

# Nonlinear Features of the Longitudinal Instability

Workshop on Instabilities  
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GSI Darmstadt

# Outline

- Linear Regime (Observation / Simulation, Limits)
- Nonlinear Saturation and Overshoot
- Self-Stabilizing Tails

co-workers: O. Boine-Frankenheim, G. Rumolo

# “History” of Longitudinal Instability

- **Longitudinal instability of coasting beams or long bunches** driven by resistive impedance is of major concern in high intensity machines  
Linear theory in 60's: *L.J. Laslett, V.K. Neil and A.M. Sessler, 1961*  
*A.G. Ruggiero and V.C. Vaccaro, 1968 etc.*  
*Keil-Schnell circle criterion*
- Numerous observations in many machines on appearance and saturation predominantly for negligible or weak space charge effects; theoretical + simulation models (overshoot etc.)
- Specific interest in connection with Heavy Ion Inertial Fusion drivers far outside Keil-Schnell circle (space charge impedance of kOhm)
  - Simulation claiming “self-stabilizing tails” (I.H., 1985)
  - Recently observation in space charge dominated regime:
    - time domain in linear e-resistive channel at Maryland (*J.G. Wang et al., PRL 1997*) generating separately localized slow and fast waves and observing linear growth
    - in frequency and time domain in ESR+SIS / GSI with initially single modes driven by passive RF cavity (*G. Rumolo et al., NIM 1998, EPAC98*, O. Boine-Frankenheim et al., *PRL 1999*) with observations and simulation in nonlinear regime

## Questions of interest

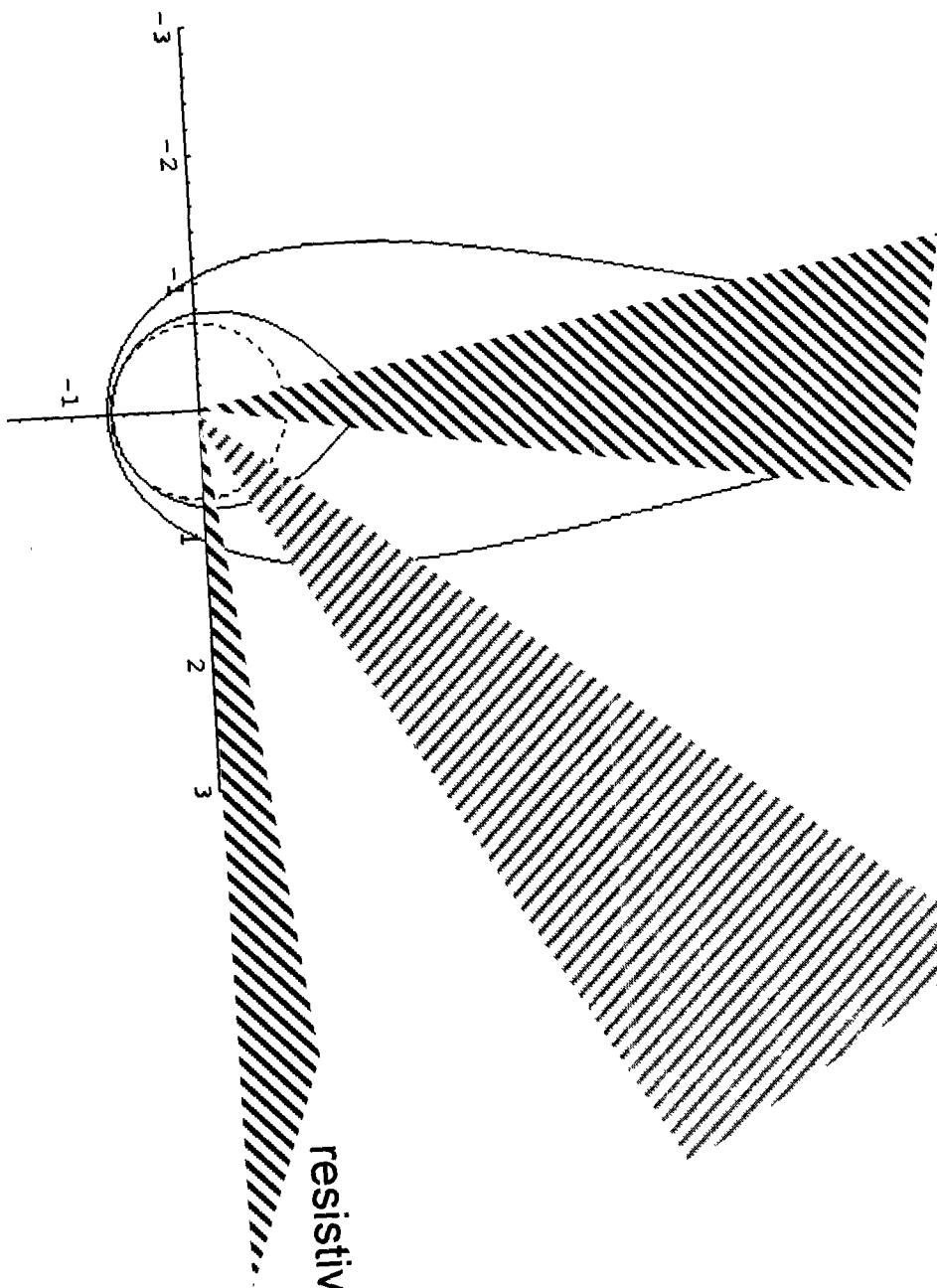
- is the linear theory confirmed by experiments?
- is linear stability necessary?
- what is the influence of the impedance distribution (over n) on nonlinear behaviour?
  - narrow-band vs. broad-band
  - space charge impedance significant factor

