Instability Workshop 1999
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Intensity Dependent Effects in RHIC

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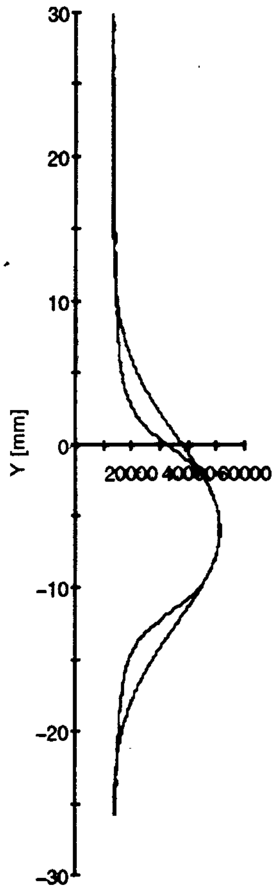
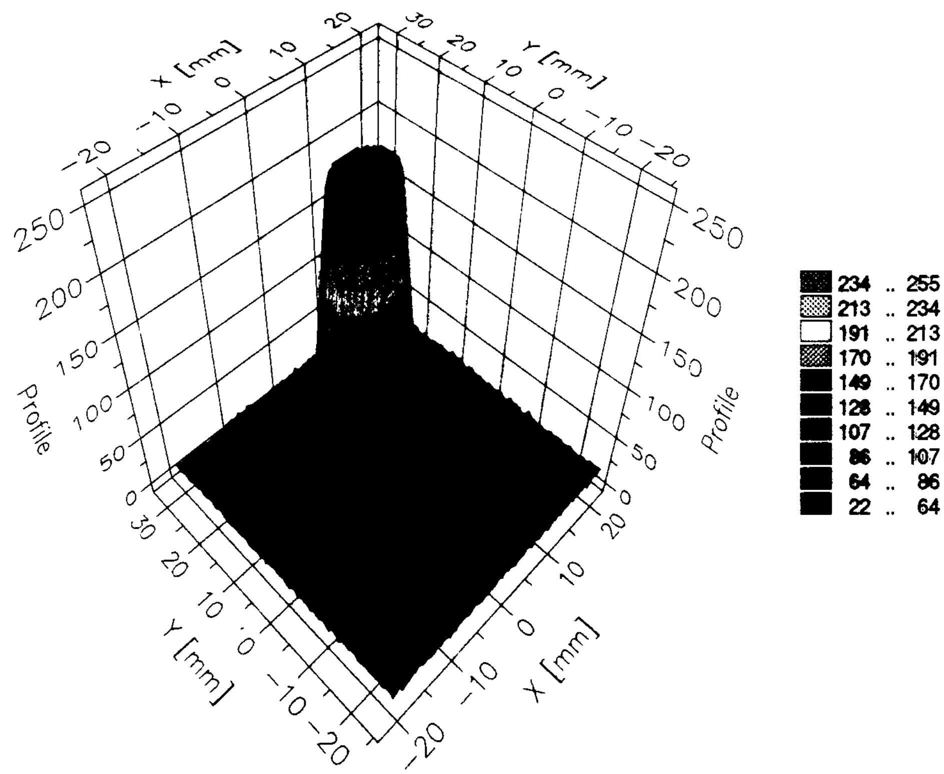
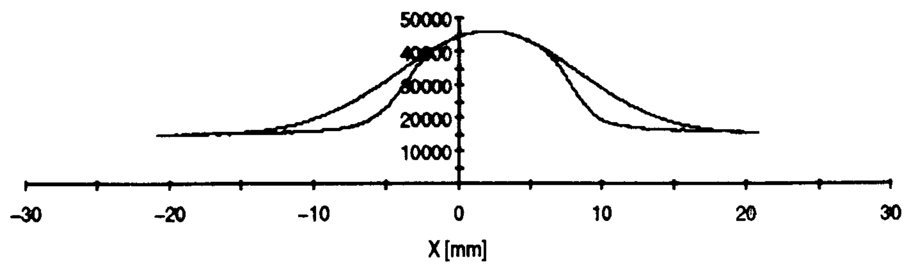
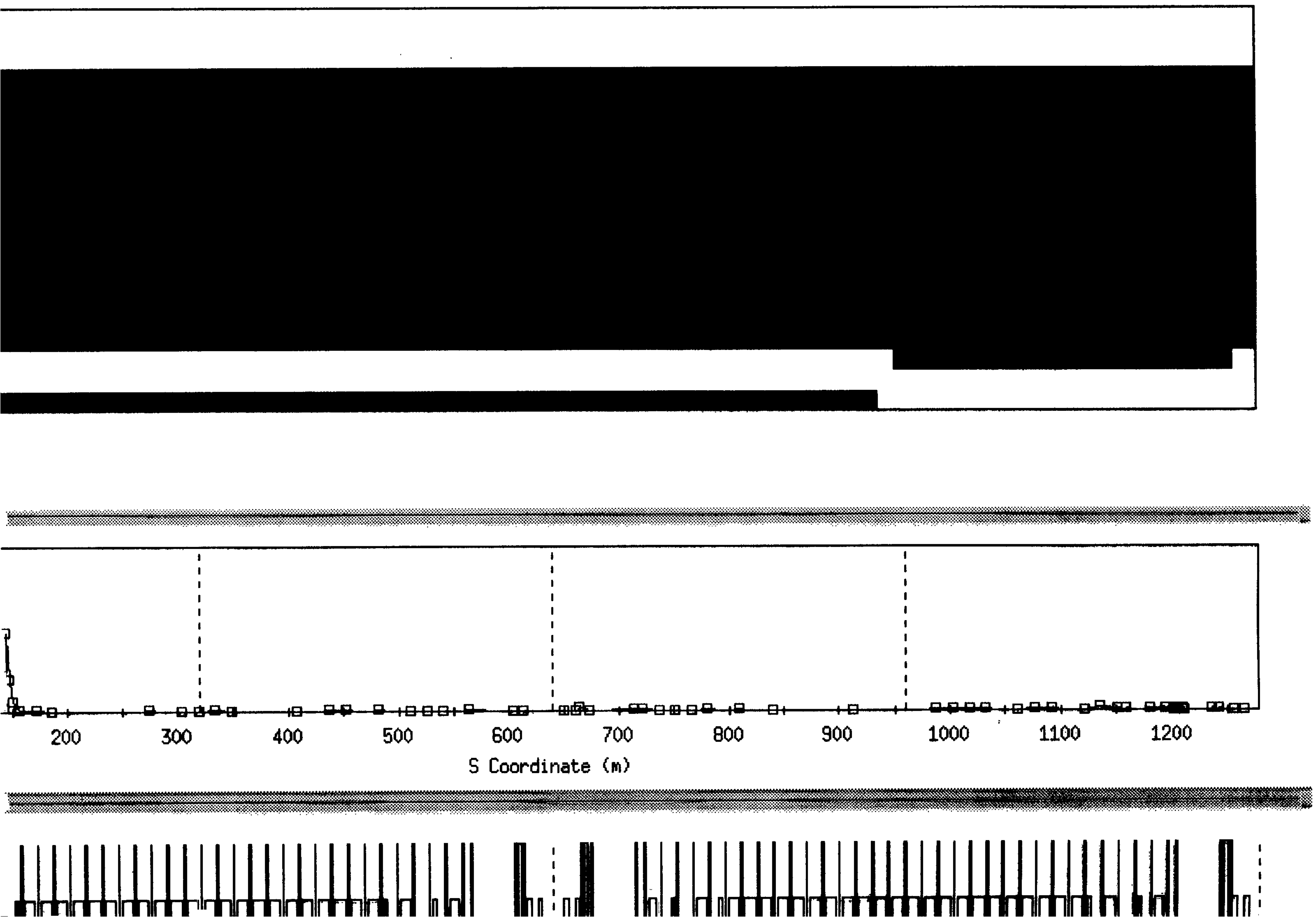
-- xf2 Wed Jun 30 04:35:20 1999 --
Framegrabber = fg2
Signal Range = 19 255
Centroid = 0.882573 -1.31507 mm
Center = 1.85381 -6.15978 mm
Sigma = 6.238 6.3902 mm
Ellipse = 6.25823 6.37684 mm
Theta = -0.124315
Intensity = 11445619
Attenuation = 0%








