in this note we discuss the correction of a distorted closed orbit in the machine.

There are many possible schemes to correct a distorted closed orbit in a circular accelerator. The harmonic correction procedure is particularly attractive since it is tailored to the periodic structure of the machine and moreover makes use of well optimized fast computer routines.[1] To correct a distorted orbit, one could physically displace some magnets or use the angular kick on the trajectory provided by correcting coils. The former method is only suitable for day-one adjustements, while the latter can be used at any time during the life of the accelerator. This note discusses the use of correcting or steering magnets.

The problem of correction is particularly easy to solve in cases like the AGS Booster, where the number of PUE's or beam monitors coincides with the number of correctors and where monitors and correctors are located in the same position and are equally spaced (!).

II. Theoy of harmonic orbit correction

The equation of motion in curvilinear coordinates of a particle in a cyclic accelerator can be written as follows

$$\frac{d^2y}{dy^2} + K(s)y = F(s), \tag{1}$$

where y(s) is the transverse displacement (horizontal or vertical) of the trajectory from the

ideal orbit and s is the distance along the circumference. Using the Floquet transformation

$$\phi(s) = \int^{s} \frac{ds'}{\nu\beta}, \tag{2a}$$

$$y[s(y)] = \sqrt{\beta}\,\eta(\phi), \tag{2b}$$

Equation (1) becomes

$$\frac{d^2\eta}{d\phi^2} + \nu^2\eta = \nu^2\beta^{3/2}F[s(\phi)] = \nu^2 f(\phi). \tag{3}$$

Here, η is a function of ϕ and the function f on the r.h.s of Equation (3) is in effect the sum of an error function f^e and a correction function f^c

$$f(\phi) = f^e + f^c. \tag{4}$$

A special solution $\eta(\phi)$ of Equation (3) with periodic boundary conditions determines the closed orbit $y = \sqrt{\beta}\,\eta$.

In practice, the orbit $y(\phi)$ is sampled at fixed angles ϕ_k, where the PUE's are located. Therefore, the input to the correction calculation is a set of measured data y or of η, i.e. orbit weighted by the local values of β_k. This input can be expanded in a discrete Fourier sum, with coefficients E_n to be calculated by a Fast Fourier Transform

$$\eta_k \equiv \eta(\phi_k) = \frac{1}{N}\sum_{n=1}^{N} E_n e^{-i(n-1)\phi_k}, \tag{5}$$

where N is the number of sample points (monitors and correctors), and

$$\phi_k = \frac{1}{\nu}\int^{s_k} \frac{ds'}{\beta} \approx \frac{2\pi k}{N}. \tag{6}$$

Note that, because of the symmetries of the β functions, the locations of monitors and correctors are approximatively equidistant also in Floquet's angles.

The function f^e of Equation (4) representing the errors in the machine is a periodic function of the Floquet angle ϕ with periodicity 1 (the whole machine) and its values at the observation points form a discrete set that can be represented by a Fourier expansion with coefficients ${F_n}^e$, i.e.

$$ {f_k}^e \equiv f^e(\phi_k) = \frac{1}{N}\sum_{n=1}^{N} {F_n}^e e^{-i(n-1)\phi_k}. \tag{7} $$

The relationship between the Fourier coefficients of the machine error function f^e and the coefficients of the distorted closed orbit function η can be found by inserting the expansions of Equations (5) and (7) into the differential Equation (3). It is found

$$ {F_n}^e = \frac{\nu^2}{\nu^2 - (n-1)^2} E_n. \tag{8} $$

.

Now, to correct the distorted orbit the correction must cancel the driving term of the errors: $f^c = -f^e$, or

$$ {F_n}^c = -{F_n}^e $$

for each n. The correction is achieved by local deflections $\delta\theta_k$ of the trajectory produced by steering coils. The relation between deflections and correcting function values is found immediately by rewriting Equation (1) as follows

$$ \frac{d\theta}{ds} + Ky = F = F^e + F^c, \tag{9} $$

that shows how F has to be interpreted as a deflecting force. The deflection produced by a steering magnet can be written

$$ \delta\theta = \int F_c ds = \nu \int_{\delta\phi} \beta^{-1/2} f^c d\phi, \tag{10} $$

the integrals being extended to the azimuthal length of the correctors. For short correctors

$$\delta\theta \approx \nu\beta^{-1/2} f^c \delta\phi, \tag{11}$$

In terms of the field integral $B\delta s$ in each corrector, the local deflections is expressed as

$$\delta\theta = \frac{B\delta s}{\bar{B}\rho}, \tag{12}$$

with $\bar{B}\rho$ the magnetic rigidity of the particle beam. For short correctors, the set of discrete deflections used to achieve orbit correction is

$$\delta\theta_k \approx \nu\beta^{-1/2} f^c(\phi_k)\, \delta\phi_k. \tag{13}$$

If we write the Fourier expansion for f^c, analogous to Equation (5) and invert it, then use Equations (8) and (10), the following relations between the deflection Fourier coefficients to correct the orbit and the orbit coefficients are finally found

$$-\frac{\nu^2}{\nu^2 - (1-n)^2} E_n = \sum_{k=1}^{N} \sqrt{\beta_k} \frac{\delta\theta_k}{\nu\delta\phi_k} e^{i(k-1)\phi_k}. \tag{14}$$

This set of linear equations can be solved by Fourier anti-Transform to give the values of needed deflections $\delta\theta_k$. From these and Equation (14) the field integral in the steering coils can be calculated.