## Discussion of regimes in stability diagram (impedance plane)

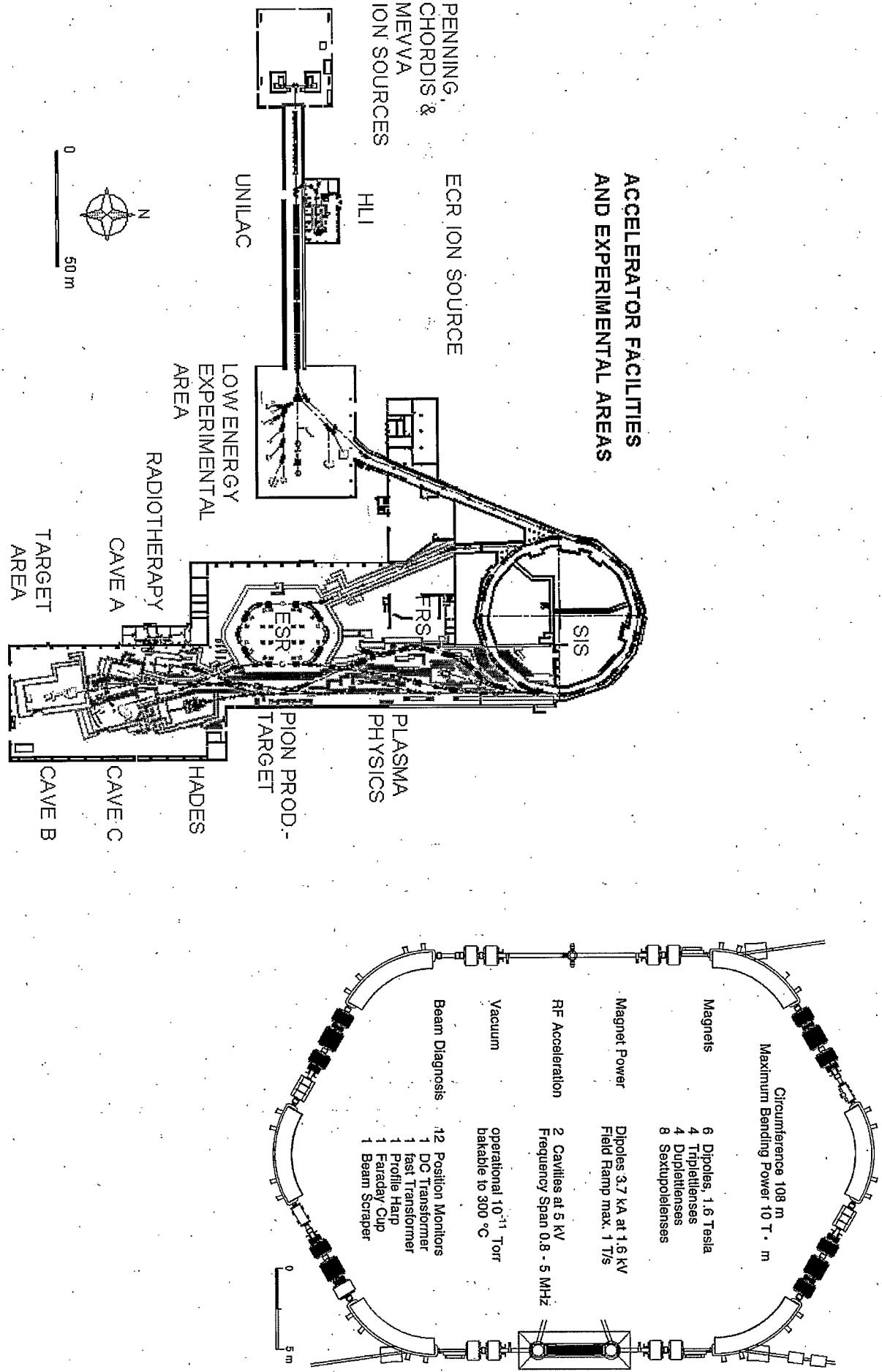
space charge impedance dominant

balanced impedances

resistive impedance



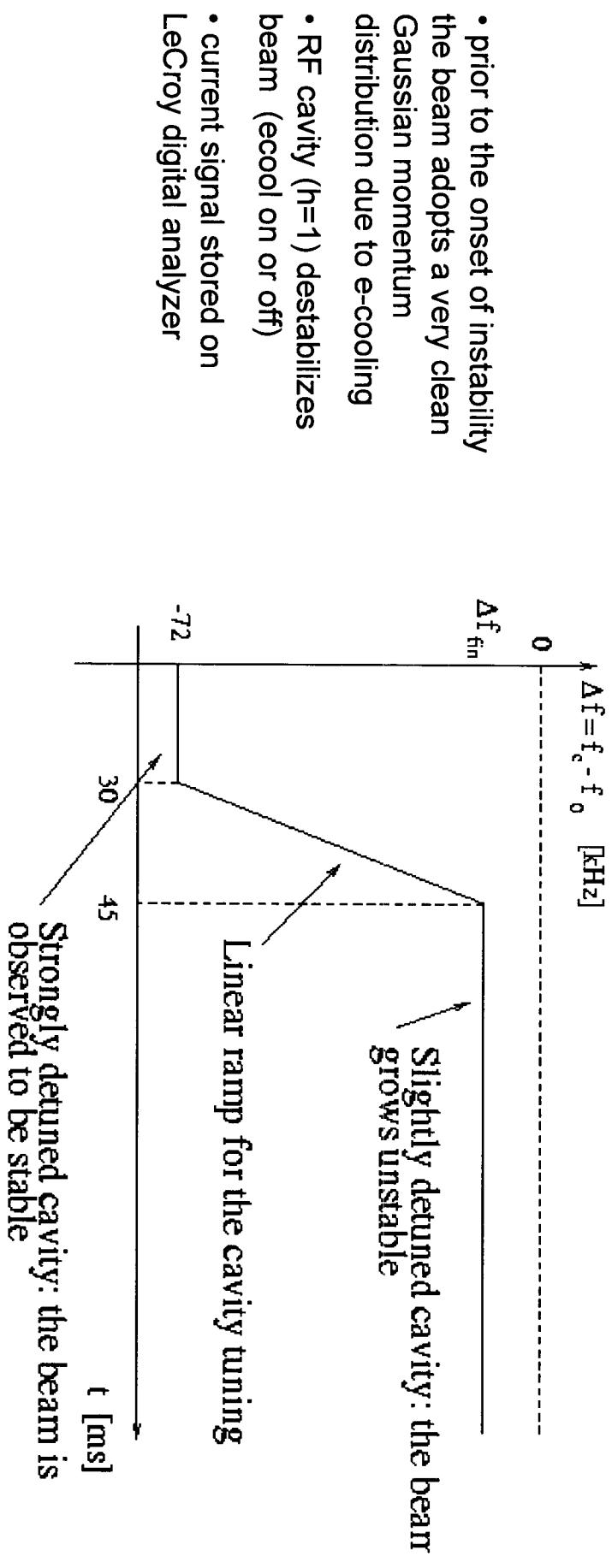
# Experimental Storage Ring for heavy ions at GSI



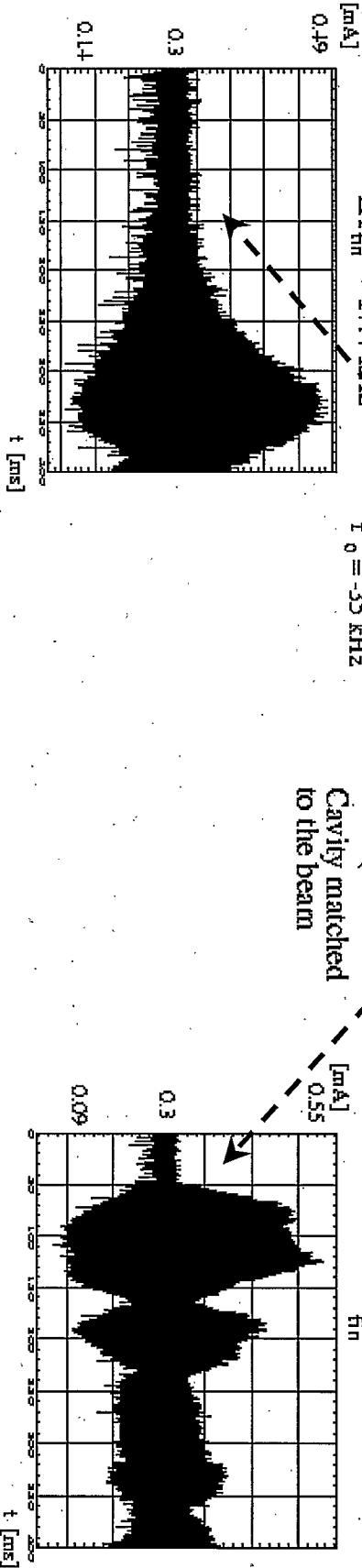
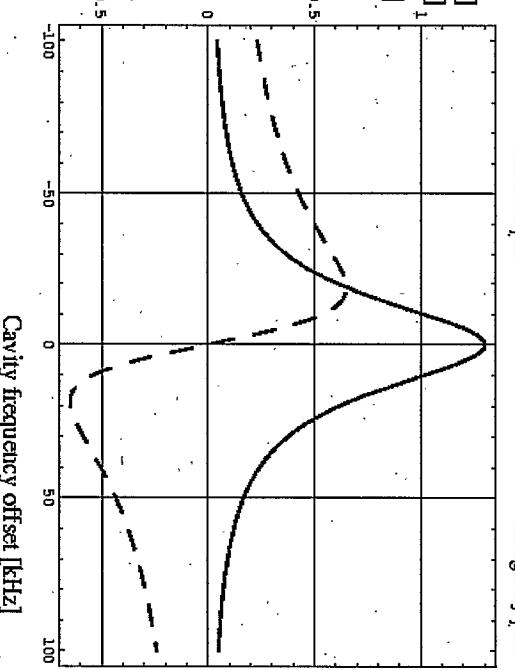
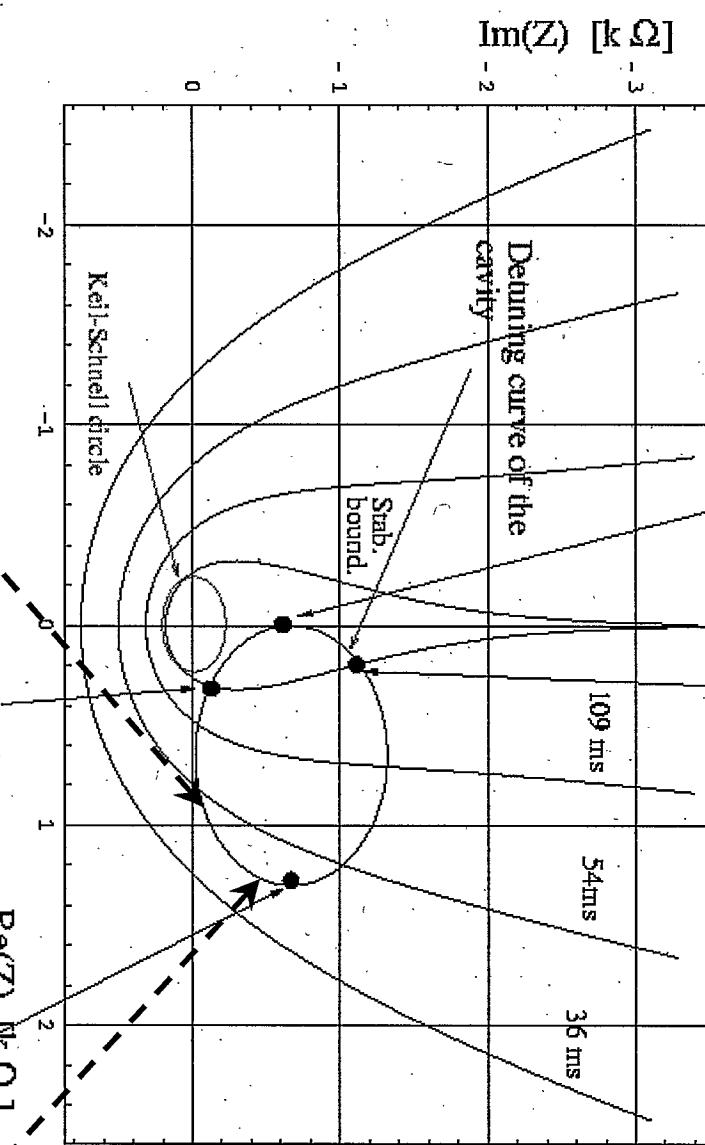
# ESR experiments with e-cooled C<sup>6+</sup> beam at 340 MeV/u