200
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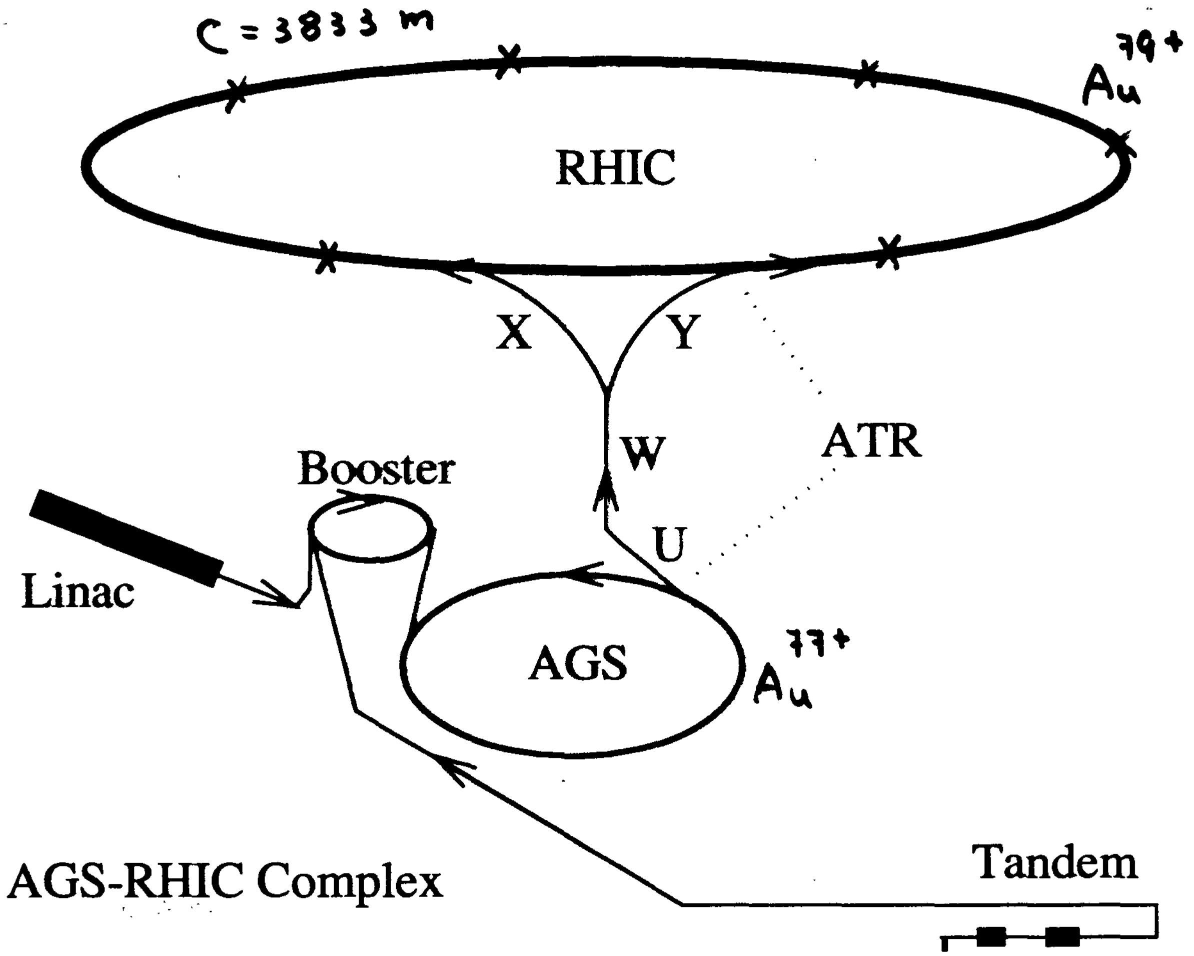
S Coordinate (m)

I. Overview

Schematic layout of the RHIC and the injector complex.



Au^{79+} : 100 GeV/u head-on collision

p^+ : 250 GeV

Intensity dependent mechanisms in RHIC:

- Intra-beam scattering
  - dominant for heavy ion beam at collision (Z^4/A^2)
  - predicted to produce 40% beam loss in 10 hours
  - luminosity improvement by more frequent re-filling, beam cooling, etc.
- Transition crossing
  - first superconducting machine to cross transition
  - slow magnet ramping, combined chromatic effect & intensity effects
  - a transition jump is necessary
- Beam-beam effects
  - moderate for gold operation ($\xi \sim 0.001$ per IP)
  - zero crossing angle for nominal 60-bunch operations
- Instabilities
  - susceptible to microwave instability at transition
  - transverse & longitudinal dampers for coupled bunch instabilities
  - electron cloud effect insignificant for nominal 60-bunch operation

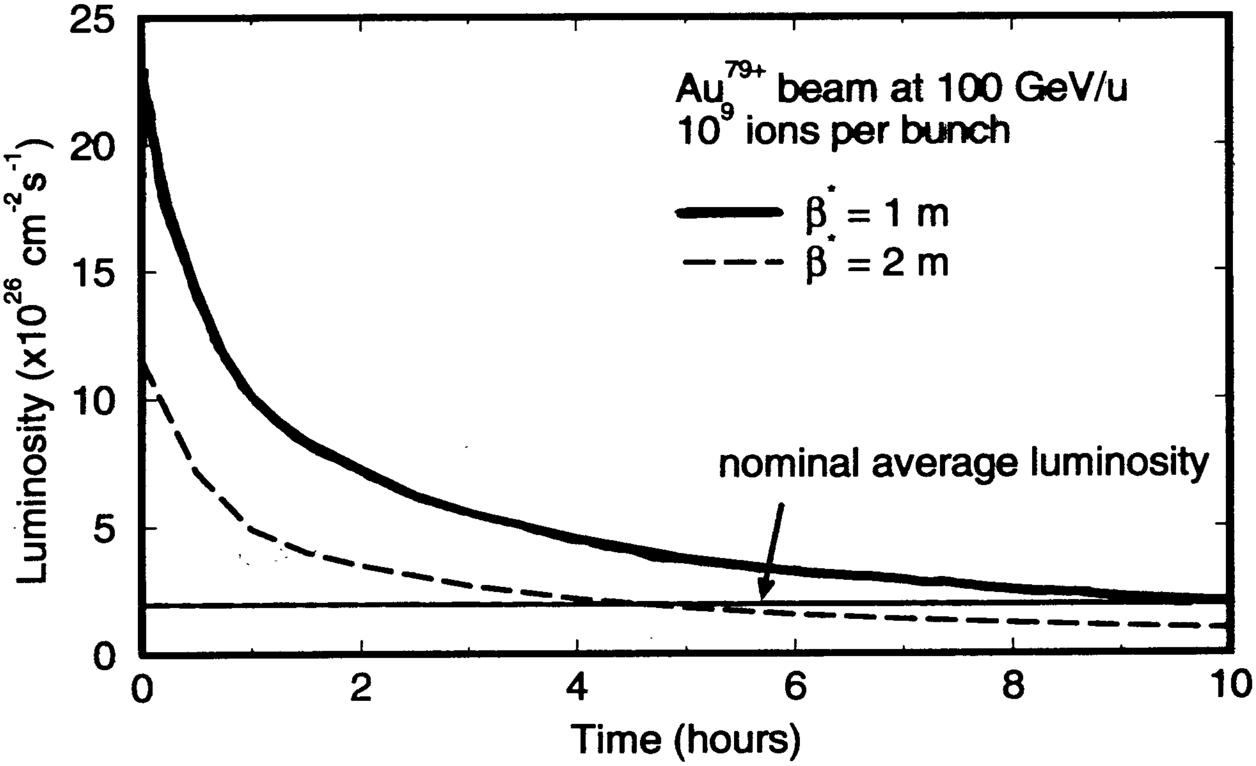
Luminosity reduction caused by intrabeam scattering during a 10-hour store for a crossing point with a β^* of 1 m and 2 m, respectively.

N_0 : ~ 40% loss in 10 hours. $V_{RF} = 6$ M

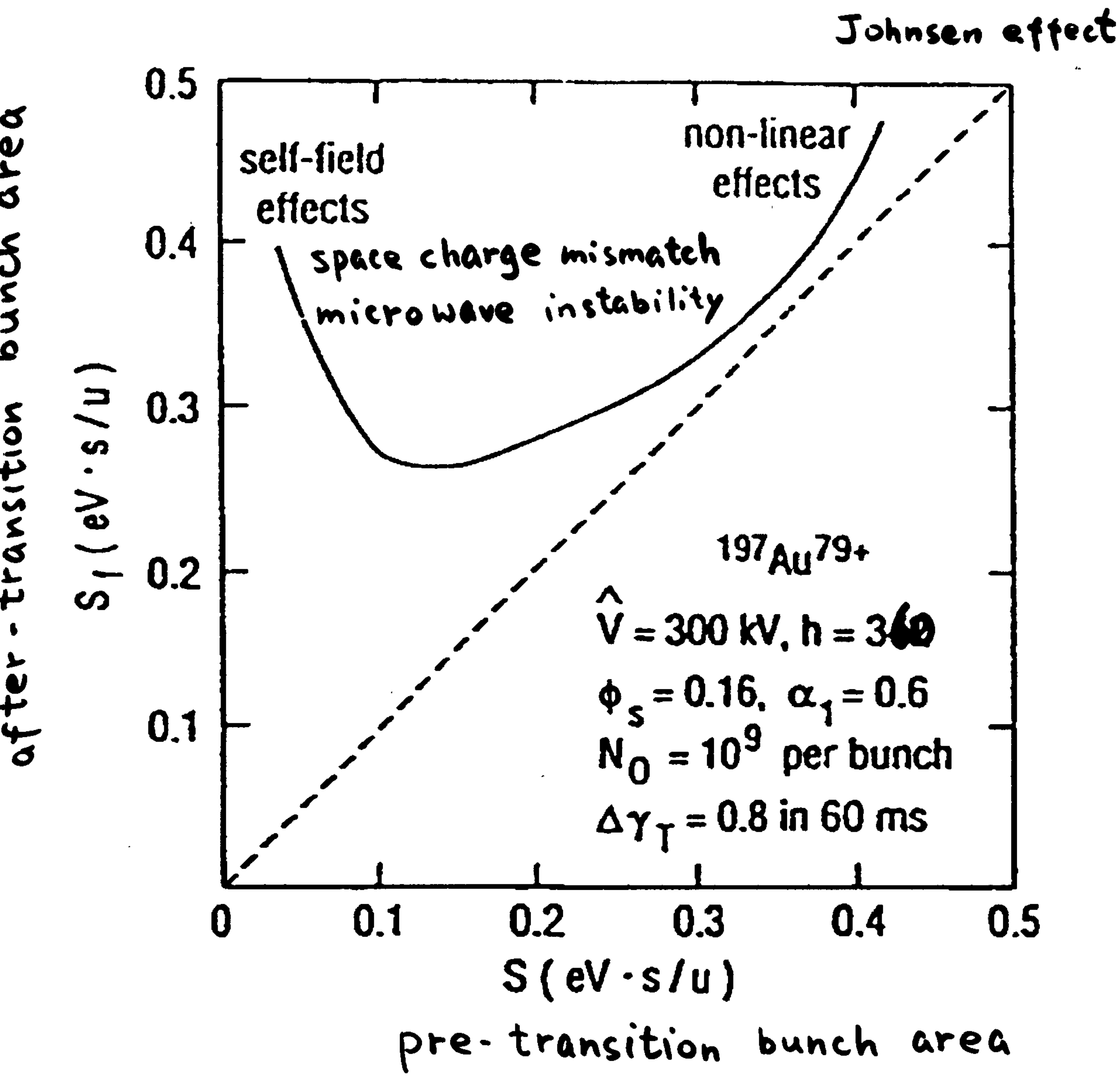
(~2% for $V_{RF} = 16$ MV, 30% higher $\mathcal{L}$)

