Impedances and Wakes in High-Energy Proton Accelerators

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Bruno Zotter

CERN, Geneva, Switzerland

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Introduction

- **High beam currents** in particle accelerators or storage rings require knowledge of impedances which lead to instabilities and/or overheating and limit performance. Impedances: measure of voltage induced by charged beam in vacuum chamber walls and other surrounding structures acting back on beam. Standard practice to prepare complete **impedance budget** for new particle accelerators, update for design changes or additions.
- Not discussed here **space charge impedance** (Shelter Island workshop), **radiation impedance** (not important for proton machines). Excellent **general reviews** of techniques to calculate impedances and wakes published over last years[1, 2, 3, 4, 5].

References

[1] P. Wilson, AIP-184 (1987) p. 525 ff.
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[3] S. Heifets, S. Kheifets, Rev. Mod. Physics 63(1991) p.631 ff.
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Introduction 2

- Important geometric impedances: **rf cavities, kicker tanks, bellows** and other **cross-section variations** of vacuum chamber. (low-frequency) impedance as well as (short-range) wake potential due to localized elements estimated by computer, use directly in **simulation codes** for single bunch instabilities, for machines with high charge per bunch (electrons).
- For proton beams, typically with many rather long bunches, **sharp resonances** important (long memory), excite multi-bunch instabilities. Calculate with 2-D (and 3-D) computer programs, **exact resonant frequencies** depend critically on structure shapes and dimensions, **change with temperature** or position of variable tuners in cavities.
- Not possible to predict total impedance when **resonances overlap** due to closely spaced modes or many similar elements (e.g. cavities or bellows). **Worst case assumption**: all elements exactly equal and resonances overlap completely - far too pessimistic, leads to exaggerated requirements.

Introduction 3

- Instabilities often cured with simple **feedback systems**, only necessary to estimate required strength, pessimistic estimate not too expensive.
- In (low energy) proton machines (e.g. spallation sources), **space charge** (negative inductance, often called "capacitive") and **finite conductivity** of surrounding structures dominant. These impedances distributed over machine circumference.
- Since impedances depend critically on specific beam parameters and structure geometries, evaluate for particular elements, **cannot be estimated in general**. Therefore, concentrate here on **few subjects** discussed recently: resistive wall effect in non-circular chambers, impedance of metallized ceramic walls and of wire cages for rf shielding.

Introduction 4: Definitions

- **Longitudinal impedance**: ratio of (Fourier transforms of) induced voltage to beam current, at higher frequencies complicated function of frequency with many sharp resonances; dimension Ω= V/A;
- **Longitudinal Wake function**: ratio of induced voltage to charge of point particle, function of distance (or time delay) behind source, dimension V/(p)C; impedance and wake function pair of Fourier transforms;
- **Transverse impedance** (or wake function); replace denominator by moment (beam current or charge multiplied by transverse offset) impedance multiplied by imaginary unit i (or -j) to allow for 90° phase shift (action of kick on slope rather than offset);
- longitudinal and transverse quantities related by **Panofsky-Wenzel theorem**.

Introduction 5: More Definitions

- Finite bunch length, **effective impedance** and **wake potential**: summation over all spectral frequencies(often replaced by integration) of product of impedance (or wake function) and (normalized) bunch distribution dimensions unchanged;
- functions of bunch length (and shape), much better behaved: sharp resonances smeared out, wakes continuous (no jumps or breaks;
- (short range) wakes can be computed directly in time domain with finite difference mesh codes (TBCI/MAFIA, ABCI, XWAKE etc);
- finite element methods : better description of structure shapes with deformable meshes, not available in time domain;
- **Loss factor** and **kick factor** (transverse loss factor) obtained by one more integration over distribution function.