BEAM PHYSICS

- key to new understanding of space charge regime

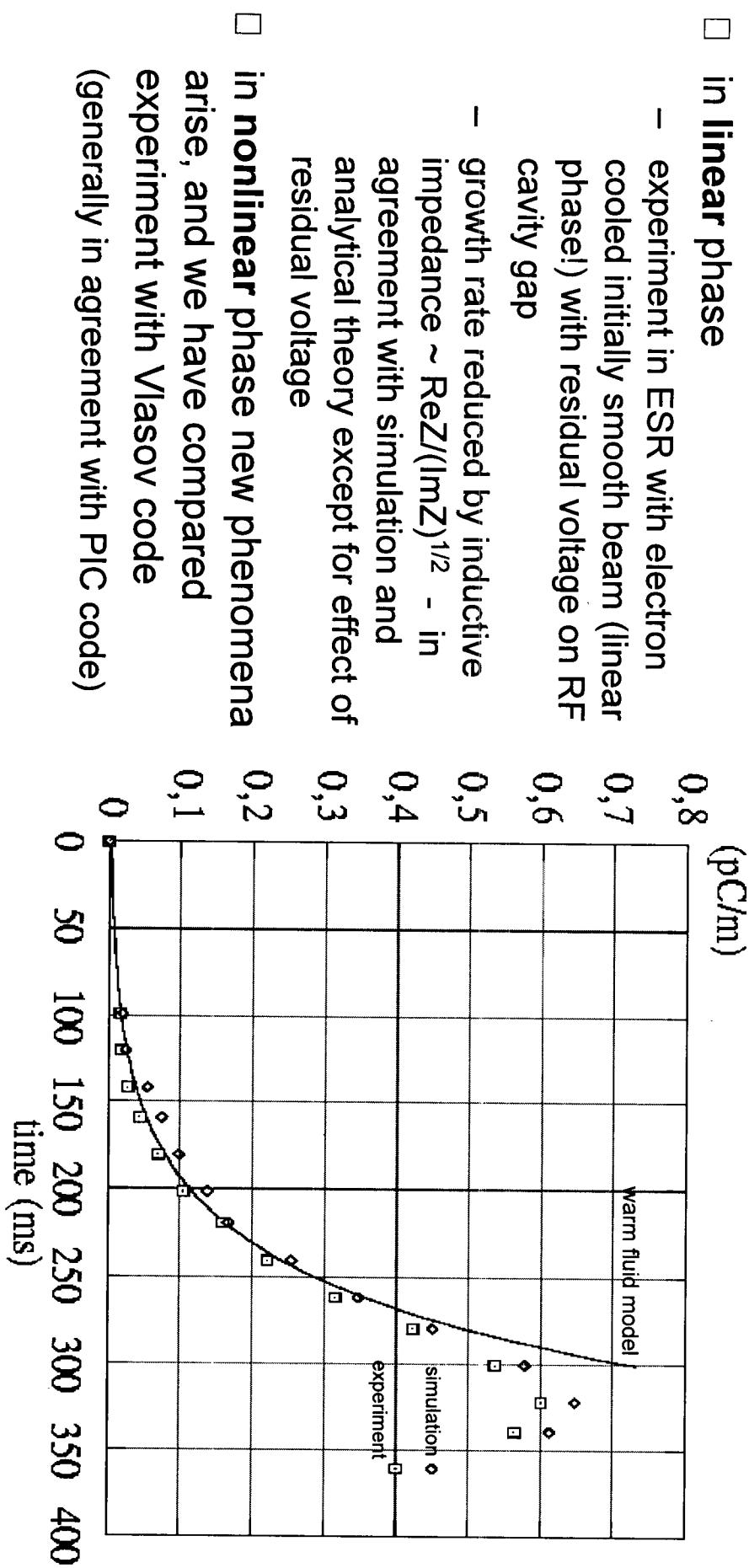


## RF cavity detuning effect on working point



# Comparison of observation and simulation

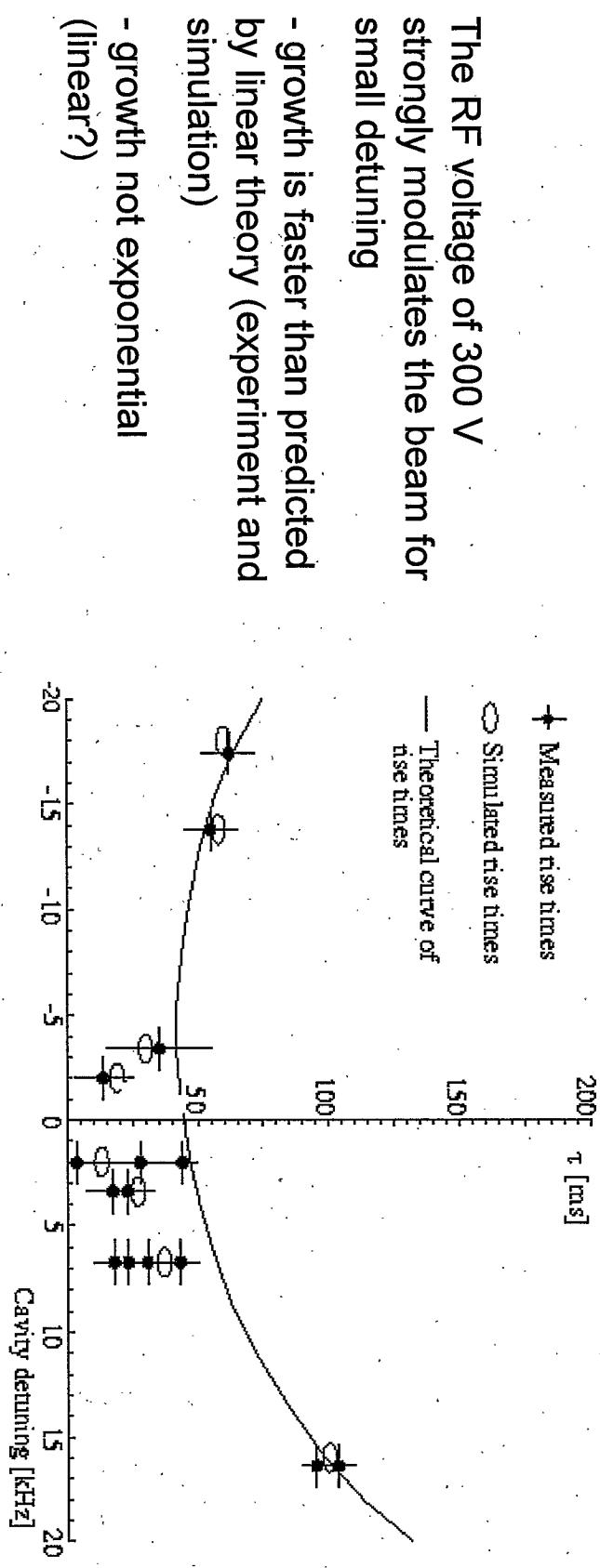
-17.4 kHz detuning: RF voltage is negligible



- in linear phase
  - experiment in ESR with electron cooled initially smooth beam (linear phase!) with residual voltage on RF cavity gap
  - growth rate reduced by inductive impedance  $\sim \text{Re}Z/(\text{Im}Z)^{1/2}$  - in agreement with simulation and analytical theory except for effect of residual voltage

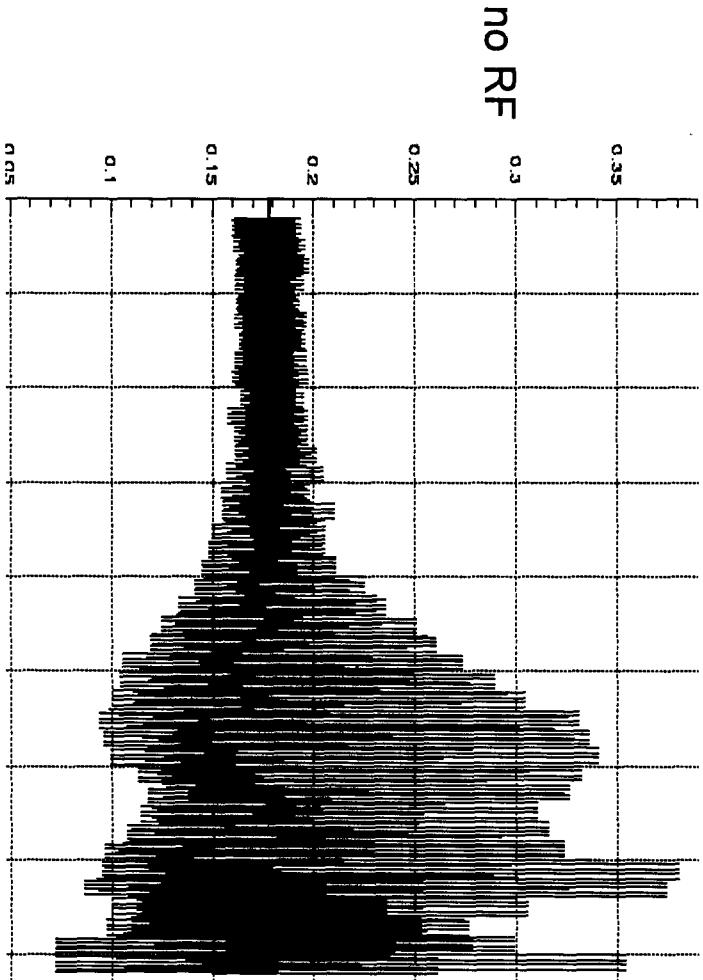
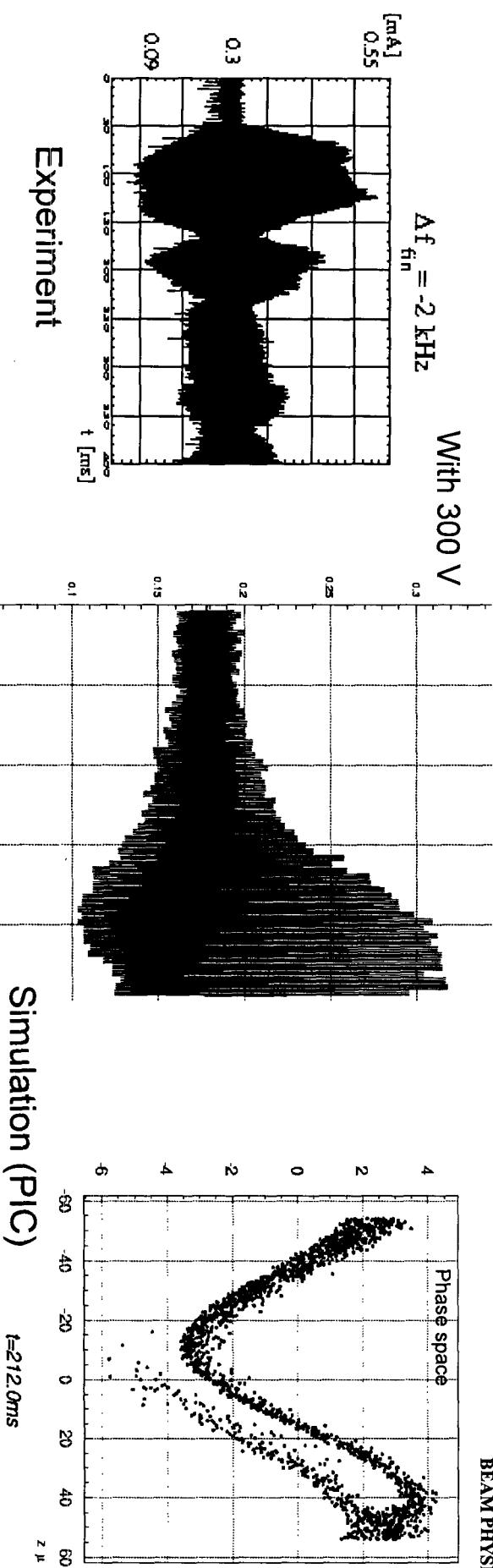
- in nonlinear phase new phenomena arise, and we have compared experiment with Vlasov code (generally in agreement with PIC code)

# Comparison of initial growth times



- The RF voltage of 300 V strongly modulates the beam for small detuning
- growth is faster than predicted by linear theory (experiment and simulation)
- growth not exponential (linear?)