ε_N : ~ 10 π μm → 40 π μm (coupled)

60 π μm (uncoupled)



Effects of chromatic nonlinearities and self fields at transition.



In the absence of γ_T jump:

* ~ 70% loss at γ_T under nominal condition
* ~ 30% loss at γ_T at low intensity

Major Parameters for the Collider.

Kinetic Ener., Inj.-Top, Au	10.8-100	GeV/u
(each beam), protons	28.3-250	GeV
No. of bunches/ring	60	
Circumference	3833.845	m
Number of crossing points	6	
β^*, injection, H/V	10	m
β^*, low-beta insertion, H/V	1	m
Betatron tunes, H/V	28.18/29.18	
Transition energy, γ_T	22.89	
Magnetic rigidity, injection	97.5	T·m
top energy	839.5	T·m
Bending radius, arc dipole	242.781	m
Coil i.d. arc magnets	8	cm
Coil i.d. triplet magnets	13	cm
Acceleration RF harmonic	360	
Storage RF harmonic	2520	

II. Intra-beam Scattering

(qualitative understanding & quantitative treatment)

Beam rest frame Hamiltonian

rotating fram

$$H = \begin{cases} \frac{1}{2}\left(P_x^2 + P_y^2 + P_z^2\right) + \frac{1}{2}x^2 \underline{- \gamma x P_z} + V_C \\ \\ \frac{1}{2}\left(P_x^2 + P_y^2 + P_z^2\right) - \frac{n_1}{2}(x^2 - y^2) + V_C + U_s \end{cases} \tag{1}$$

$$V_C = \sum_j \frac{1}{\sqrt{(x_j - x)^2 + (y_j - y)^2 + (z_j - z)^2}} \tag{2}$$

in terms of dispersion and betatron motion

$$\boxed{\bar{H} = \frac{1}{2}\left(\bar{P}_x^2 + \bar{P}_y^2\right) + \frac{1 - \gamma^2 F_z}{2}\bar{P}_z^2 + \bar{V}_C,} \tag{3}$$

$$F_z = \begin{cases} D + DD'' + (D')^2 \quad \text{(bending section)} \\ \\ DD'' + (D')^2 \quad \text{(straight section).} \end{cases} \tag{4}$$

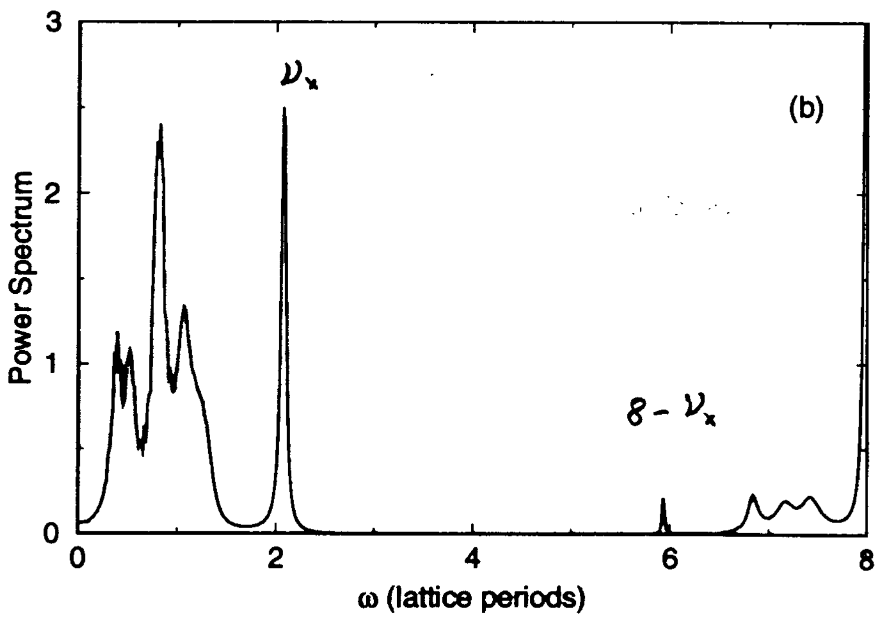
with

Wei, Li, Sessler
PRL 73, 3089 (1994)

$$\langle F_z \rangle = \frac{1}{\gamma_t^2}, \tag{5}$$

- Below transition (positive mass regime):
  - The system is conserved ("thermal equilibrium" exists) only if the machine lattice is uniform (artificial)
  - In general, the beam sees a time dependent potential, modulated by the lattice frequency. The beam structure absorbs "phonon" and heats up.
  - IBS behaves as a mixture of thermal equalization and temperature (emittance) growth
- Above transition (negative mass regime):
  - The Hamiltonian is not bounded.
  - Beam emittances (temperature) grow in both the longitudinal and transverse directions.

extreme IBS case



ring lattice superperiod : 8

- Below transition:
  - asymptotic state: equal velocity in the beam rest frame

$$\langle \frac{\sigma_x}{\beta_x} \rangle \approx \langle \frac{\sigma_y}{\beta_y} \rangle \approx \frac{\sigma_p}{\gamma}, \quad \gamma \ll \gamma_T$$

  - "optimum" operating point

- Above transition:
  - intensionally coupling horizontal and vertical motion to confine horizontal emittance growth
  - asymptotic state:

$$\sqrt{n_b n_c} \langle \sigma_x \rangle \approx \langle D_p \rangle \sigma_p, \quad \gamma \gg \gamma_T$$

$n_c = 1$ (uncoupled); 2 (fully coupled)
$n_b = 1$ (bunched); 2 (coasting)

$$\begin{bmatrix} \frac{1}{\sigma_p} \frac{d\sigma_p}{dt} \\ \frac{1}{\sigma_x} \frac{d\sigma_x}{dt} \end{bmatrix} = \frac{Z^4 N}{A^2} \frac{\pi r_0^2 m_0 c^2 L_c}{16 \gamma_T \epsilon_x \epsilon_y S} \begin{bmatrix} n_b (1 - d^2)/d \\ d/n_c \end{bmatrix} \tag{7}$$

$$d = D_p \sigma_p / (\sigma_x^2 + D_p^2 \sigma_p^2)^{1/2} < 1$$

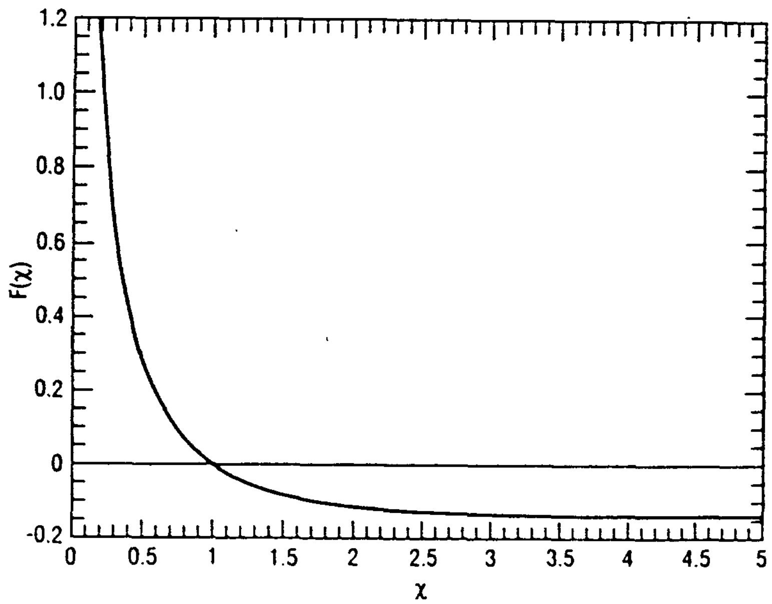
Intra-beam scattering scaling laws

(one-line formula with confidence within a factor of 2)