Resistive Wall Effect in Chambers with Non-Circular Cross Section

- Cross section of vacuum chamber often non-circular (elliptic, oval, rectangular). Also circular chambers with holes or slots (break rotational symmetry).
- Interesting results for wakes in such structures found as by-product of 3-D simulation[1]. Transverse wake no longer simply proportional to offset of leading particle (**dipolar component**), also part proportional to offset of trailing (test) particle (**quadrupolar component**).
- Figs. 1 and 2 show total transverse kick as function of offset of test particle for 4 different positions of the beam in LEP shielded bellows and separators. Figs. 3 and 4 show difference between kicks for offset and on-axis test particles.

References

[1] A. Wagner, thesis, CERN Report SL/AP 95-070 (1995)
[2] S. Heifets, A. Wagner, B. Zotter (1997), SLAC Report AP110(1998)

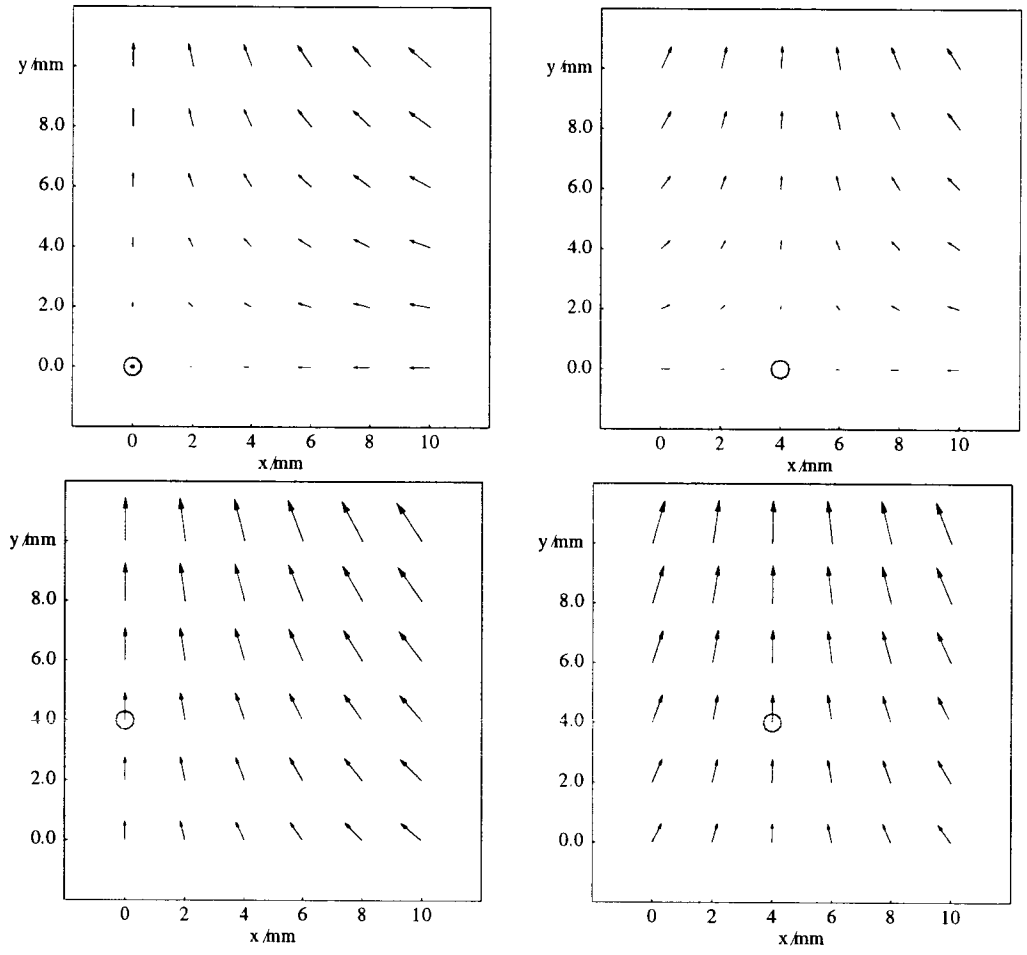

Figure 1: Transverse kicks as function of position of test particle in **LEP shielded bellows** for different beam offsets (small circles).