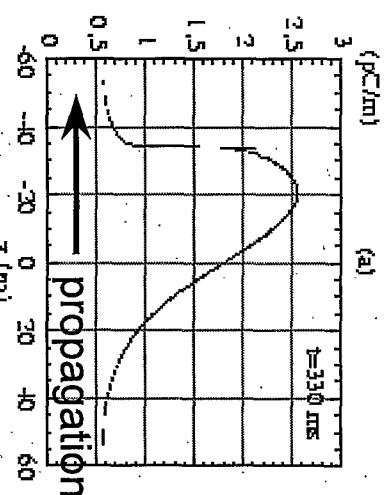
# Influence of RF Modulation



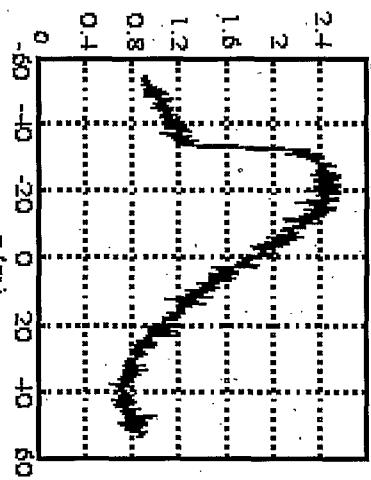
# Wave steepening described well by PIC and cold/warm fluid model



(a)



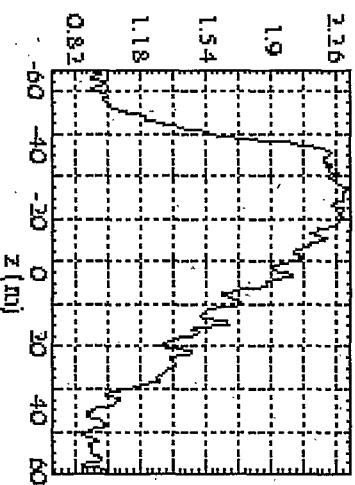
(b)



## ESR experiment

- cannot explain maximum that self-bunching reaches saturation
- fluid model

(c)



## PIC simulation

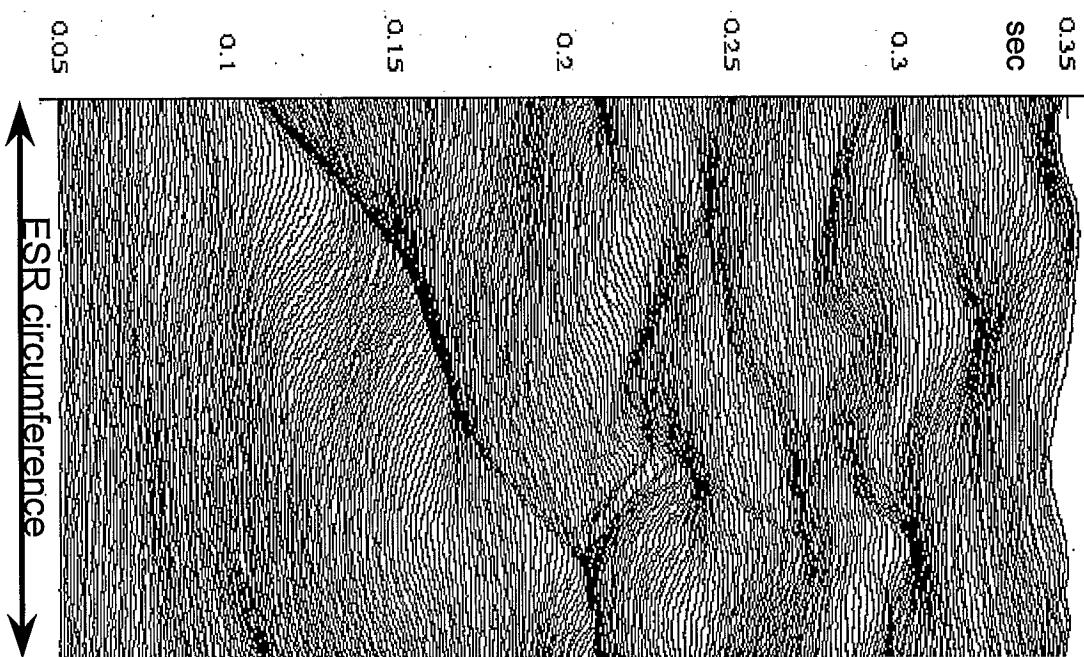
## Direct Vlasov solvers are advantageous for long-time studies

- experiment: no measurements on momentum distribution during evolution of instability - require simulation
- Vlasov Solver: partial differential equation for distribution function  $f(z, p_z, t)$  integrated on a grid :
  - 1D for longitudinal dynamics
- uses 3D Poisson solver with transverse rms envelope equations
  - smooth distribution functions (on grid)
  - nearly no granularity
  - resolve fine structures in phase space
- have gained better insight into evolution than with PIC (in good agreement)

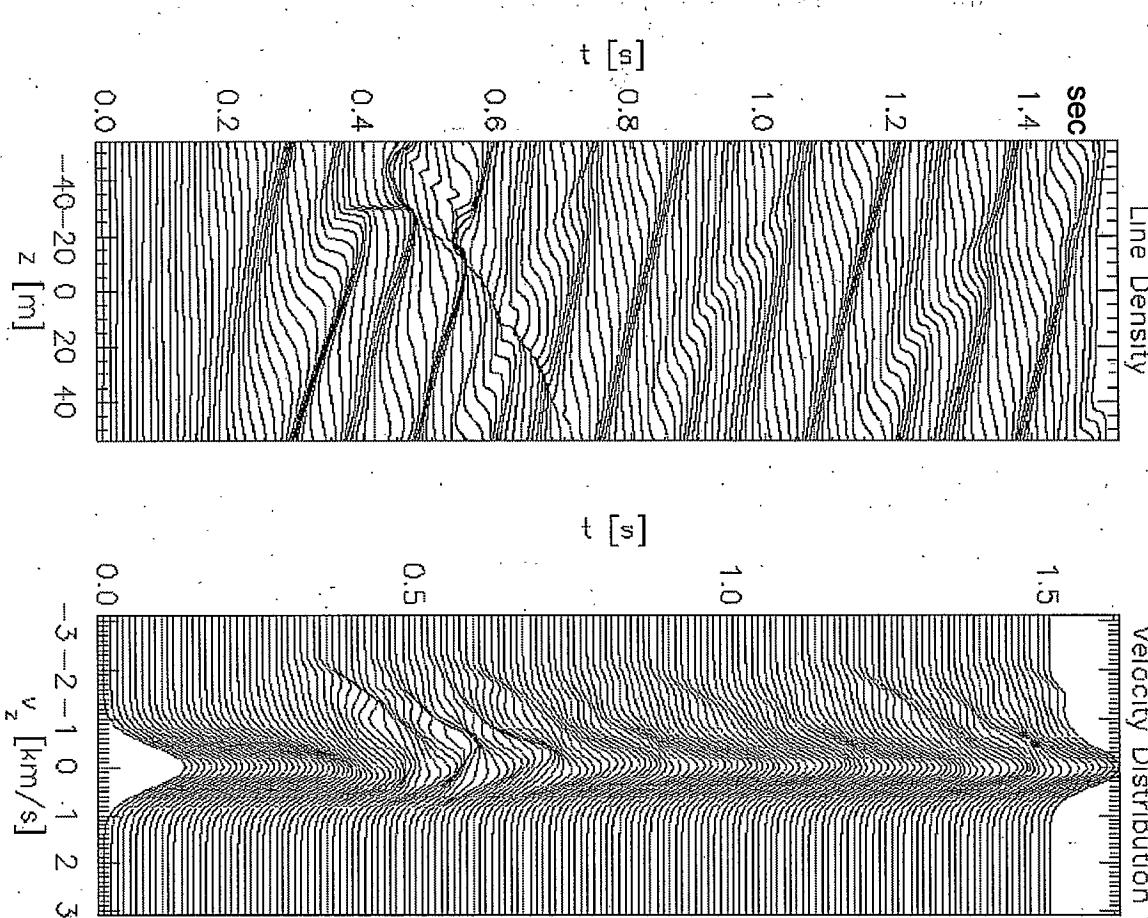
# Mountain range diagrams have shown absence of real saturation (Landau damping) - memory effect