- Gaussian distribution in all dimensions
- Coulomb logarithm $L_C \gg 1$
- $D_x/\beta_x^{1/2}$ is near constant

$$\begin{bmatrix} \frac{1}{\sigma_p}\frac{d\sigma_p}{dt} \\ \frac{1}{\sigma_x}\frac{d\sigma_x}{dt} \\ \frac{1}{\sigma_y}\frac{d\sigma_y}{dt} \end{bmatrix} = \frac{Z^4 N}{A^2}\frac{r_0^2 m_0 c^2 L_c}{8\gamma\epsilon_x\epsilon_y S}F(\chi)\begin{bmatrix} n_b(1-d^2) \\ -a^2/2+d^2 \\ -b^2/2 \end{bmatrix} \tag{6}$$

- proportional to Z^4/A^2
- linearly proportional to 6-D phase space density



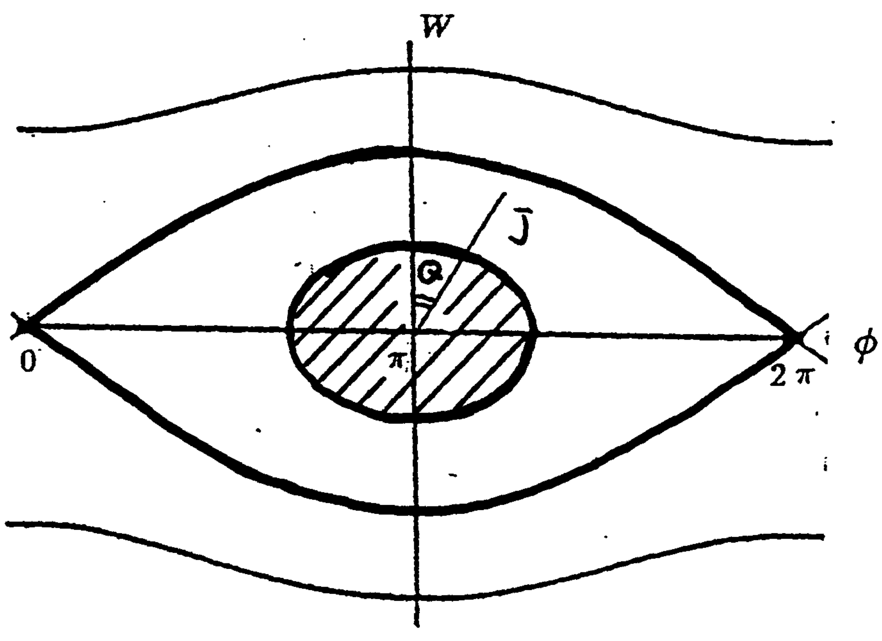
derived from Piwinsky's & Parzen's formalism

Fokker-Planck approach and IBS beam loss

- non-Gaussian distribution
- evaluation of beam loss through boundary

$\Rightarrow$

- Fokker-Planck approach in the longitudinal direction
- using action-angle variable, derive a 1-D transport equation for the evolution os the density function $\Psi(J)$



1. Fokker-Planck Eq. in action variable

Distribution function:

$$\Psi_{T,L}(x,x',y,y',\underline{Q,J};t) = \rho_H(x,x';t)\rho_V(y,y';t)\underline{\Psi_L(Q,J;t)}$$

Fokker-Planck equation:

$$\underline{\frac{\partial \Psi_L}{\partial t} = -\frac{\partial}{\partial x^\mu}\left(\Psi_L \frac{\langle \Delta x^\mu \rangle_{C,T}}{\Delta t}\right) + \frac{1}{2}\frac{\partial^2}{\partial x^\mu \partial x^\nu}\left(\Psi_L \frac{\langle \Delta x^\mu \Delta x^\nu \rangle_{C,T}}{\Delta t}\right)}$$

$$\mu,\nu = Q,J$$

- Average over collision events $\langle\ \rangle_C$
- Average over all transverse variables $\langle\ \rangle_T$
- Average over syn. osc. angle $\langle\ \rangle_Q$

$\boxed{\text{correlation time}} \ll \boxed{\text{syn. osc. period}} \ll \boxed{\text{IBS relaxation time}}$

Simplified F-P equation:

$$\frac{\partial \Psi}{\partial t} = -\frac{\partial}{\partial J}(F\Psi) + \frac{1}{2}\frac{\partial}{\partial J}\left(D\frac{\partial \Psi}{\partial J}\right)$$

Boundary condition:

$$\begin{cases} J = 0: \; -F\Psi + \dfrac{D}{2}\dfrac{\partial \Psi}{\partial J} = 0 \\ J = \hat{J}: \; \Psi = 0 \end{cases}$$

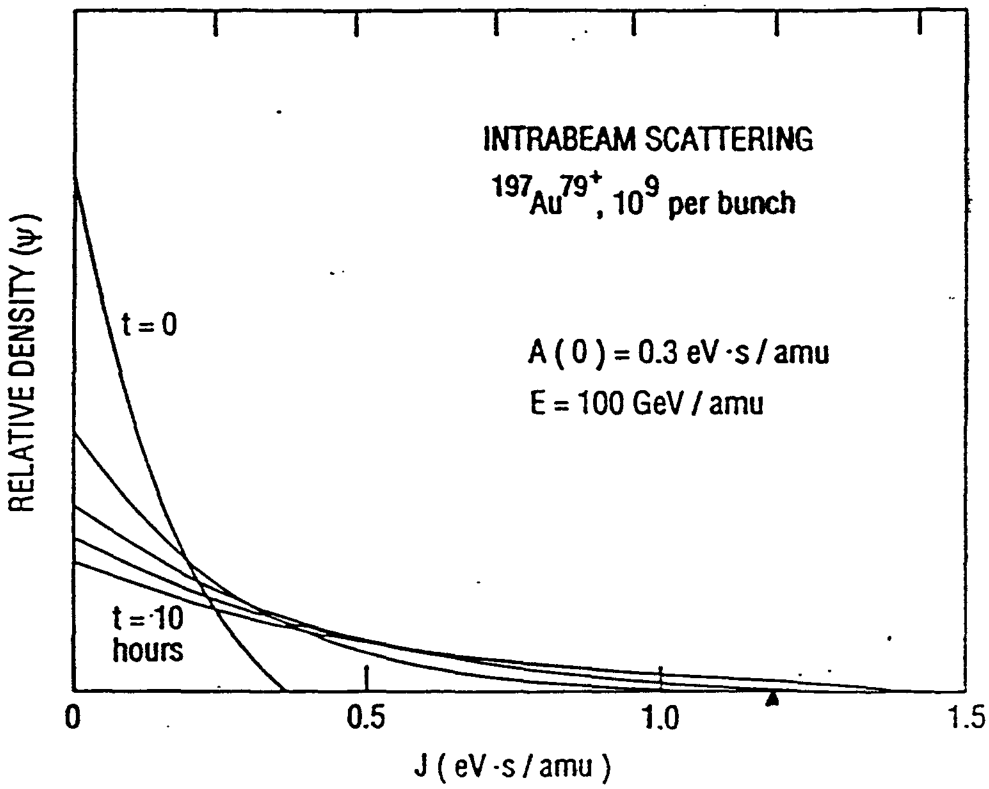
Dynamic friction:

$$F(J;t) = \int_0^1 dQ \left.\frac{\langle \Delta J \rangle_{C,T}}{\Delta t}\right|_0$$

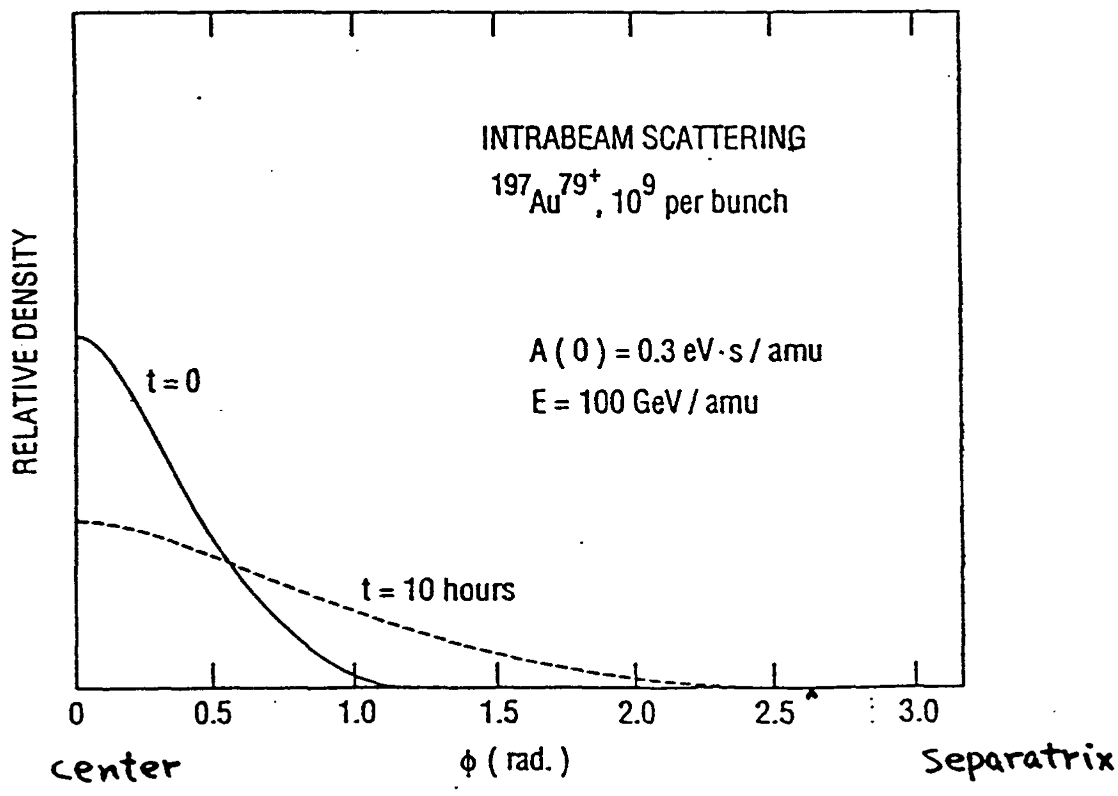
Diffusion:

$$D(J;t) = \int_0^1 dQ \left.\frac{\langle (\Delta J)^2 \rangle_{C,T}}{\Delta t}\right|_0$$

Evolution of density distribution $\Psi(J)$ (IBS):

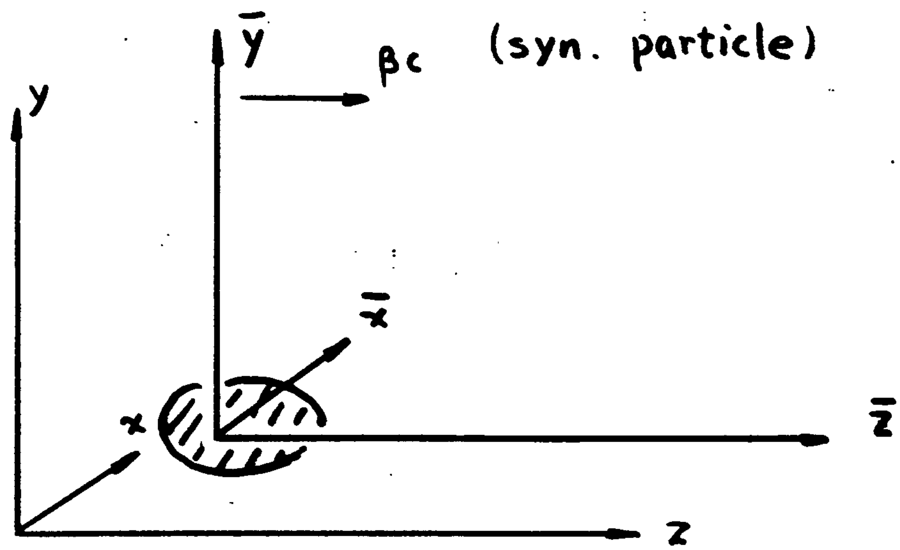


Projection in azimuthal direction (ϕ):



Rutherfold scattering in the rest frame

Lorentz transformation:



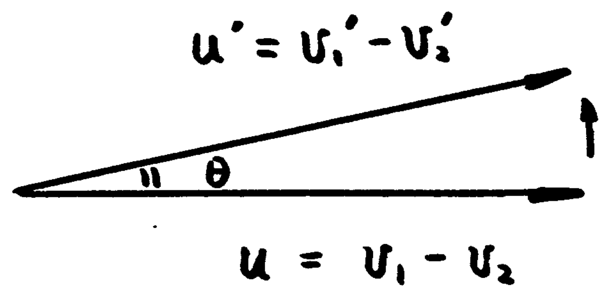
Relative velocity between 1 (test particle) and 2 (media):

$$|\mathbf{u}| \approx \beta c \sqrt{\left(\frac{\Delta p_1}{p} - \frac{\Delta p_2}{p}\right)^2 + \gamma^2 \left(x_1' - x_2'\right)^2 + \gamma^2 \left(y_1' - y_2'\right)^2}$$

Motion in the rest frame is non-relativistic.

Coulomb scattering cross section:

$$\sigma(u,\theta) = \frac{q^4 e^4}{A^2 m_0^2 u^4 \sin^4 \theta/2}$$



$$\langle \Delta \overline{v}_{z1} \rangle_\Omega = -2\Gamma \, \frac{u_z}{u^3} \,, \qquad \langle (\Delta \overline{v}_z)_1^2 \rangle_\Omega = \Gamma \, \frac{u_x^2 + u_y^2}{u^3}$$

solid angle

$$\Gamma \equiv \frac{4\pi q^4 e^4 \mathrm{Log}}{A^2 m_0^2}, \quad \mathrm{Log} \equiv -\ln \sin \frac{\theta_{min}}{2}$$

θ_{min}: minimum scattering angle

Log $\gg$1 $\Longrightarrow$ F-P approach O.K.

o Average over collision events $\langle\ \rangle_C$

$$\langle U_W\rangle_C = \frac{1}{\gamma}\frac{\beta E}{h\omega_0 c}\int dx'_{\beta 2}\rho_{x_\beta x'_\beta}(x_{\beta 2}, x'_{\beta 2}; t)\int dy'_2\rho_{yy'}(y_2, y'_2; t)$$
$$\times\frac{h}{\gamma R}\int dW_2\Psi\left[J(\phi_1, W_2)\right]\langle\Delta\overline{v}_{z1}\rangle_\Omega$$

solid angle

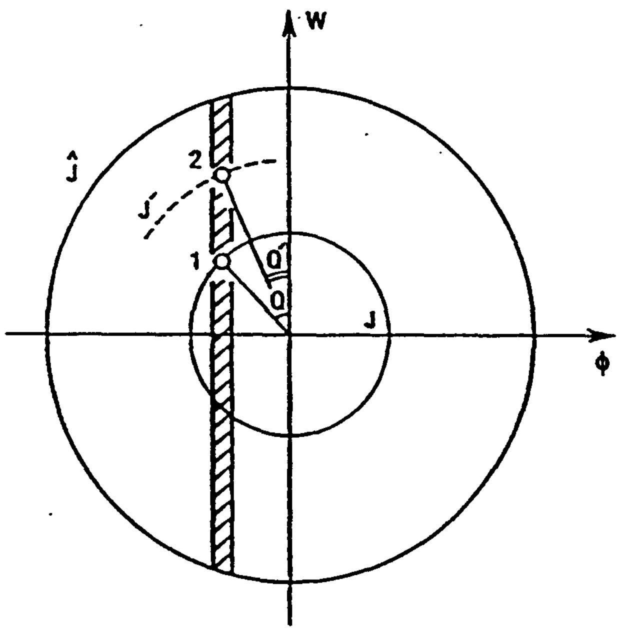
o Average over all transverse variables $\langle\ \rangle_T$

Assuming transverse distributions are time-varying Gaussians:

$$\rho_{yy'}(y, y'; t) = \frac{\sqrt{1+\alpha_y^2}}{2\pi\sigma_y\sigma_{y'}}\exp\left[-\frac{1+\alpha_y^2}{2}\left(\frac{y^2}{\sigma_y^2}+\frac{2\alpha_y yy'}{\sqrt{1+\alpha_y^2}\sigma_y\sigma_{y'}}+\frac{y'^2}{\sigma_{y'}^2}\right)\right]$$

$$\sigma_{x_\beta,y} = \sqrt{\frac{\beta_{x,y}\epsilon_{Nx,y}(t)}{6\beta\gamma}},\quad \sigma_{x'_\beta,y'} = \sqrt{\frac{(1+\alpha_{x,y}^2)\epsilon_{Nx,y}(t)}{6\beta\gamma\beta_{x,y}}}$$

o Average over angle variable $\langle\ \rangle_Q$



F-P coefficients in accessible forms:

$$
\begin{aligned}
F(J) \; &= \; \int \frac{2dz}{\pi R} \int_0^{\frac{1}{4}} dQ \, \frac{\partial W}{\partial J}\bigg|_{\phi}^{-1} (Q, J) \int_{J_{min}}^{\hat{J}} \frac{\partial W}{\partial J}\bigg|_{\phi} (Q', J') \\
&\quad \times [A_F(\lambda_1) + A_F(\lambda_2)] \, \Psi(J') dJ'.
\end{aligned}
$$

$$
\begin{aligned}
D(J) \; &= \; \int \frac{2dz}{\pi R} \int_0^{\frac{1}{4}} dQ \left[\frac{\partial W}{\partial J}\bigg|_{\phi}^{-1} (Q, J) \right]^2 \int_{J_{min}}^{\hat{J}} \frac{\partial W}{\partial J}\bigg|_{\phi} (Q', J') \\
&\quad \times [A_D(\lambda_1) + A_D(\lambda_2)] \, \Psi(J') dJ'
\end{aligned}
$$

where

$$
k(J') \sin 2\pi Q' \approx \sin [\phi(Q, J)/2]
$$

$$
\lambda_{1,2} = \frac{h\omega_0 a}{\gamma \beta^2 E} (W \mp W'), \qquad a = \frac{1}{2} \sqrt{\frac{6\beta\gamma\beta_{x,y}}{\epsilon_{Nx,y}}}
$$

$$
\begin{aligned}
A_F(\lambda) \; &= \; -\frac{2\Gamma}{\gamma} \frac{\beta E}{h\omega_0 c} \frac{h}{\gamma R (\beta c \gamma)^2} \frac{I_F(\lambda)}{4\pi \sigma_{x_\beta} \sigma_y} \\
A_D(\lambda) \; &= \; \frac{\Gamma}{\gamma} \left(\frac{\beta E}{h\omega_0 c} \right)^2 \frac{h}{\gamma R \beta c \gamma} \frac{I_D(\lambda)}{4\pi \sigma_{x_\beta} \sigma_y}
\end{aligned}
$$