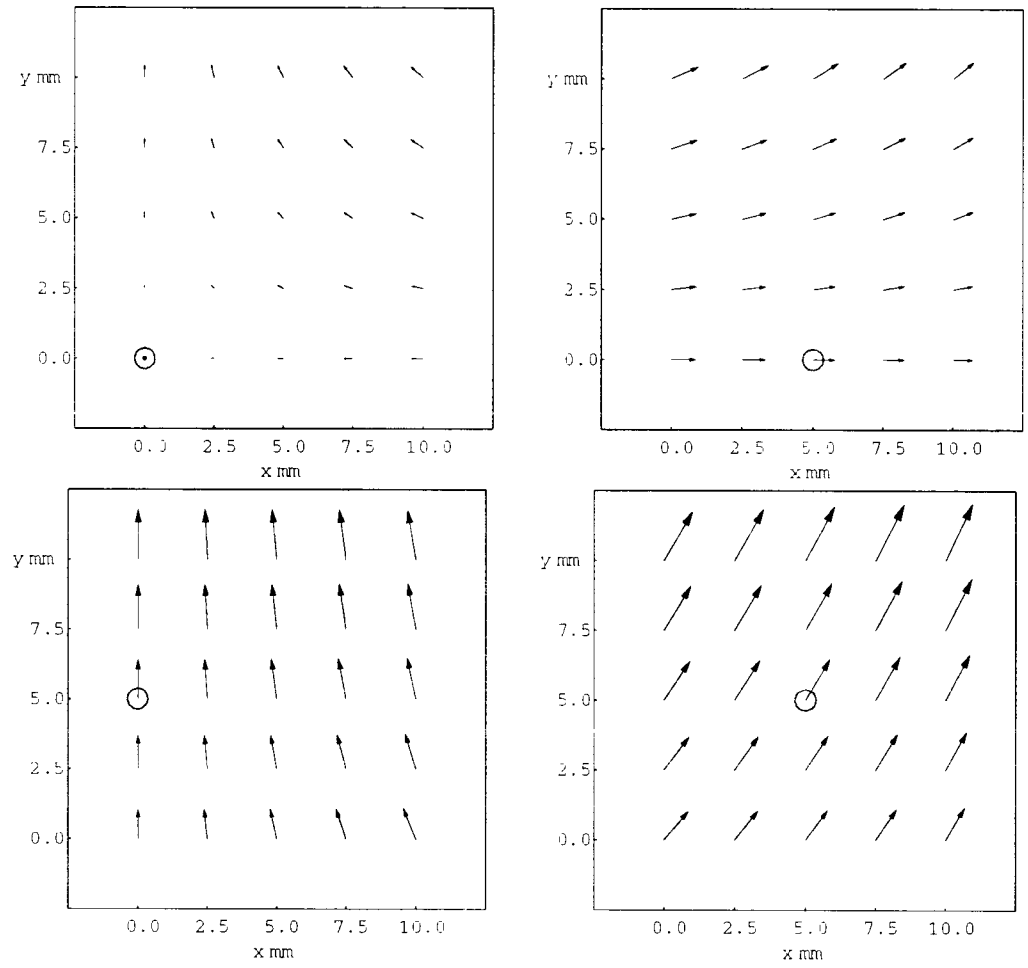

Figure 2: Transverse kicks as function of position of test particle in **LEP separator** for different beam offsets (small circles).