## Experiment

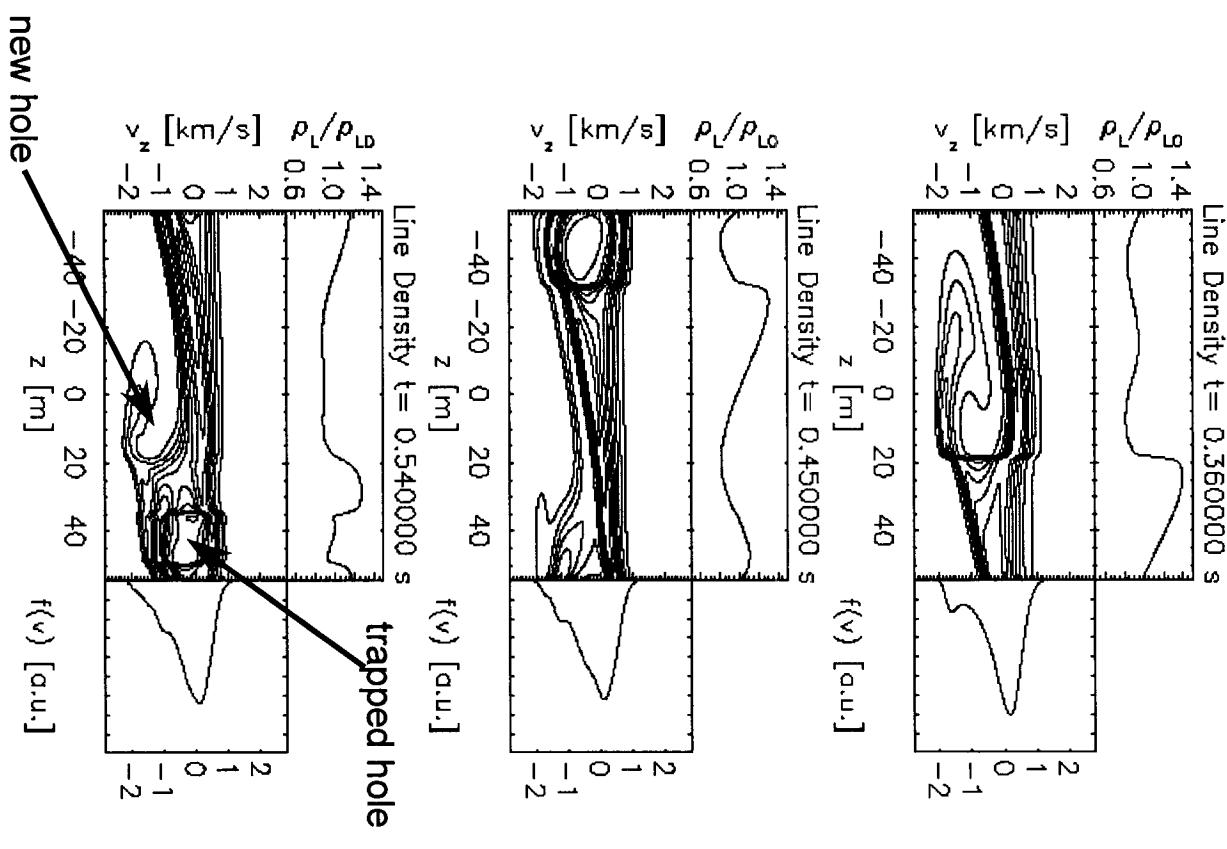


## Vlasov Simulation

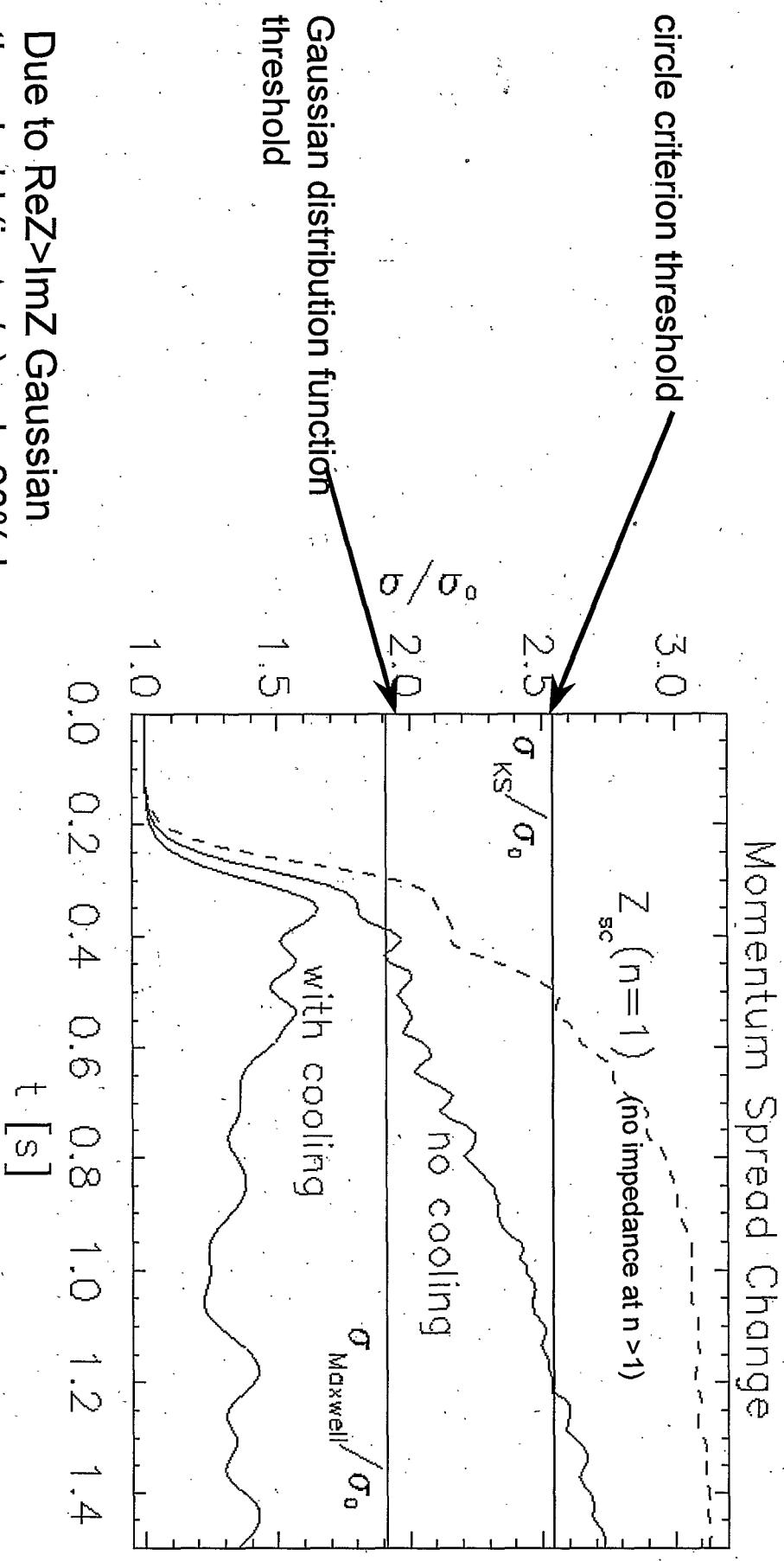


# Holes are continuously trapped in phase space

- wave is generated at lower momentum edge
- the fast instability leads to hole structures traveling in the direction of larger momentum (self-buckets)
- result is a momentum broadening and slight deceleration of beam (energy source=longitudinal motion)
- broad band impedance nature of space charge modifies results