For round beam with $\beta_x x'_p + \alpha_x x_p \sim 0$:

$$I_F(\lambda) = 2a^2 \mathrm{sgn}(\lambda)\chi\left\{1 - \sqrt{\pi}|\lambda|e^{\lambda^2}\left[1 - \Phi(\lambda)\right]\right\}$$

$$I_D(\lambda) = a\chi\left\{\sqrt{\pi}(1 + 2\lambda^2)e^{\lambda^2}\left[1 - \Phi(\lambda)\right] - 2|\lambda|\right\}$$

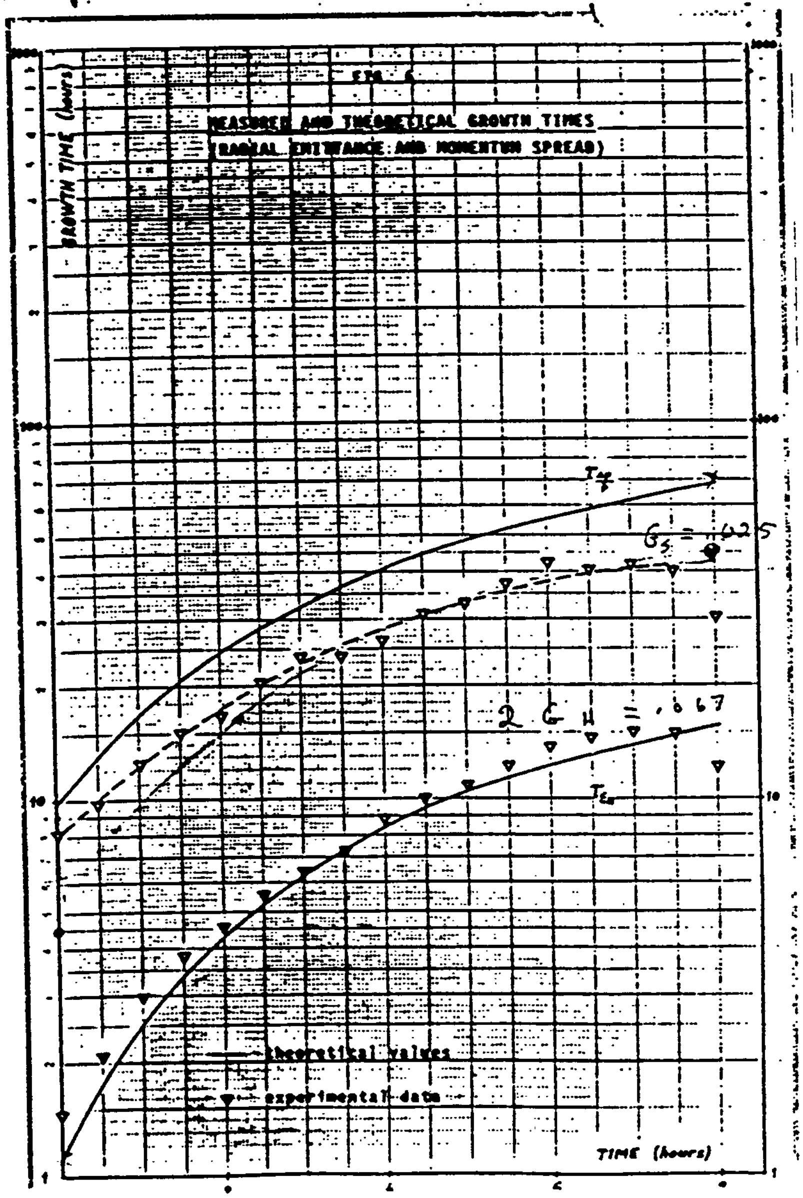
Φ: error function

Dispersion x_p effectively increases the horizontal beam size, thus decreasing the growth rate:

$$\chi = \exp\left[-\left(\frac{x_p\gamma\lambda}{2\sigma_{x_\beta}a}\right)^2\right]$$

Comparison with previous growth-rate calculation & experimental data

* When boundary is far away, agreement between Fokker-Planck approach and previous calculation ~ 20%
* No information on beam loss



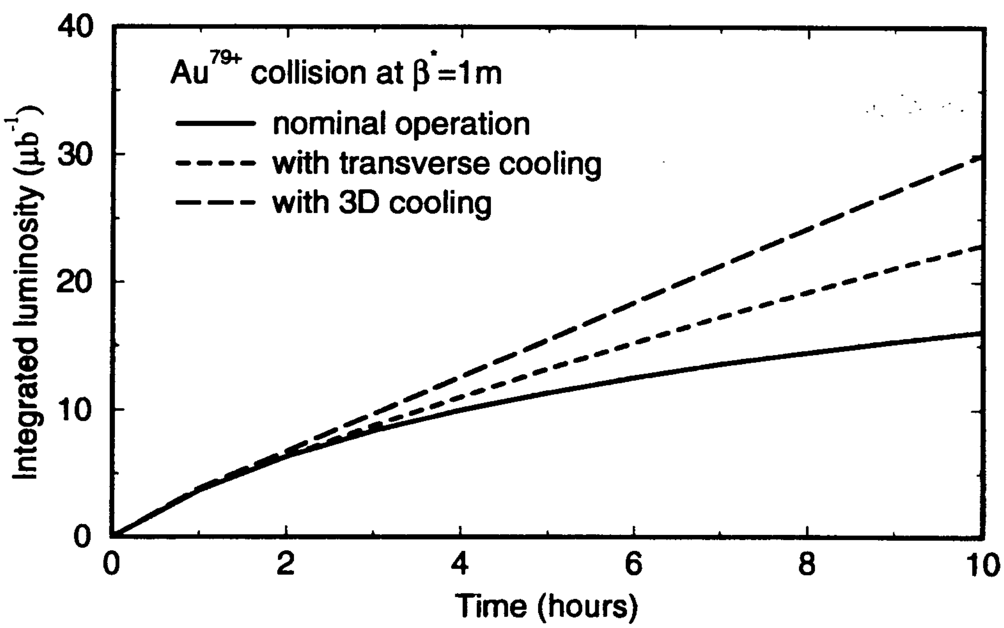
Martini

Colestock

IBS compensation with beam cooling

- **Stochastic cooling:**
  - feasible with a cooling system of 4 – 8 GHz bandwidth
  - present experiments with bunched beam experience large coherent signal suspected to be caused by instability
- **Electron cooling:**
  - needs separate electron storage ring for electron damping

Burov, Debenev, Ng, Lee, Colestock et. al.



III. Transition Crossing

Non-adiabatic regime formalism

- transition crossing is often the "bottle neck" during acceleration
- analysis is complicated by the non-adiabatic nature of the motion
- chromatic nonlinearity (Johnsen effect), self-field mismatch, microwave instability enhance each other
- $\Rightarrow$ longitudinal amplitude function β_L

$$T_c = \left(\frac{\pi E_s \beta_s^2 \gamma_T^3}{qeV|\cos\phi_s|\dot{\gamma} h \omega_s^2}\right)^{\frac{1}{3}}$$

$\sim \pm 40\,\mathrm{ms}$

Synchrotron frequency:

$$\Omega_s = k\beta_L^{-1}$$

Maximum excursions in ϕ and W:

$$\hat{\phi} = \sqrt{2\gamma_L J}; \quad \hat{W} = \sqrt{2\beta_L J}$$

$$\begin{cases} \hat{\sigma}_\phi = 0.52\,(S/kT_c)^{1/2} \\ \hat{\sigma}_\delta = 0.71 h\omega_s\,(kT_c S)^{1/2}/E_s\beta_s^2. \end{cases}$$