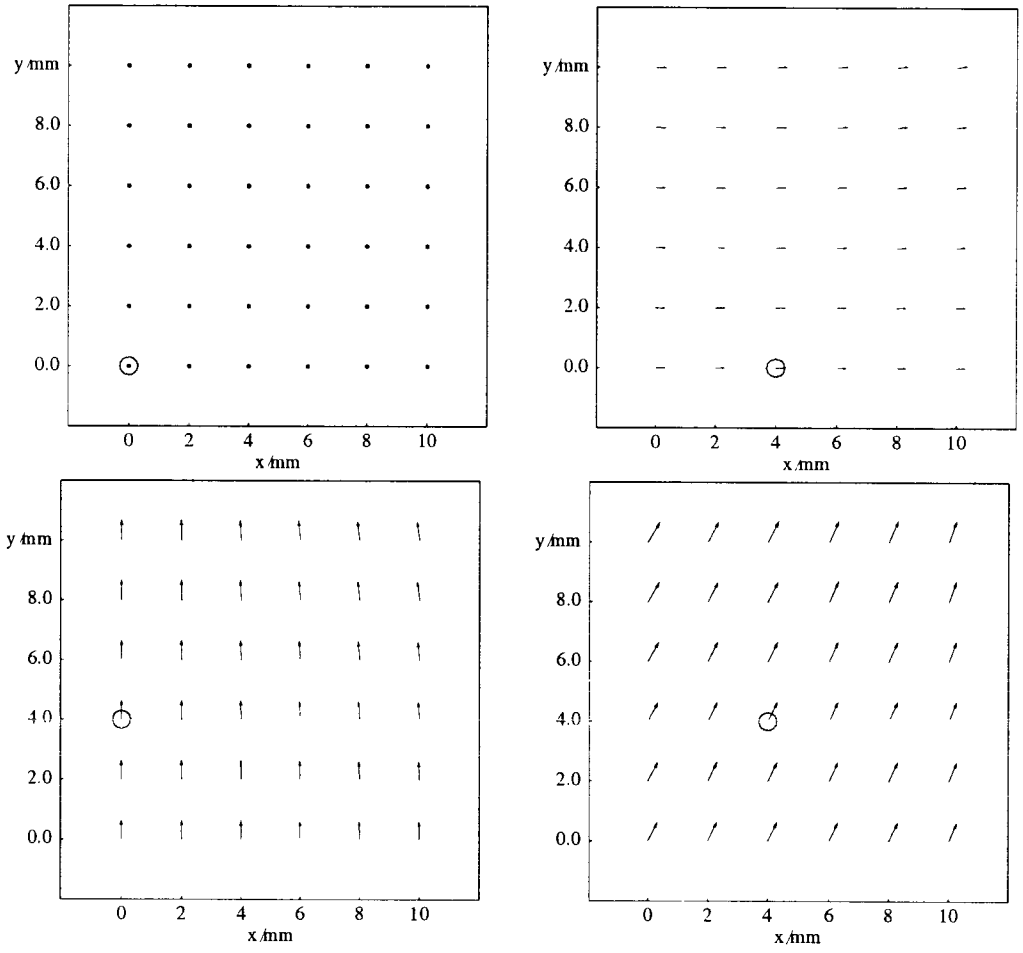

Figure 3: Transverse kicks reduced by kicks due to an on-axis beam in **LEP shielded bellows** for different beam offsets (small circles).