III. Computer solution. Examples

A computer code to calculate the harmonic correction of a distorted closed orbit in the present assumption, valid for the AGS Booster, of equal number of monitors and correctors does the following.

The input is: a set y_k, (k=1,N) of observed beam positions; the values of the relevant β function and of the tune ν at the same points; the value of $B\rho$; the corrector strengths ${K_k}^c$.

The code weights the numbers y_k with the values of $\sqrt{\beta_k}$, calculates the coefficients E_n, (n=1,N), by Fast Fourier Transform and then the l.h.s. of Equation (14).

The next step is to perform a Fast Fourier anti-Transform of the E_n set to reconstruct the orbit and an anti-Transform to calculate the coefficients of the r.h.s. of Equation (14). From these, using the set of ${K_k}^c$ and Equation (12) the required field integrals in the correcting magnets are calculated.

The AGS Booster has 24 monitors and correctors for each transverse plane. They are located at the entrance of each quadrupole and are equally spaced. In the present calculation we have studied the horizontal orbit correction, with monitors and correctors at the entrance of the focusing quads. Lacking a real machine, the orbit readings were simulated by random numbers corresponding to a closed orbit with up to 5 mm distortions and average value zero. To check the Fourier Transform and to simulate an hypothetical periodicity in the errors, a sine modulation with frequency 6 (the number of periods in the lattice) with variable amplitude and random overall phase was added to the pure white noise random values for the orbit.

β and ν values were calculated using the machine design code MAD[2], running interactively on the Apollo Workstation, with standard data for the Booster, with results shown in Figures 1 and 2.

Fourier Transforms, forward and backwards, were executed using Fast FT routines of the package SLATEC on the Apollo. A Fast FT produces a spectrum of amplitudes extending only to a frequency equal to half of the sampled points (here, only 12 frequency values). FFT was chosen because of its speed, since it is planned to repeat these calculations for control of the real machine in quasi-real time. Before executing the antitransform, we tried some low-pass filtering of the spectrum, by cutting frequencies above some thresh-

old. We found, as expected that harmonic correction is more effective on sine modulated distorted orbit data than on the random part, that in general contains high frequencies.

Figure 3 shows the results of the calculation for a pure random orbit simulation. The lower graph pair represents the distorted orbit before and after correction. In the calculation we assumed that the corrector strength parameters were all the same, that the correction itself was affected by a 5% error and that the corrected orbit could not be read better than $\pm 0.5mm$. The upper curve represents the correction $B\delta s$ needed at each corrector location along the ring in [gauss-cm]. Figure 4 shows the Fourier spectrum for this case (amplitude and phase).

Figure 5 shows what happens when a sinusoidal variation with frequency $f = 6\omega_o$ of the same aplitude is added to the random values. The orbit is being corrected with about half the corrector strengths needed for the previous case, since this is a kind of correction that Fourier seems to enjoy. Figure 6 shows the spectrum for this case.

REFERENCES

[1] E.Gianfelice, R.Giannini and J.Jäger *Methods of Closed-Orbit Correction in Lear .....*
Report CERN/PS/LEA/Note 83-4. July 1983.

[2] F.Ch.Iselin and J.Niederer, *The MAD Program Version 6.*
Report CERN/LEP-TH/87-33. April 1987.