* Longitudinal amplitude function β_L

$$\frac{1}{2}\beta_L\beta_L'' - \frac{1}{4}\beta_L'^2 + K\beta_L^2 = 1$$

with boundary conditions

Normalised amplitude function $\beta_L(\tau) = |k_1^{-1}|T_c\hat{\beta}_L(x)$

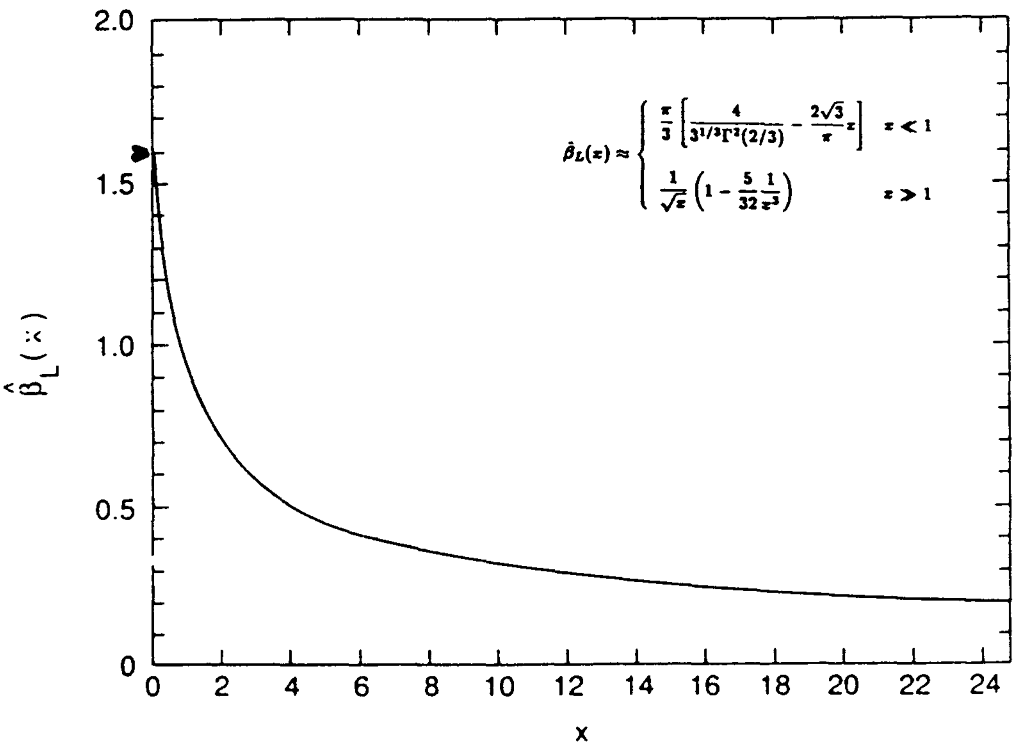
$$\frac{1}{2}\hat{\beta}_L\hat{\beta}_L'' - \frac{1}{4}\hat{\beta}_L'^2 + x\hat{\beta}_L^2 = 1$$

general solution (with $x = |t|/T_c$ and $y = \frac{2}{3}x^{\frac{3}{2}}$):

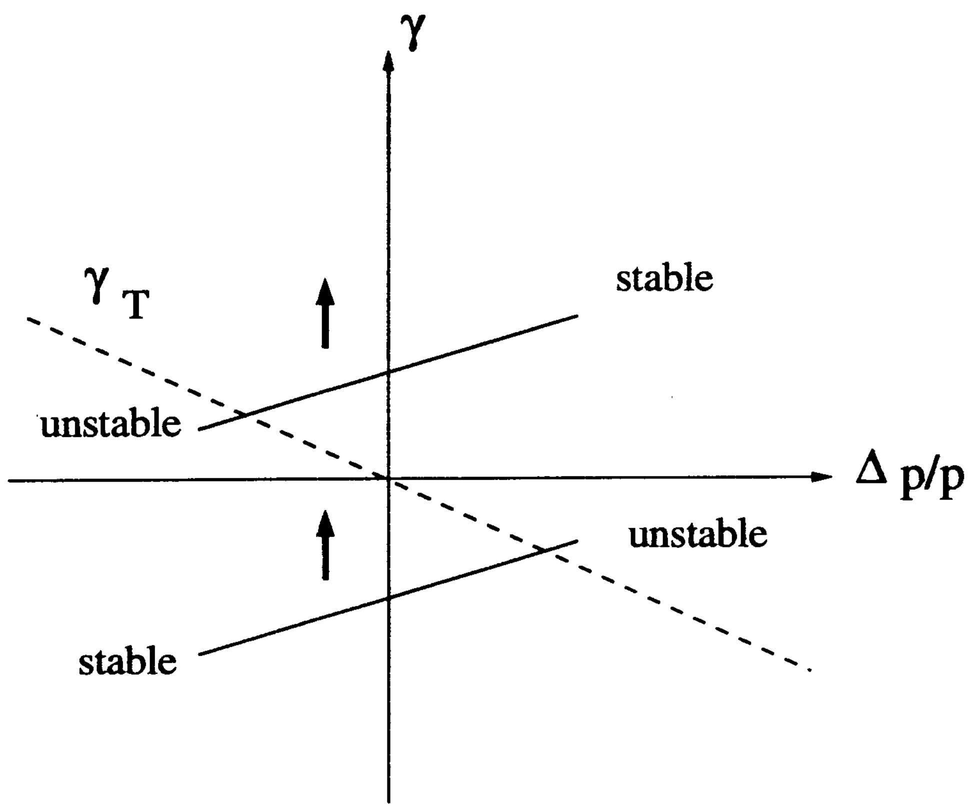
$$\hat{\beta}_L(x) = \frac{\pi}{3}x\left[C_1\mathrm{J}^2_{-\frac{1}{3}}(y) + C_2\mathrm{N}^2_{-\frac{1}{3}}(y) - 2(C_1C_2 - 1)^{\frac{1}{2}}\mathrm{J}_{-\frac{1}{3}}(y)\mathrm{N}_{-\frac{1}{3}}(y)\right]$$

$\hat{\beta}_L$ is positive and monotonic $\Longrightarrow C_1 = C_2 = 1$

$$\underline{\hat{\beta}_L(x) = \frac{\pi}{3}x\left[\mathrm{J}^2_{-\frac{1}{3}}(y) + \mathrm{N}^2_{-\frac{1}{3}}(y)\right], \quad 0 \le x < \infty}$$



Johnsen effect



- non-linear time T_{nl}

$$T_{nl} = \left|(\alpha_1 + \frac{3\beta_s^2}{2})\right| \frac{\sqrt{6}\hat{\sigma}_\delta \gamma_T}{\dot{\gamma}}$$

$$\frac{\Delta S}{S} \approx \begin{cases} 0.76\,\frac{T_{nl}}{T_c}, & \text{for } T_{nl} \ll T_c; \\ e^{\frac{4}{3}\left(\frac{T_{nl}}{T_c}\right)^{3/2}} - 1, & \text{for } T_{nl} \geq T_c, \end{cases}$$

$\frac{140 \text{ ms}}{40 \text{ ms}}$

in action-angle variables

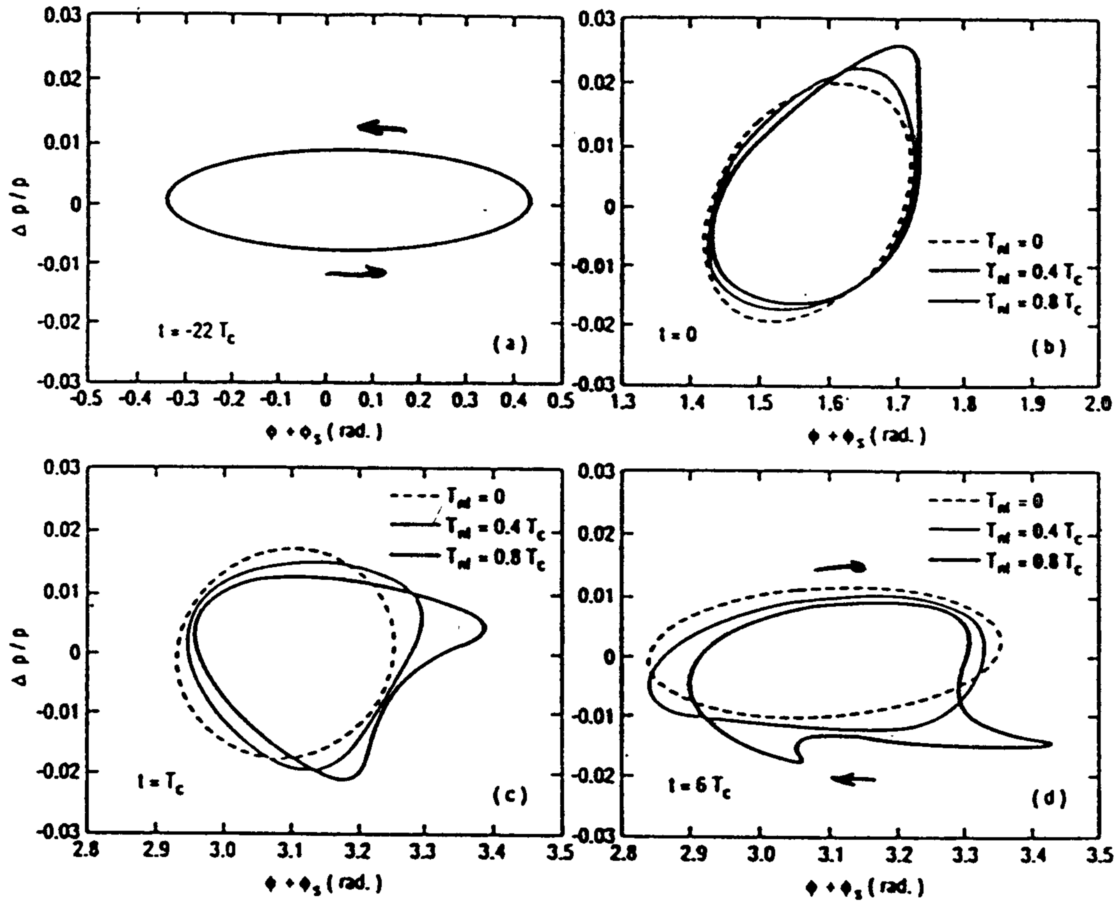
$$H(\varphi, J, \tau) = J/\beta_L + \sqrt{8}\, \epsilon_3 (J \beta_L)^{3/2} \cos^3\varphi$$

$$\epsilon_3 = - \frac{2\pi h^3 \omega_s^3 \eta_1}{3 Z e \hat{V} \cos\phi_s E^2 \beta^4}$$

canonical transformation $F_2(\varphi, \tilde{J}, \tau) = \varphi \tilde{J} + G(\varphi, \tilde{J}, \tau)$