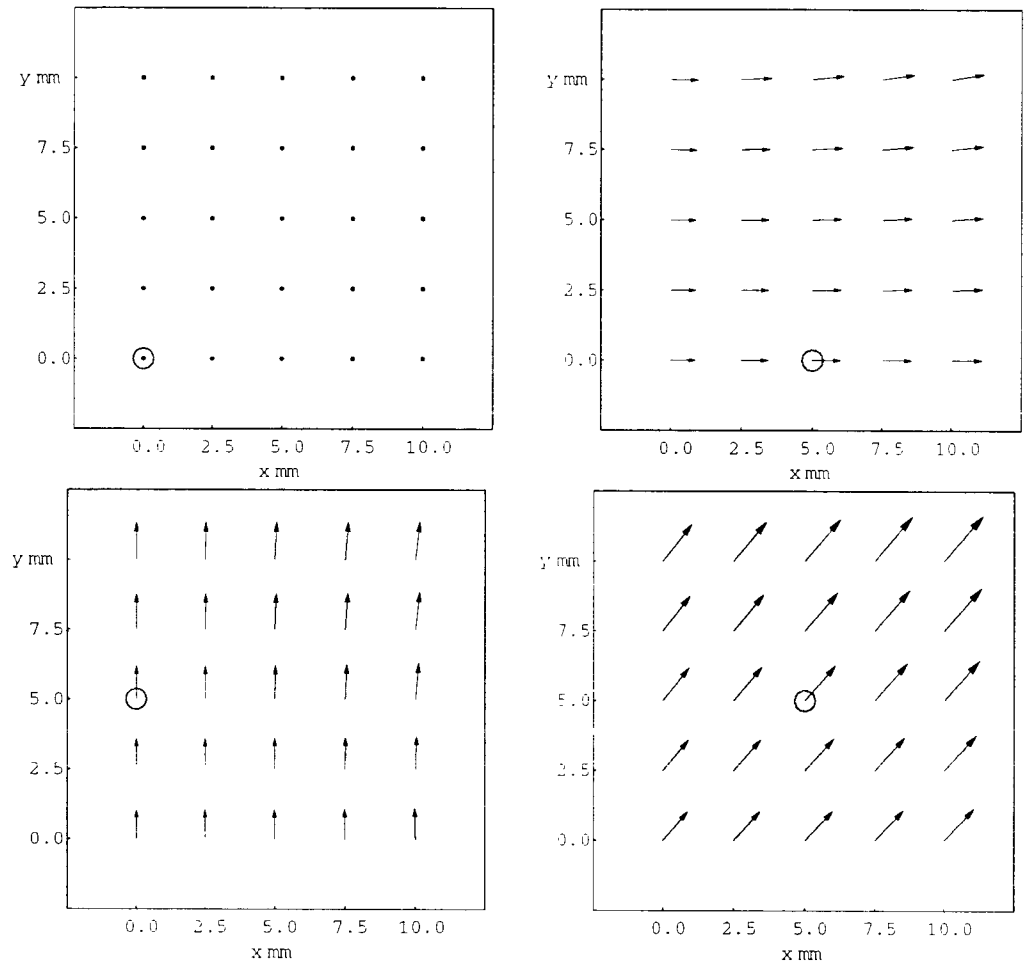

Figure 4: Transverse kicks reduced by kicks due to an on-axis beam in **LEP separator** for different beam offsets (small circles).

Resistive Wall Effect 2

- Long-range transverse resistive wall wake proportional to **inverse square root** of distance from source. Fields decay slowly, in circular machine need to sum over all previous turns. In structures with axial symmetry effect proportional to offset of leading particle, terms change sign due to betatron oscillations, infinite sum converges.
- Quadrupolar part of wake: leading **and** trailing particles same oscillations, offsets change sign together. Terms all same sign, infinite sum over $1/\sqrt{s}$ diverges.
- Unphysical result, correct: diffusion of fields through metal wall of finite thickness, cutoff when they reach outer wall[1] or are reflected back to beam[2]. However, square root decay only approximate, at large distances changes to exponential[3].

References

[1] A. Chao, private communication (1999)
[2] S. Heifets, SLAC-Pub-7985 (Oct.1998)
[3] B. Zotter, talk at SLAC AP group (Feb.1999)

Resistive Wall Effect 3

- Direct impedance calculation for non-circular chambers: first paper on resistive wall effect for beams in rectangular chamber[1], then parallel plates[2, 3];
- Calculations also for elliptic chambers[4, 5, 8], results generalized to arbitrary cross sections[6, 7] and beams with finite offsets. Apparent difference between results explained as different integrals over e.m. forces[9].
- Transverse kick always defocusing in axially symmetric structures, but for asymmetric structures may become **focusing** (negative kick factor) in one of two transverse planes.