Appendix. List of the Fortran code

```
      program ORBIT

C 23-Sep-1988
C random orbit for correction

      parameter (m=48)
      integer   rand_c,seed
      real      x(m),y(m),u(m),beta_x(m),
     &          eta(m),work(2*m+15),c(m),phi(m),
     &          p(m),dy(m),b_ds(m)

      data  AA,bias,ds,pi /.005,10.,.2286,3.141592654/
c-----------------------------------------------------------------------
      open      (10, file='orbit_beta.dat', status='unknown')
      read      (10,*) NN
        N6      = NN/6
      do 100 i  = 1,N6
100   read      (10,*,err=120) x(i),beta_x(i),dummy          {read beta
120   read      (10,*) qx                                    {read Qx
      read      (10,*) b_rho
      close     (10)

        dx      = x(2) -x(1)
      do 130 i  = N6+1,NN
        x(i)    = x(i-1) +dx
        beta_x(i) = beta_x(i-N6*((i-1)/N6))
130   continue
c-----------------------------------------------------------------------

      write     (6,6000)
6000  format    ('enter i_filtr, mod_depth : ',$)
      read      (5,*) i_filtr,depth

c               call srand_c(seed)

        y_ave   = 0
        rr2     = 2*(rand_c()+1)/32768. -1                   {random number
      do 200 i  = 1,NN
        rr1     = 2*(rand_c()+1)/32768. -1                   {simulated orbit
        y(i)    = AA*(rr1 +depth*sin(12*i*pi/NN +rr2))/(1+depth)
        y_ave   = y_ave +y(i)
200   continue
        y_ave   = y_ave/NN
        y_2ave  = 0
      do 210 i  = 1,NN
        y(i)    = y(i) -y_ave
        eta(i)  = y(i)/sqrt(beta_x(i))
        u(i)    = eta(i)
        y_2ave  = y_2ave +y(i)*y(i)
210   continue
        y_2ave  = sqrt(y_2ave)/NN
c-----------------------------------------------------------------------
                call rffti(NN,work)                          {FFT initial.
                call rfftf(NN,eta,work)                        {FFT transform

      do 300 i  = 1,NN/2-1
        b       = eta(2*i)                                   {spectrum
        a       = eta(2*i+1)                                 {
        c(i)    = sqrt(a*a +b*b)                             {amplitude
        if (a.ne.0 .or. b.ne.0) phi(i)  = atan2(a,b)         {phase
300   continue

      do 320 i  = i_filtr,NN
320     eta(i)    = 0.

      do 330 i  = 1,NN                                       {FFT coeff
```

```
        p(i)     = -eta(i)*qx*qx/(qx*qx-(1-i)**2)            {of correction
330   continue

                 call rfftb(NN,eta,work)                      {FFT anti-
                 call rfftb(NN,p,work)                       {transform

        dy_2av   = 0
        b_ds_2av= 0
      do 400 i   = 1,NN
        deta     = u(i) -eta(i)/NN
        rr       = 2*(rand_c()+1)/32768. -1
        dy(i)    = deta*sqrt(beta_x(i)) +0.10*rr*AA          {orb. correction
        rr       = 2*(rand_c()+1)/32768. -1
        ds       = ds*(1 +0.05*rr)
        p(i)     = p(i)/NN                                    {corr. strength
        dtheta   = ds/(beta_x(i)*sqrt(beta_x(i)))*p(i)
        b_ds(i)  = b_rho*dtheta
        dy_2av   = dy_2av +dy(i)*dy(i)
        b_ds_2av= b_ds_2av +b_ds(i)*b_ds(i)
400   continue
        dy_2av   = sqrt(dy_2av)/NN
        b_ds_2av= 1.e6*sqrt(b_ds_2av)/NN                     {gauss-cm
c-----------------------------------------------------------------
      open       (20, file='orbit.dat', status='unknown')
      open       (22, file='orbit.cor', status='unknown')
        x_prior = x(1) -N6*dx
      do 410 i   = 1,N6-1
        j        = NN-(N6-1)+i
        x_prior = x_prior +dx
        y_mm     = 1.e3*y(j)
        dy_mm    = 1.e3*dy(j)
410   write      (20,'(3g13.5)')
     &           x_prior,y_mm,dy_mm
      do 420 i   = 1,NN
        b_ds_gdm= 1.e5*b_ds(i) +bias
      write      (22,'(3g13.5)') x(i),b_ds_gdm,bias            {write correction
        y_mm     = 1.e3*y(i)
        dy_mm    = 1.e3*dy(i)
420   write      (20,'(3g13.5)') x(i),y_mm,dy_mm               {write orbit
        x_next   = x(NN)                                       {to file
      do 430 i   = 1,N6
        x_next   = x_next +dx
        y_mm     = 1.e3*y(i)
        dy_mm    = 1.e3*dy(i)
430   write      (20,'(3g13.5)') x_next,y_mm,dy_mm
      write      (20,*) i_filtr-1,' harmonics; mod_depth=',depth
      write      (20,*)
     &           ' <y2>=',y_2ave,'; <B_ds>=',b_ds_2av,'; <dy>=',dy_2av
      write      (6 ,*)
     &           ' <y2>=',y_2ave,'; <B_ds>=',b_ds_2av,'; <dy>=',dy_2av
      write      (22,*) 'eod'
      write      (20,*) 'eod'
      close      (22)
      close      (20)
c-----------------------------------------------------------------
      open       (21, file='orbit.spc', status='unknown')
      do 500 i   = 1,NN/2-1
        ri       = i
        c1000    = 1.e3*c(i)
500   write      (21,'(6g13.5)') ri,c1000,phi(i),dummy1         {write spectrum
      write      (21,*) 'eod: spectrum'                        {to file
      close      (21)
c-----------------------------------------------------------------
      stop       'files ** orbit.dat ; orbit.cor ; orbit.spc ** written'
      end
```