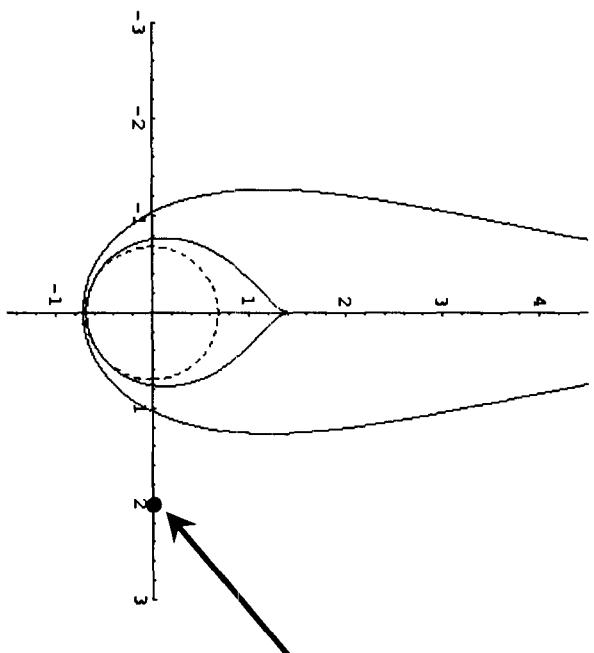


# Overshoot and long-time behaviour

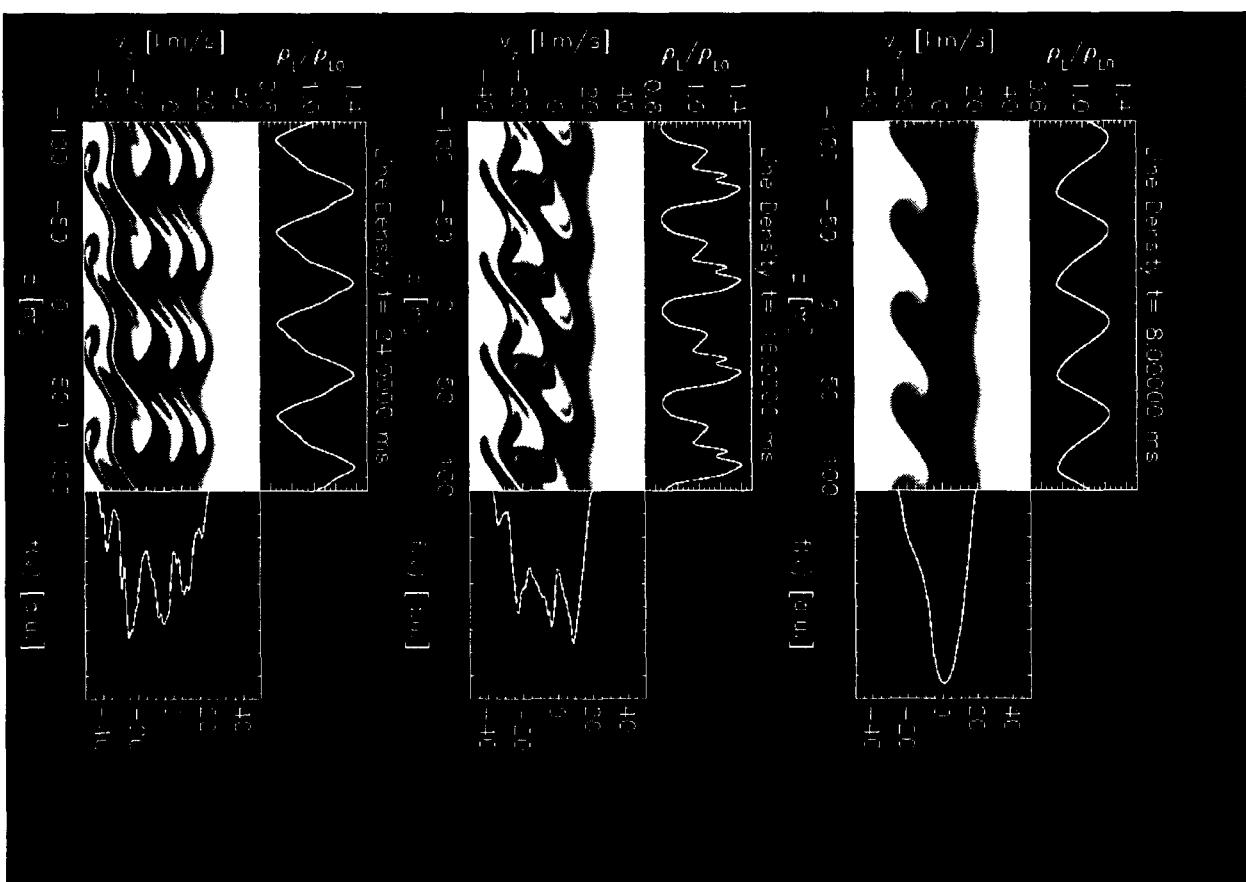


Due to  $ReZ > ImZ$  Gaussian threshold (in  $\Delta p/p$ ) only 20% lower

# Purely resistive impedance: broadening effect

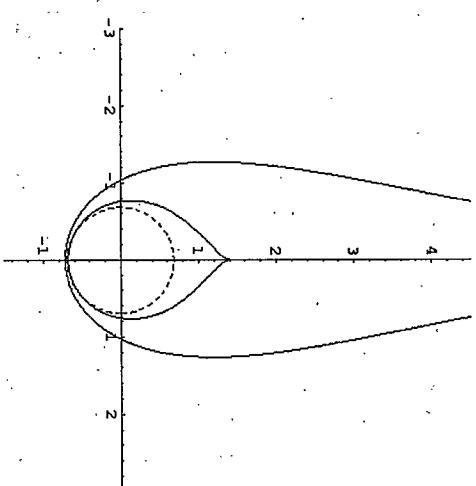
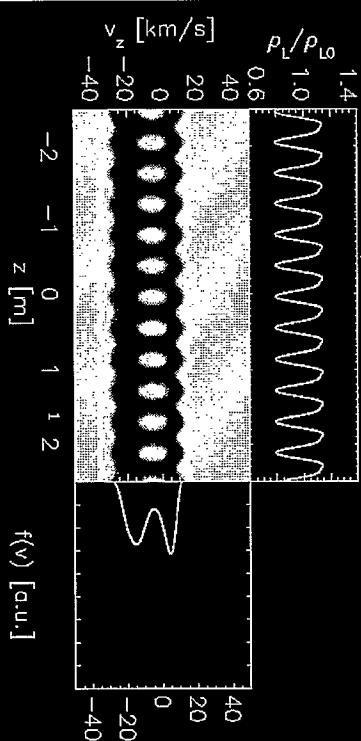
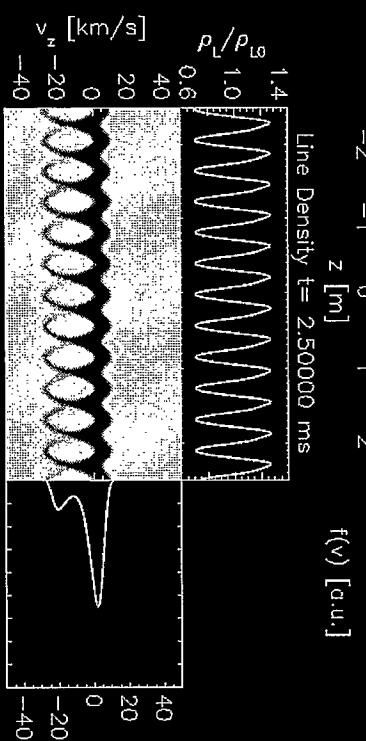
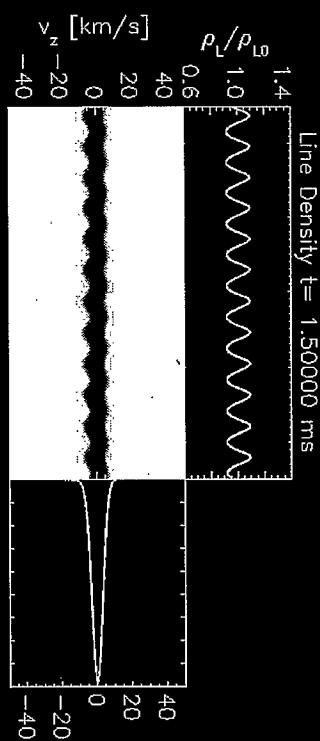


- typical behaviour at high energy
- layers of self-trapped buckets generated at lower edge of momentum space with strong progressive filamentation
- results in broadening (to lower momenta)



# $\text{Im}Z \gg \text{Re}Z$ on one harmonic ( $n=12$ ) only

$U=25; V=2$

isolated impedance on imaginary axis  
degrades beam quality by single  
harmonic instability

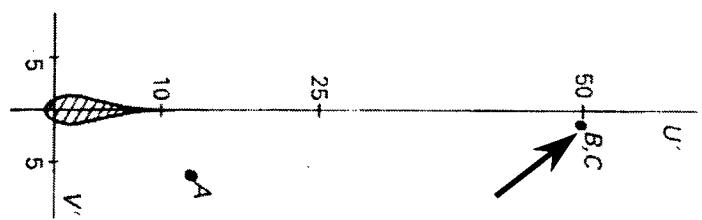
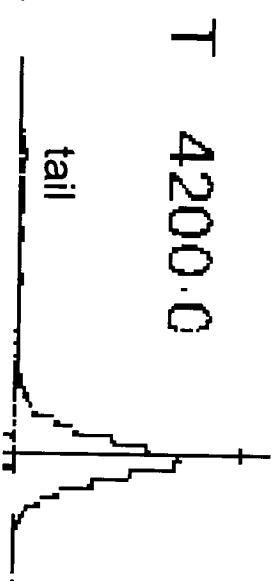
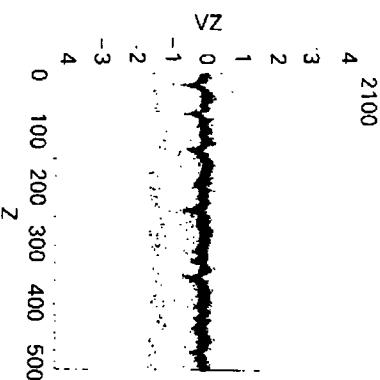
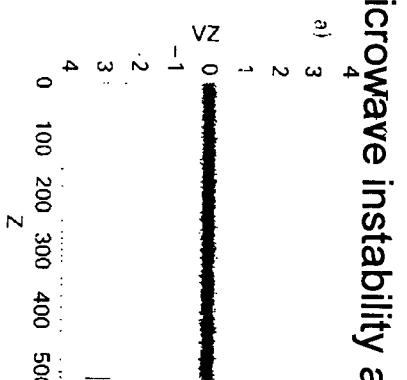
# Self-stabilization by tail with dominant space charge impedance

- assume  $\text{Re}Z \ll \text{Im}Z$  and very dense, initially unstable beam:

in 1985 we claimed self-stabilization occurs with tail to smaller momenta(I.H., "Suppression of Microwave Instabilities", Laser and Particle Beams, 1985)

- broad-band nature of space charge impedance is crucial to obtain this stabilization by coupling to higher order modes (cutoff, where  $Z/n$  drops to zero)

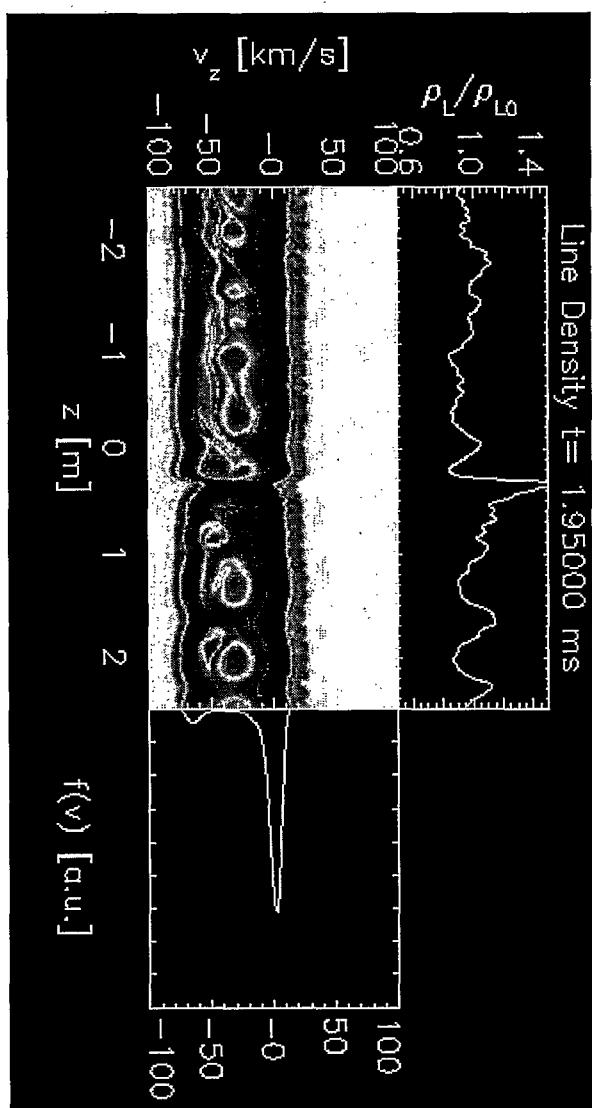
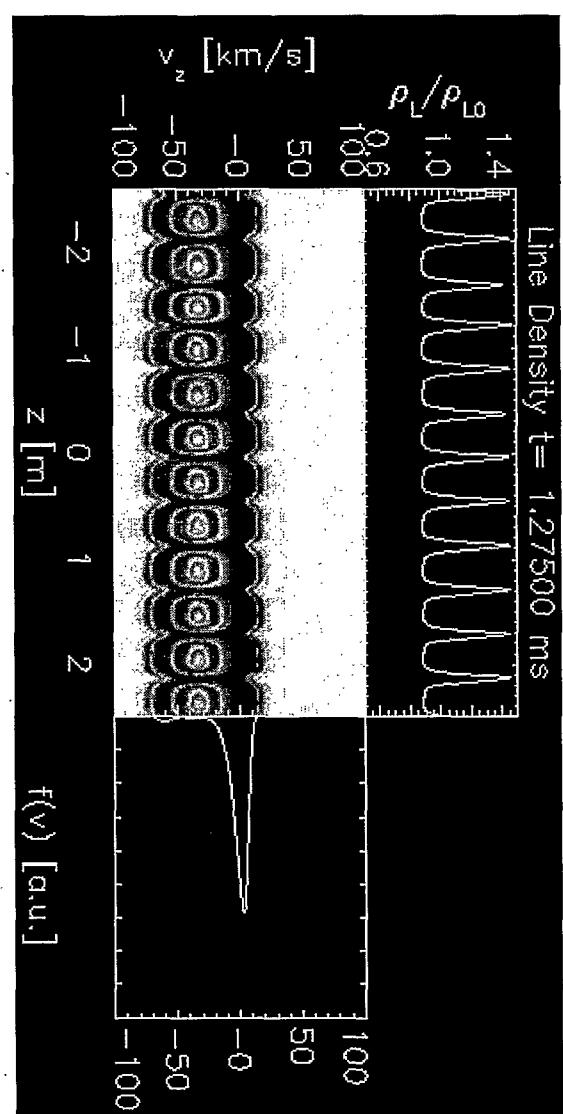
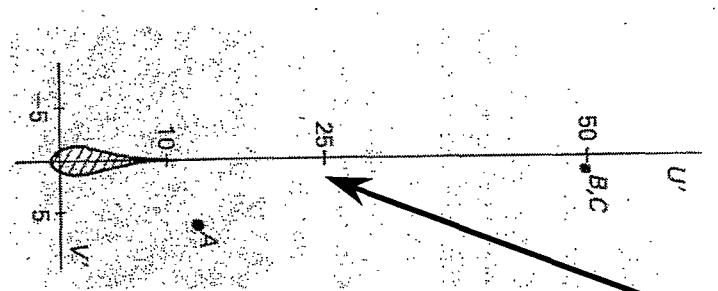
- also applies to microwave instability at high  $n$