References

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[7] F. Ruggiero, CERN Report SL/AP 95-95 (1995)
[8] V. Balbekov, Proc. 3-rd EPAC, Berlin 1992, p.792
[9] K. Yokoya, KEK Preprint 92-296 (1993)

Metallized Ceramic Wall

- Fast change of applied magnetic field in rapidly cycling synchrotrons requires vacuum chambers of badly or non-conducting materials to reduce eddy current losses.
- Walls of insulating material (ceramic) need (thin) conducting layer to avoid accumulation of static charges, continuous metallization or stripes parallel to beam. Small width of stripes reduces eddy currents further, field penetration remains small if gap between them is narrow.
- Required thickness d to avoid penetration of EM fields through metal layer, compared to skin depth (at revolution frequency) δ: controversy, several opinions:
  - a) thicker than skin depth $d > \delta$;
  - b) thicker than "effective skin depth" $\delta_{eff} = \delta^2/d$;
  - c) thicker than $\delta_{eff} = \delta^2/b$ where b is inner radius of vacuum chamber;
- recent measurements on LHC kicker seem to agree with smallest thickness as obtained from c).

Metallized Ceramic Wall 2

- Ceramic chambers of finite length (e.g.LHC kicker tanks[2, 3]). Structure behind may form resonant cavity, field penetration strong at resonant frequencies[7], reduce by metal layer on ceramic. Recent measurements have shown that metal layers much thinner than the skin depth are sufficient to shield the e.m. fields[4].
- Impedance of thin metal walls: for dielectric outside,same problem as metallized ceramic walls[1]. For pure vacuum on outside, specify its extent and outer boundary. Analysis simplified[2] when vacuum extending to infinity assumed. Results of limited practical use, as small susceptibility ($\epsilon - 1$) of air (0.0004) sufficient to slow down light velocity below beam velocity when γ exceeds 16. Then (weak) Cerenkov radiation occurs and behavior as with ceramic.

References

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[3] S. Kurennoy, Proc. PAC (1993) p.3420
[4] F. Caspers et al, PAC New York (1999)
[5] G. Lambertson, LBL report 42818 (1998)
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Impedance of rf-cages

- Outer boundary good conductor: impedance reduced, but increased for (ferro-)magnetic material. This occurs for chambers inside magnets, Even worse if magnetic pole shoes penetrate into vacuum chamber, as in some early accelerator designs in order to minimize the magnet gap and hence the costs of magnets.
- Beam tubes made of two different metals, such as in copper-clad SS liners for LHC. Different results were obtained for impedance assuming different outer boundaries - perfect conductor or vacuum to infinity.
- **Alternate solution** to metallization: metal wire cage, pioneered by designers of ISIS. Studied also for TRIUMF K-factory and AUSTRON spallation source; radius of cage changes (slowly) with beam radius to minimize space-charge impedance. Ratio of cage radius to beam radius small and approximately constant, both longitudinal and transverse impedance terms $\log(b/a)$ and $(1/a^2 - 1/b^2)$ are reduced. Permits more current in beam before reaching limits due to tune shift into resonances.

rf-cages 2

- several **practical problems**: ceramic chambers available only in short pieces(1-2 m), need reliable mechanical support and good electric connection between cage sections (in ISIS capacitive), DC grounding in several places through wall.
- **Analysis of impedance** of simplified model (no outer chamber, cage constant radius) in Austron proposal[1], revised in Shelter Island workshop[2], finally extended to include outer conducting chamber[3];
- impedances of N equally spaced rods of thickness d at the cage radius $r_c = \kappa b$ where $b = r_t$ is the tube radius. The resulting expressions can be written:

$$\begin{aligned} Z_{\parallel} &= \frac{ikLZ_0}{4\beta\gamma^2}\left[1 + 2\ln\frac{b}{a} + C_{\parallel}\right] , \\ Z_{\perp} &= \frac{iLZ_0}{2\pi\beta^2\gamma^2}\left[\frac{1}{a^2} - \frac{1}{b^2} + \frac{C_{\perp}}{b^2}\right] \end{aligned} \tag{1}$$

References

[1] T.S. Wang, CERN Report PS 94-08 (1994)
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rf-cages 3

- In each of these expressions one can recognize the two space charge terms for a uniform beam of radius a and the additional dimensionless contributions

$$
\begin{aligned}
C_{\parallel} &= \frac{2N[\ln\kappa]^2}{N\ln\kappa + \ln\frac{Nd}{r_c} - \ln\left(1-\kappa^{2N}\right)}\,, \\
C_{\perp} &= [1-\kappa^2]\frac{[\kappa^2+\kappa^{-2}]\ln\left(1-\kappa^{2N}\right) - 2\ln\frac{Nd}{r_c}}{N[1-\kappa^2] - 2\ln\frac{Nd}{r_c} + [\kappa^2+\kappa^{-2}]\ln\left(1-\kappa^{2N}\right)}\,. \qquad (2)
\end{aligned}
$$

- Additional transverse impedance quite small when number of wires increases, while longitudinal approaches constant (negative) value depending on ratio of cage to tube radius κ.