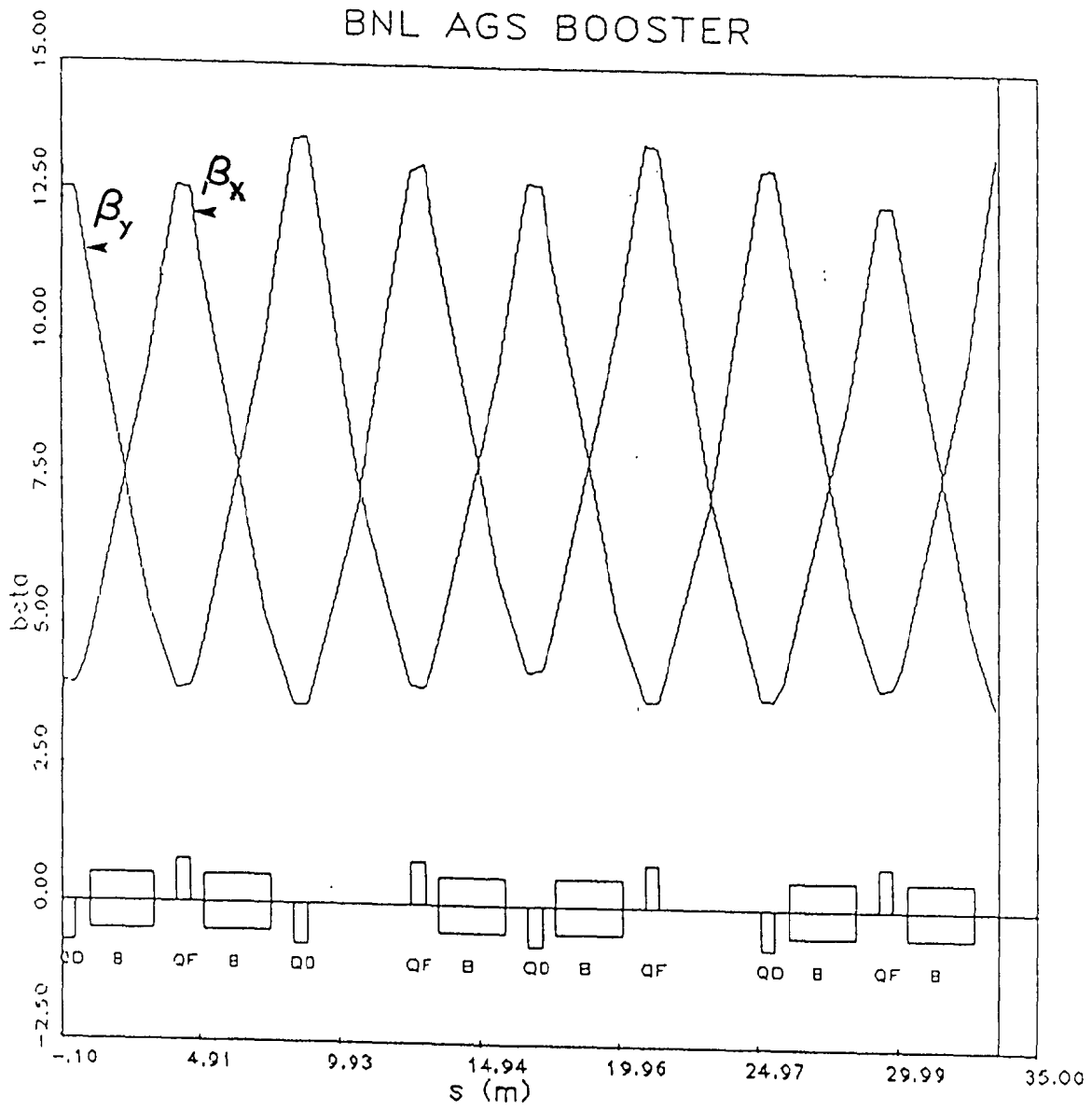

Fig. 1. Layout of one superperiod of the AGS Booster. Machine functions β_x and β_y calculated by MAD are also shown.