# Vlasov simulation of self-stabilization



confirmed by recent Vlasov  
simulation



# Influence of Impedance Distribution

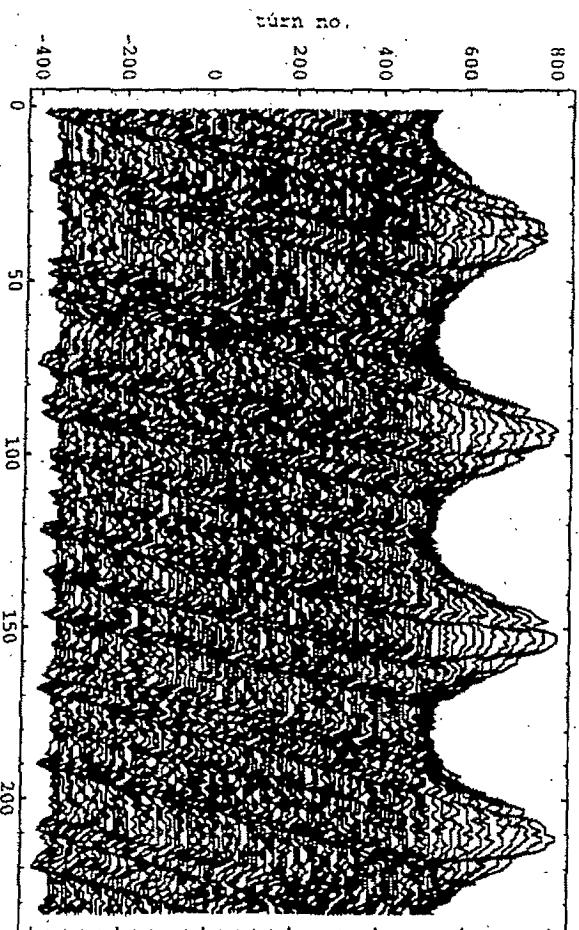
- for initial linear behaviour single harmonic relevant
- for nonlinear behaviour distribution of impedances essential  
(coupling between modes)
- stabilizing tails require broad-band nature and gentle drop  
of space charge impedance near cut-off frequency
- assume broadband machine impedance has similar effect

# SIS measurement after injection

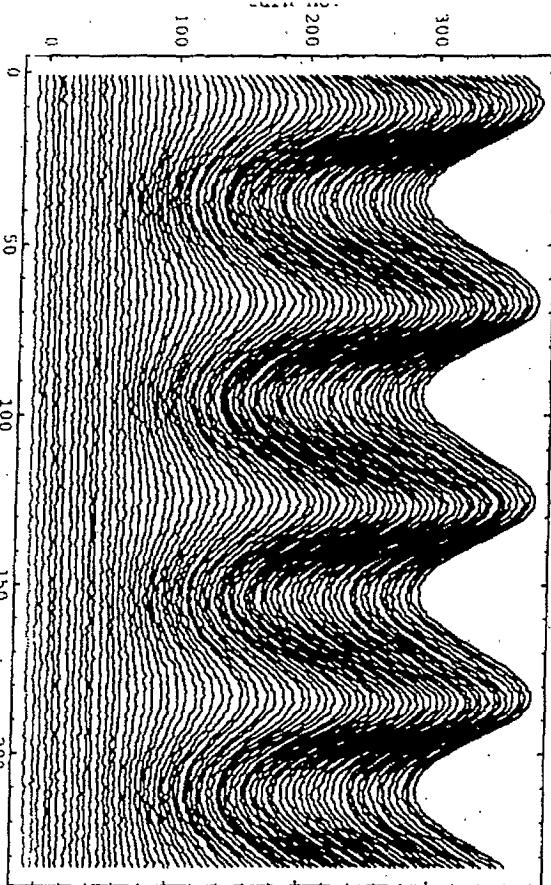


RTI/f1pb sc1.002 #100000-146000

early after injection of  
1.5  $10^{10}$  ions of  $\text{Ne}^{10+}$  at 11 MeV/u

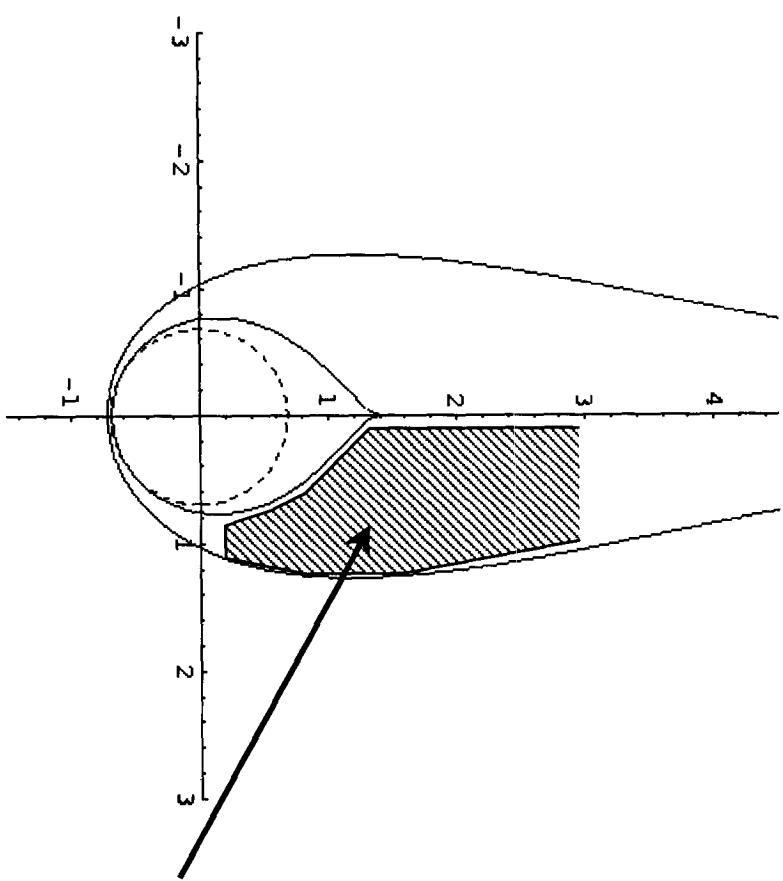


assume beam was near stability  
boundary of Gaussian



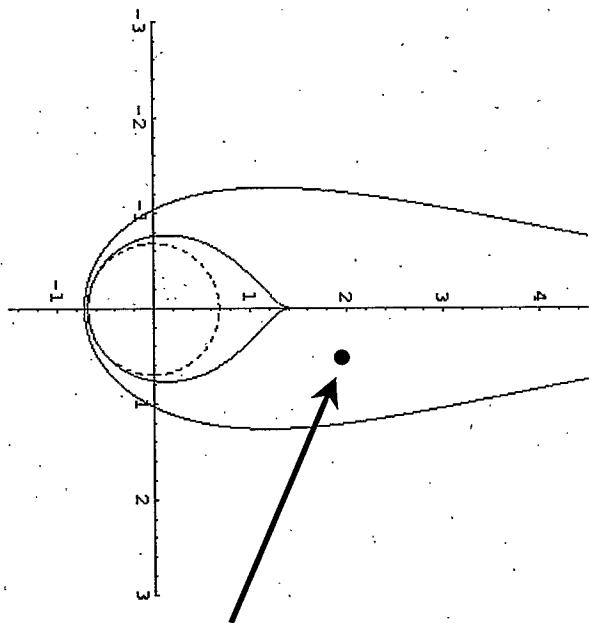
allowed 200 ms injection flat-top  
-> smoothing by IBS or Landau  
damping