- Fig.6 shows impedances as function of number of wires for constant filling factor, i.e. decreasing wire diameter.

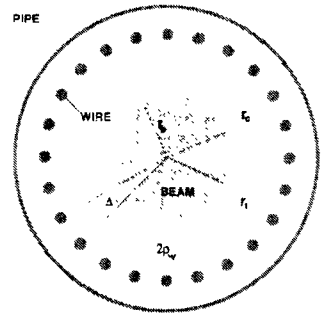

Figure 5: Schematic geometry of rf cage used in impedance calculation.

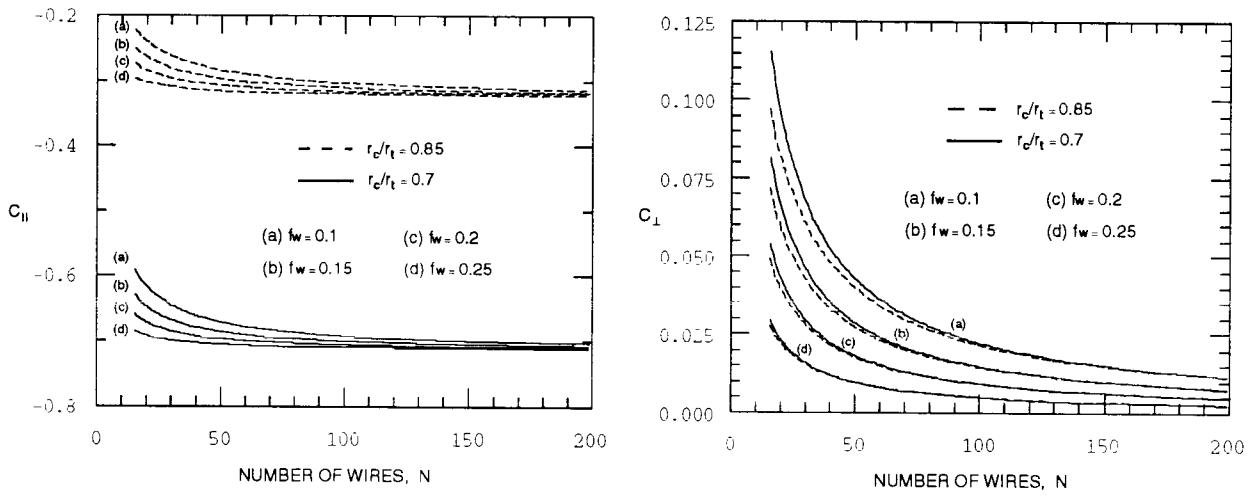

Figure 6: Longitudinal and transverse impedances of rf cage for various ratios of cage to tube radius as function of number of wires N.

Summary

- For each new particle accelerator - in particular those aiming at high beam currents - the careful establishment of an **impedance budget** is prerequisite for reaching desired performance. Verify all design changes by **impedance police**, include all structures seen by beam with either large impedances or present in large numbers.
- For multi-bunch and coasting beam instabilities **narrow resonances** important - depend critically on exact shapes and dimensions of structures; higher resonant frequencies **cannot be predicted exactly**, depend on momentary dimensions of critical elements, i.e. they change with temperature or tuner position in rf cavities;
- for (low energy) proton rings, **space-charge** and **resistive wall impedance** usually dominant, can be estimated better. Use 3-D programs and decomposition in dipolar and quadrupolar components for non axially symmetric structures to give more reliable current limits.