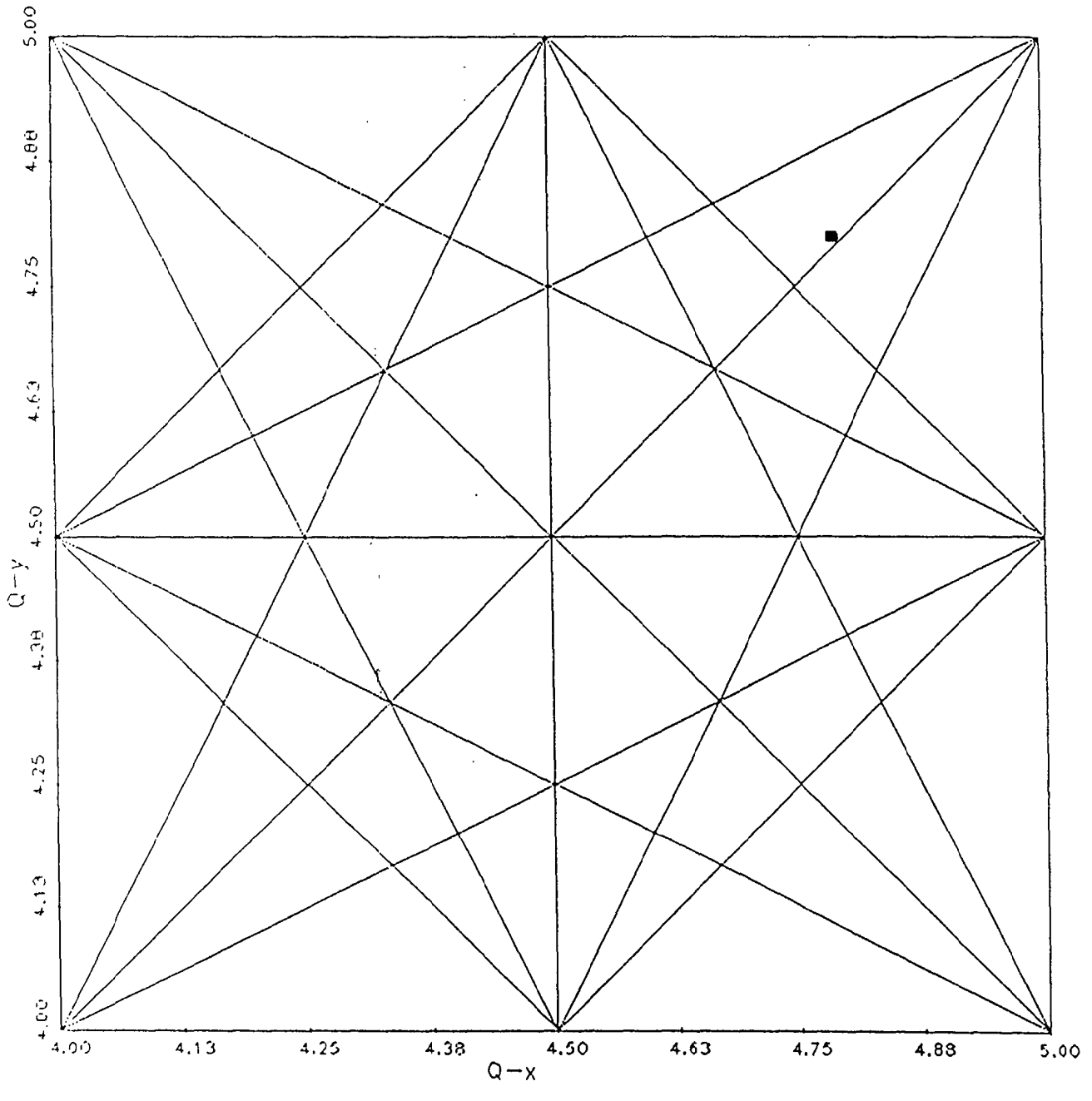

Fig. 2. Resonance chart for the AGS Booster. The calculated tune working point (ν_x, ν_y) is also shown.