# Near stability boundary

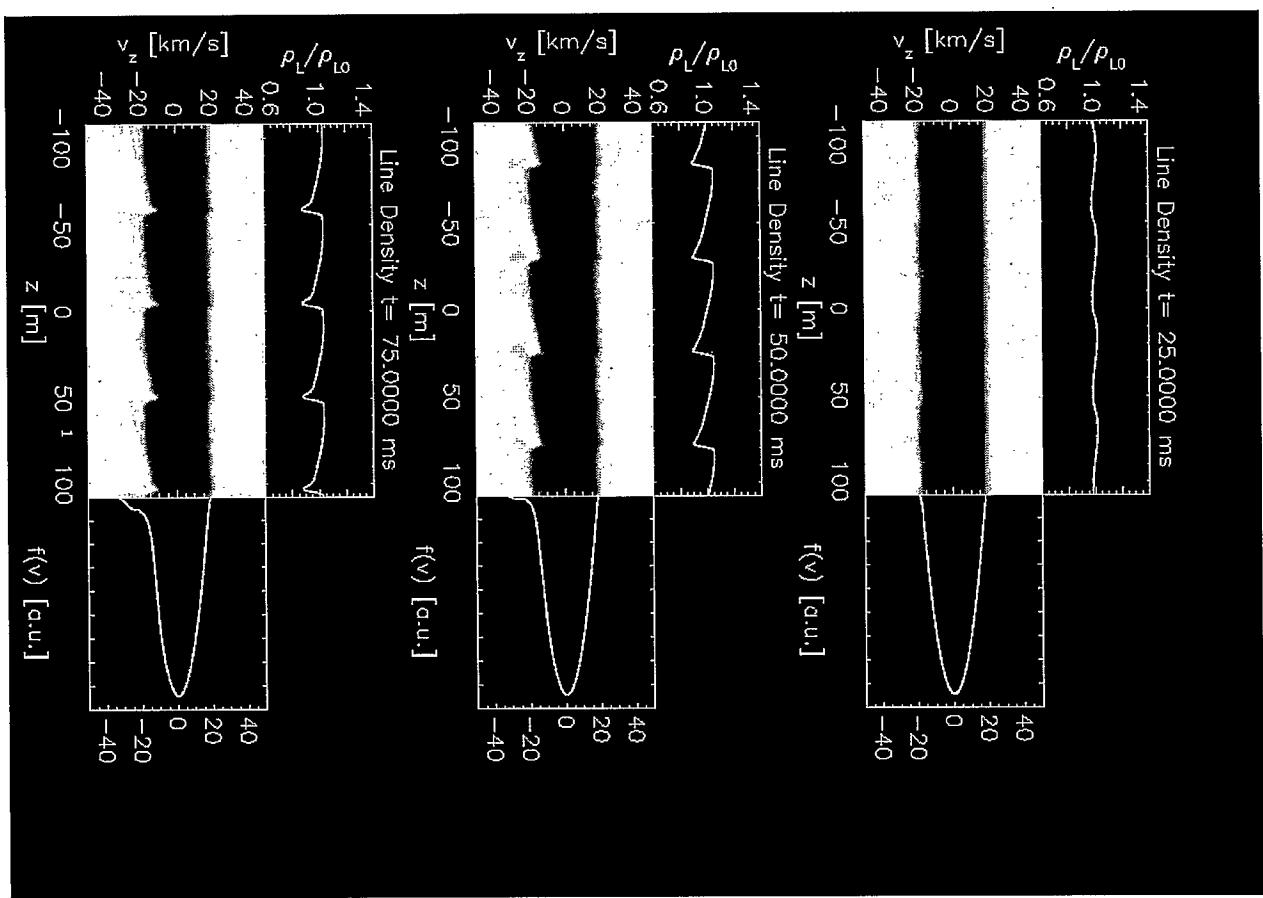


quadratic unstable,  
but Gaussian stable

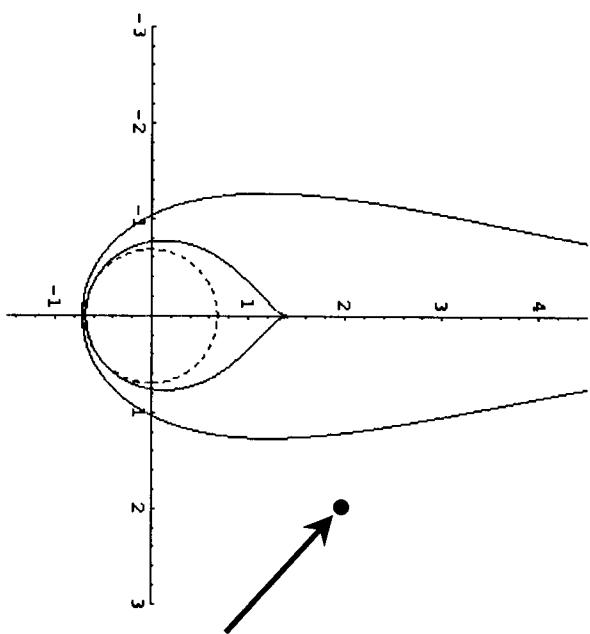
# Simulation of marginally unstable distributions



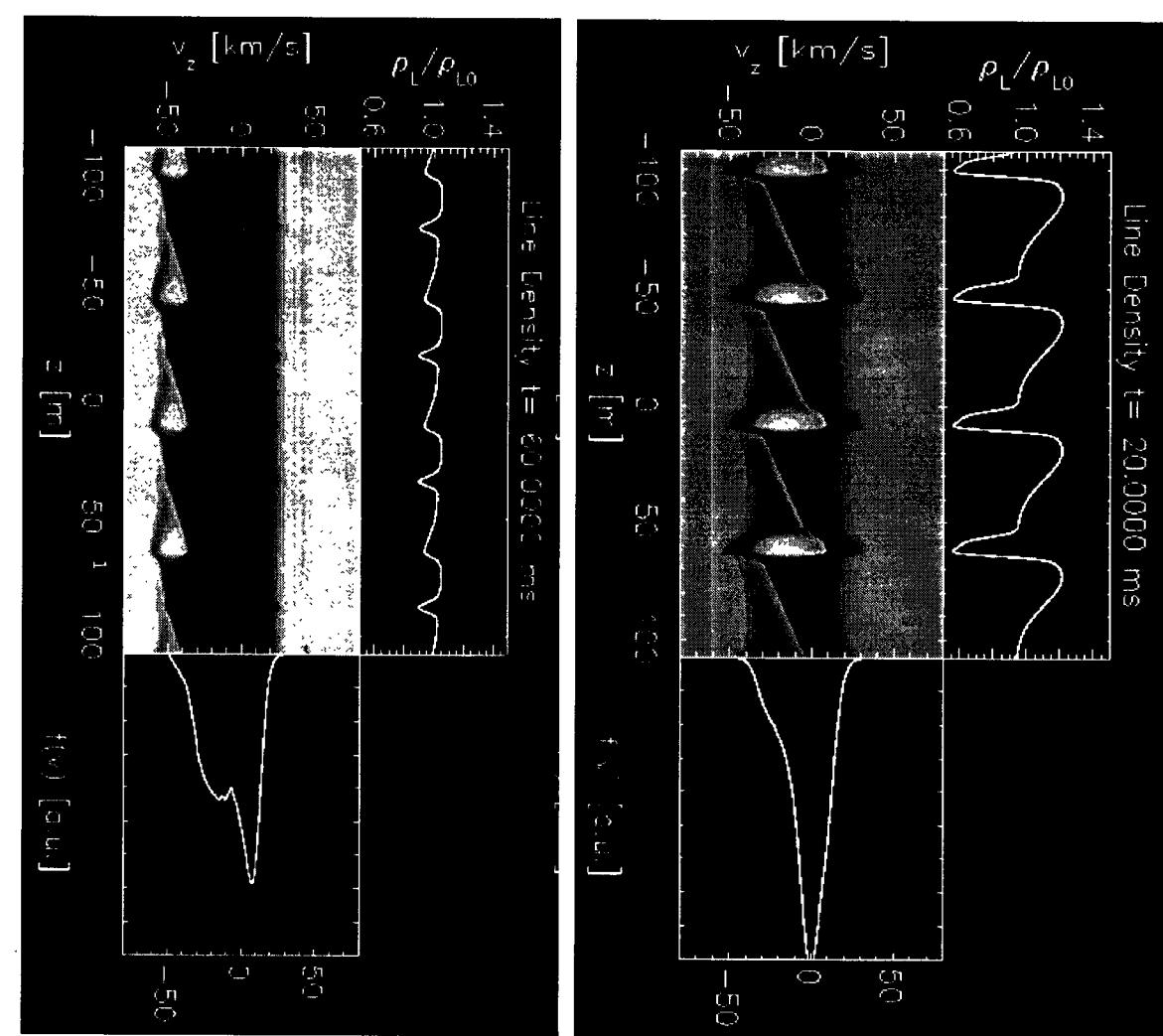
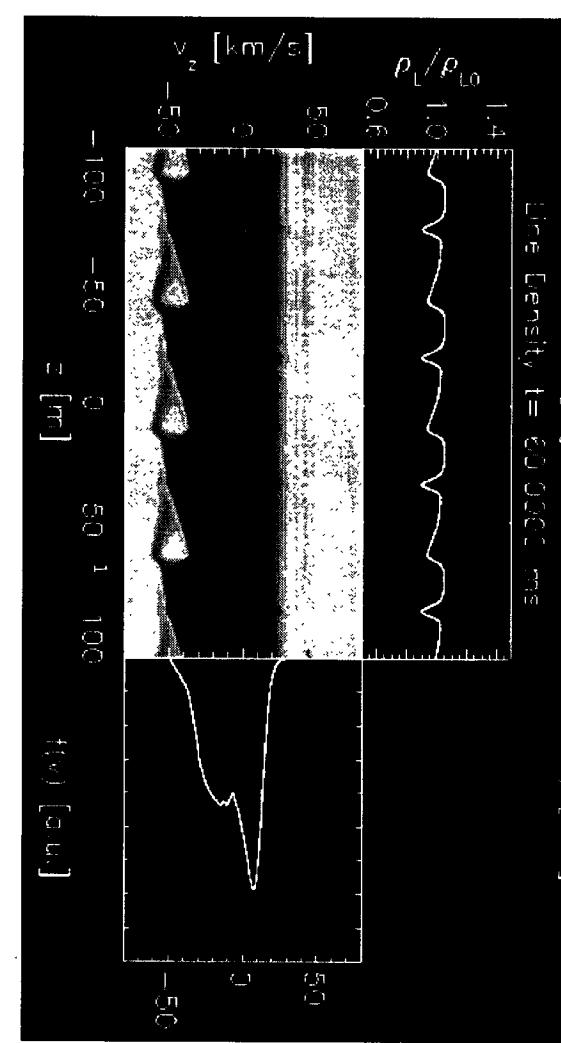
- initially parabolic (unstable)
- stabilizing tail also valid here



# Balanced impedance (ReZ~ImZ)



similar to ESR experiment: layers of holes traveling through momentum space



# Nonlinear phase strongly modified by space charge

in nonlinear phase some new features appear:

- nonlinear steepening (faster phase velocity for short waves)
- generation of lower (Fermilab work) and higher harmonics
- overshoot modified by space charge: distribution function not simply broadened
- only for mainly resistive impedance (or negative mass):  
$$(dp/p)^2_{\text{final}} \sim (dp/p)_{\text{initial}} (dp/p)_{\text{threshold}}$$
- for balanced impedances  $(dp/p)^2_{\text{final}}$  can be smaller but slowly growing
- for dominant space charge impedance:  $(dp/p)_{\text{final}} \sim (dp/p)_{\text{initial}}$  (fwhm) but tails developing towards lower momenta;
- often no thermalization of fluctuations by Landau damping
- close to stability boundary we find notches in line current affecting BTF and Schottky diagnostics - in SIS they disappear after  $\sim 200$  ms.

# Conclusions

- Presently reached good agreement experiment-simulation
  - limited comparison since detailed momentum space information only from simulation
- in nonlinear regime complex behaviour:
  - operation in unstable regime possible for  $\text{Re}Z \ll \text{Im}Z$
  - credible long-time simulation not trivial (smoothing by IBS?)
- diagnostics: density fluctuations due to initially unstable behaviour may persist and influence Schottky noise measurements
- more systematic measurements planned