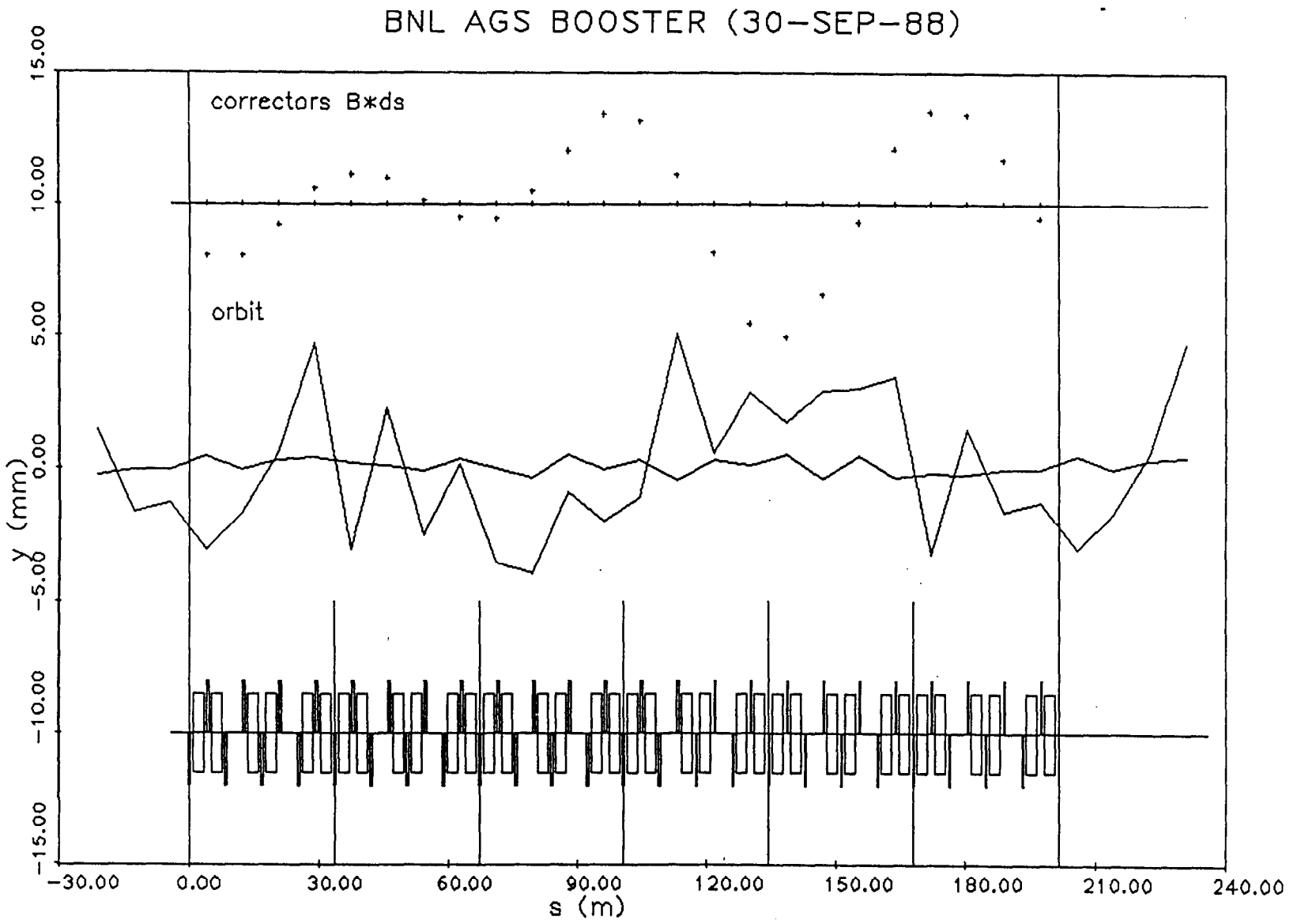

Fig. 3. Complete layout of the AGS Booster. The lower pair of curves represent a distorted closed orbit generated by random number extraction, before and after harmonic correction. The upper curve represent the corrector strength needed to achieve this result, in arbitrary units.

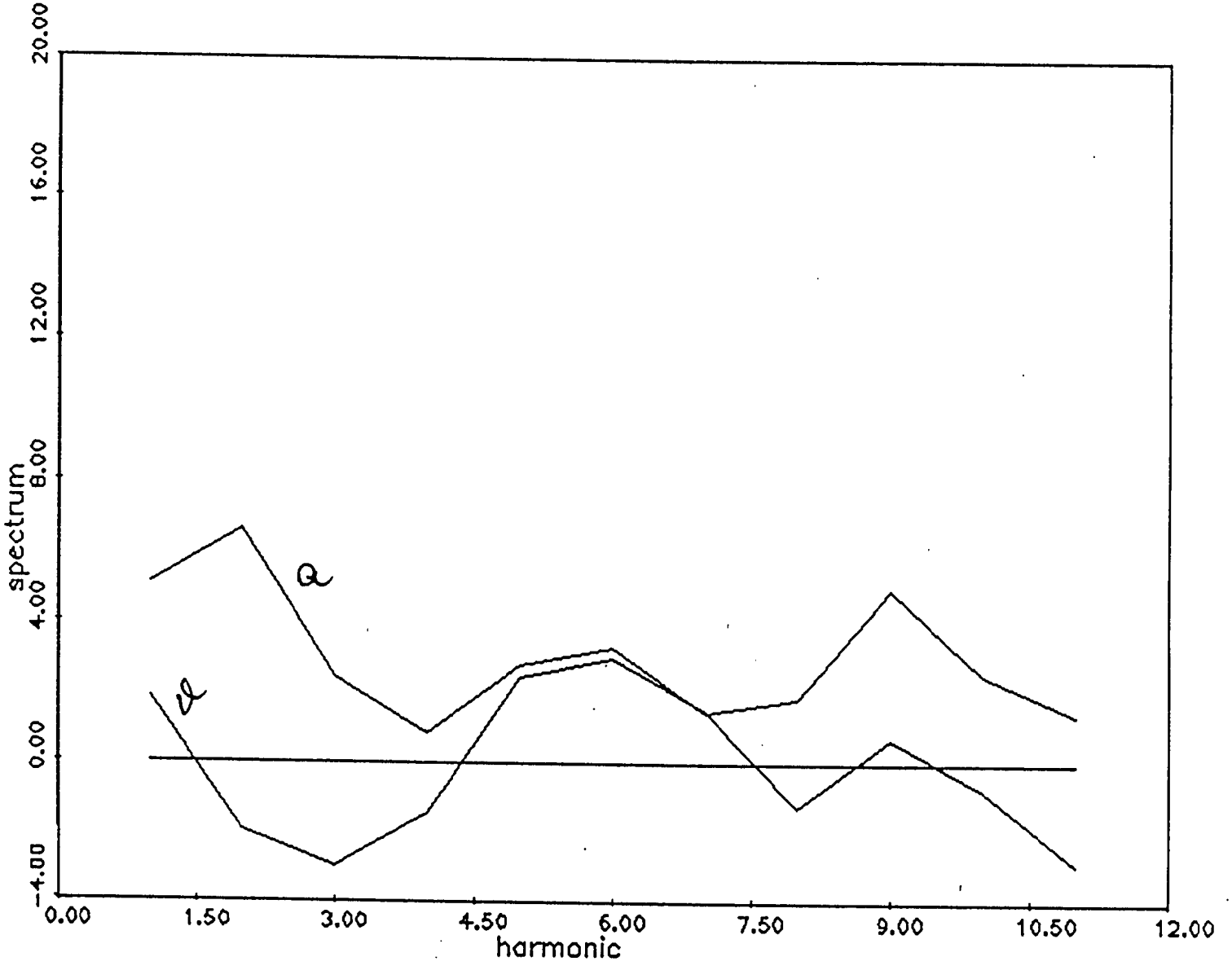

Fig. 4. Fourier spectrum, amplitude and phase, for the random orbit of Figure 3.

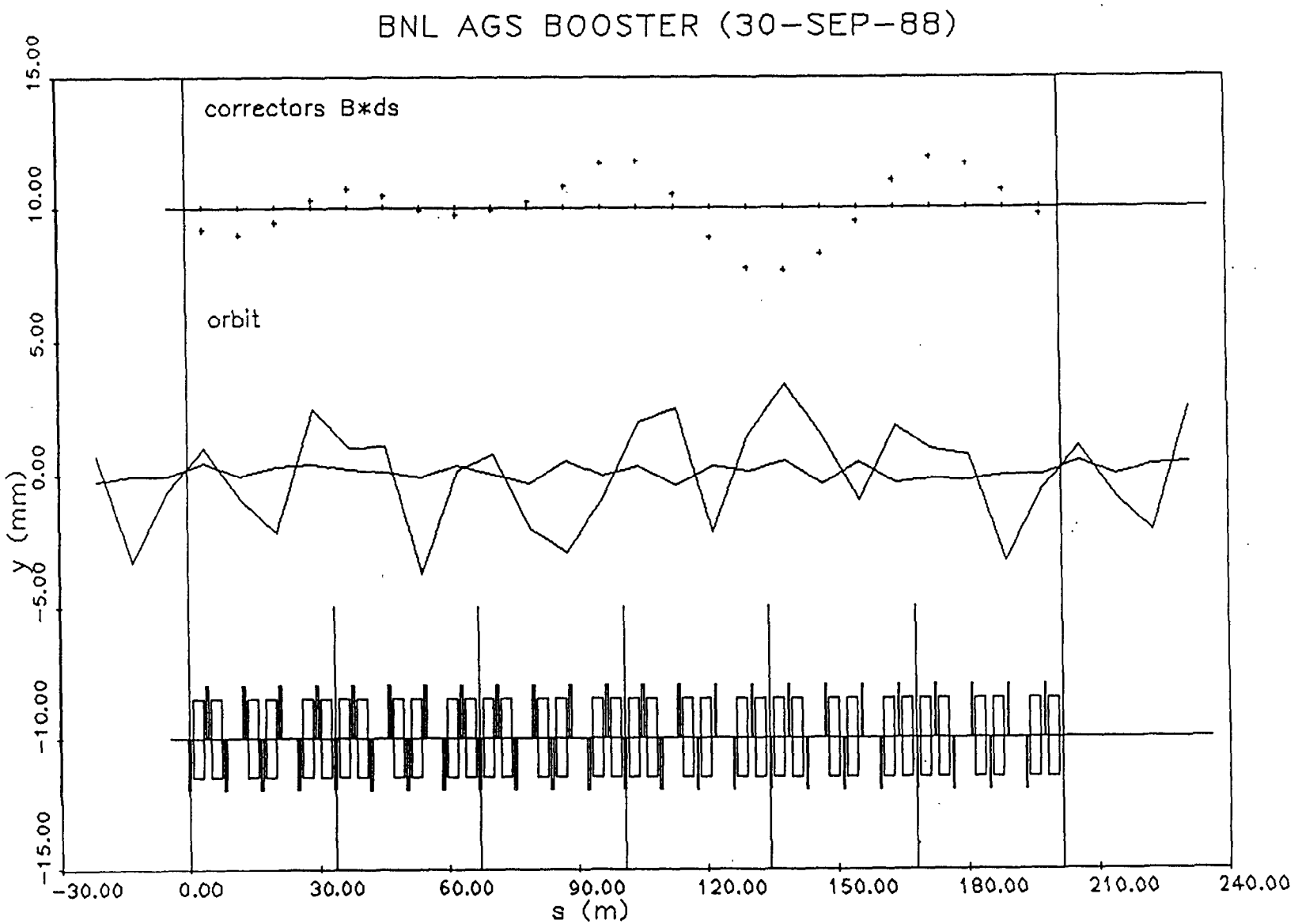

Fig. 5. Same as Figure 3, but with a sinusoidal modulation of equal amplitude added to the random numbers to generate the distorted orbit.

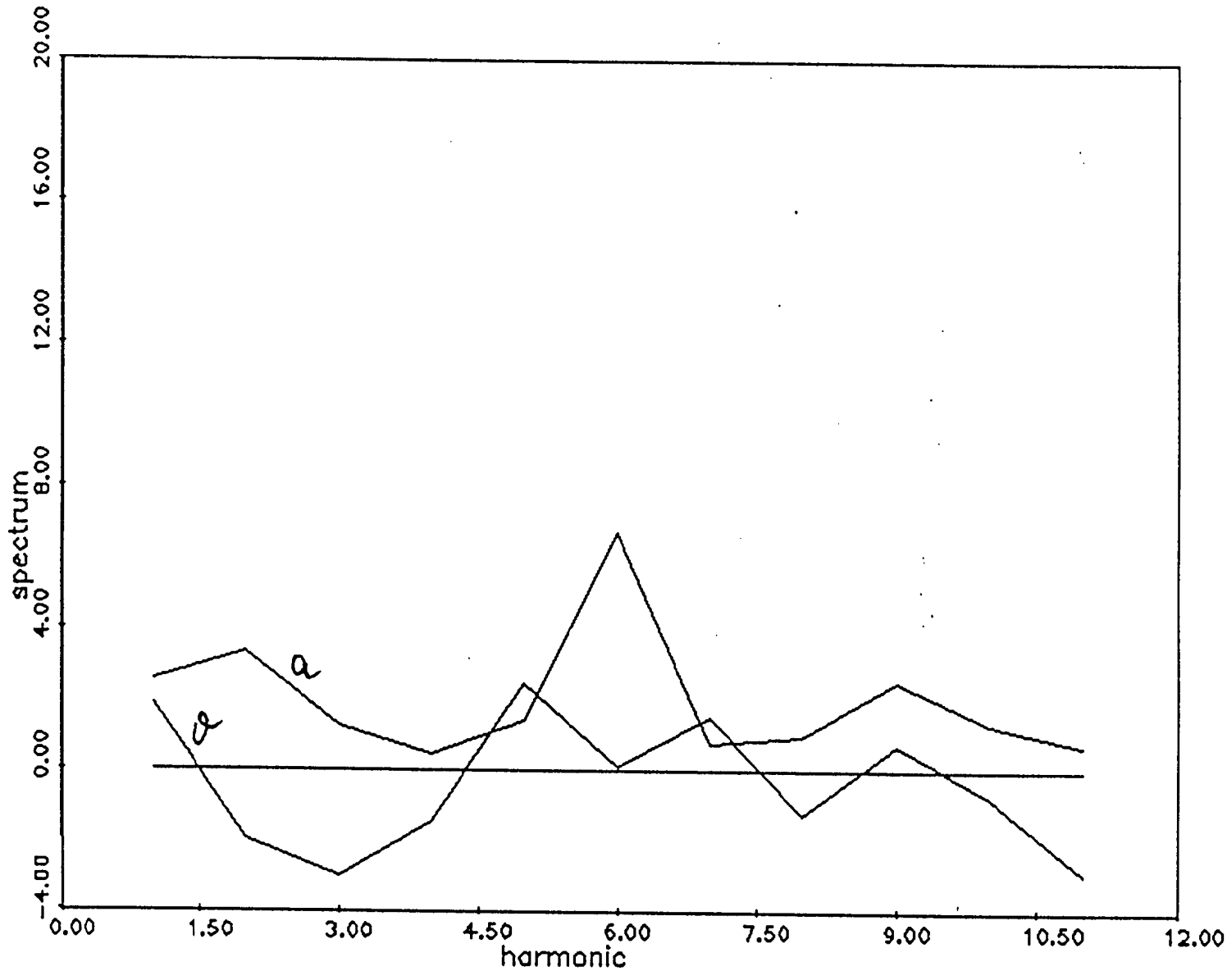

Fig. 6. Fourier spectrum for Figure 5. A peak at harmonic 6 is showing.