$\Rightarrow$ new $\tilde{H}$, independent of $\tilde{\varphi}$ to 1st order in ϵ_3

$\Rightarrow$ $\Delta J/J$



2. mismatch due to self fields

* reactive coupling

$$H = H_0 + \frac{Ze\hat{V}\cos\phi_s}{2\pi h} \cdot \frac{\epsilon_z}{2} \left[\lambda(\phi) - \lambda(0) \right]$$

$$\epsilon_z = - \frac{2 Z e h^2 \omega_s \, |Z_\parallel/n| \, \mathrm{sgn}(Z_\parallel)}{\hat{V} \cos\phi_s}$$

e.g. parabolic potential, $\lambda(\phi) - \lambda(0) \sim \phi^2$

increase / decrease focusing strength when above / below transition $\Rightarrow$ mismatch

$$\boxed{\frac{\Delta J}{J_0} = \frac{2h \hat{I} \, |Z_\parallel/n|}{\hat{V} \, |\cos\phi_s| \, \hat{\phi}^2(0)}}$$

ratio of self field to rf focusing field

$$\sim Zeh^2 \omega_s \cdot \frac{N_0}{\hat{\phi}^2} \cdot \left|\frac{Z_\parallel}{n}\right| \Big/ \hat{V} \, |\cos\phi_s| \, \hat{\phi}$$

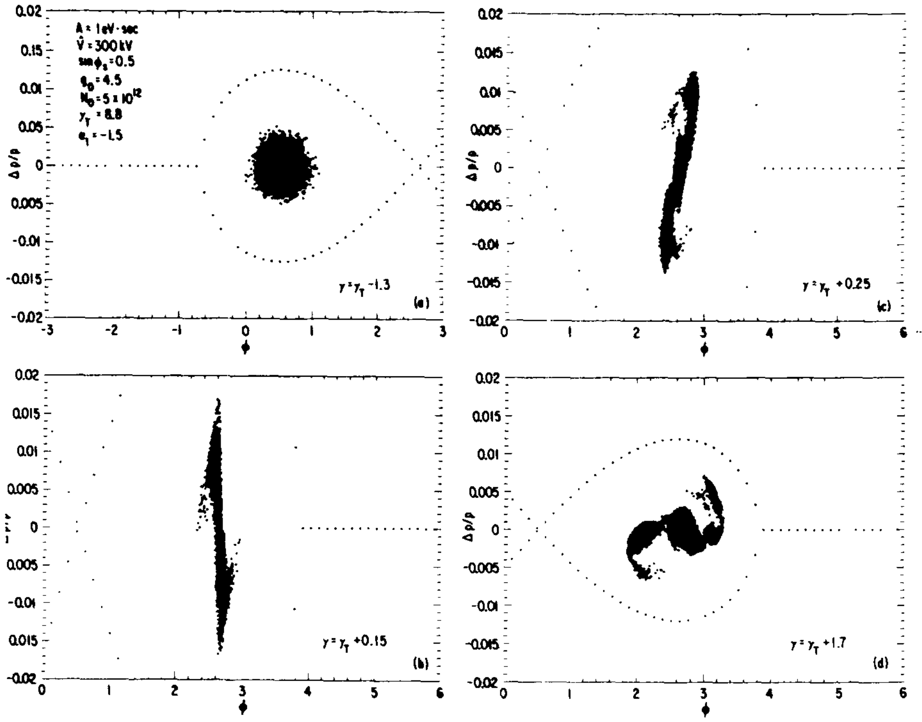
Microwave instability

- microwave instability threshold scales with σ_{ϕ}^{-3}
- Vlasov equation exactly solvable for a parabolic bunched beam
- unstable time T_{mw} around transition:

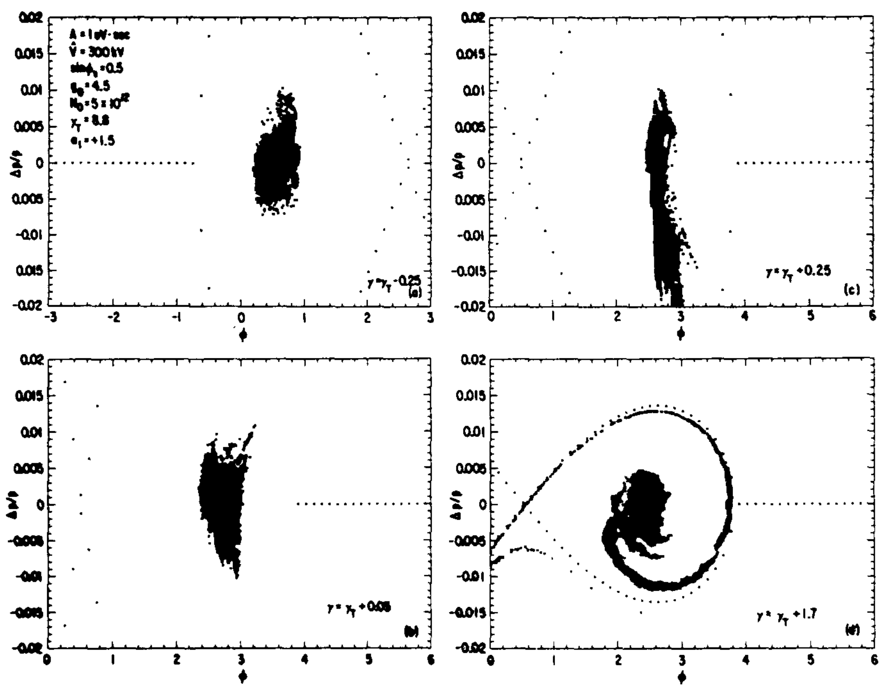
$$T_{mw} \approx 1.37\ (D_{\parallel} - 1)\ T_c$$

$$D_{\parallel} \approx \frac{4h\hat{I}\,|Z_{\parallel}/n|}{9V|\cos\phi_s|\ \hat{\sigma}_{\phi}^2} \geq 1 \tag{8}$$

* effect of space charge at γ_T



* when combined with chromatic non-linearity



* broad-band resistive impedance

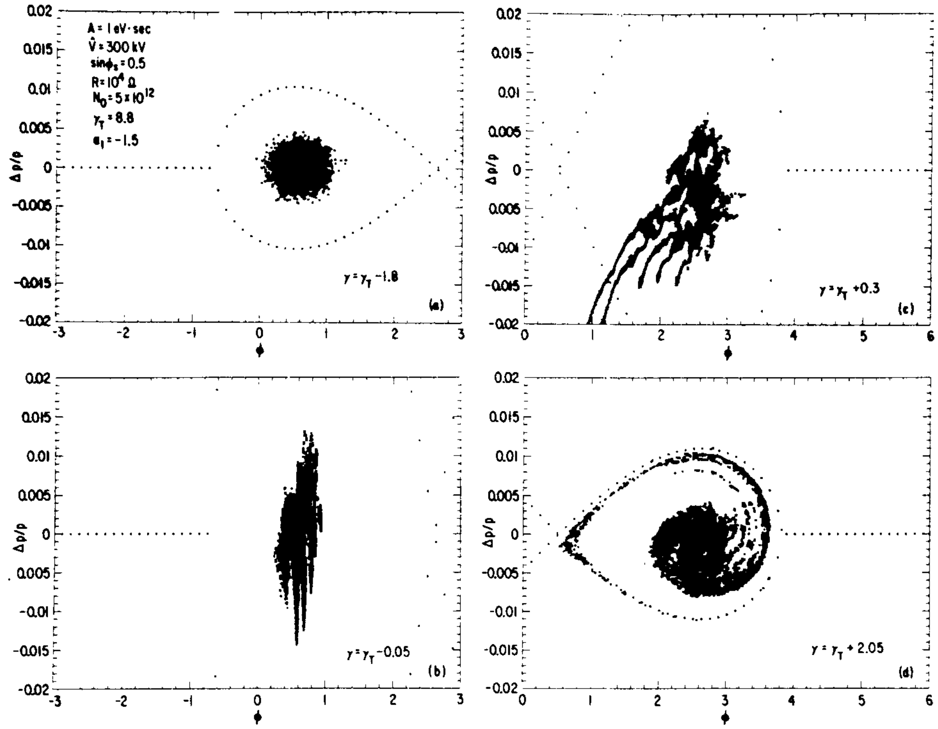


Figure 5.7: Transition crossing in the presence of a broad-band resistive coupling.

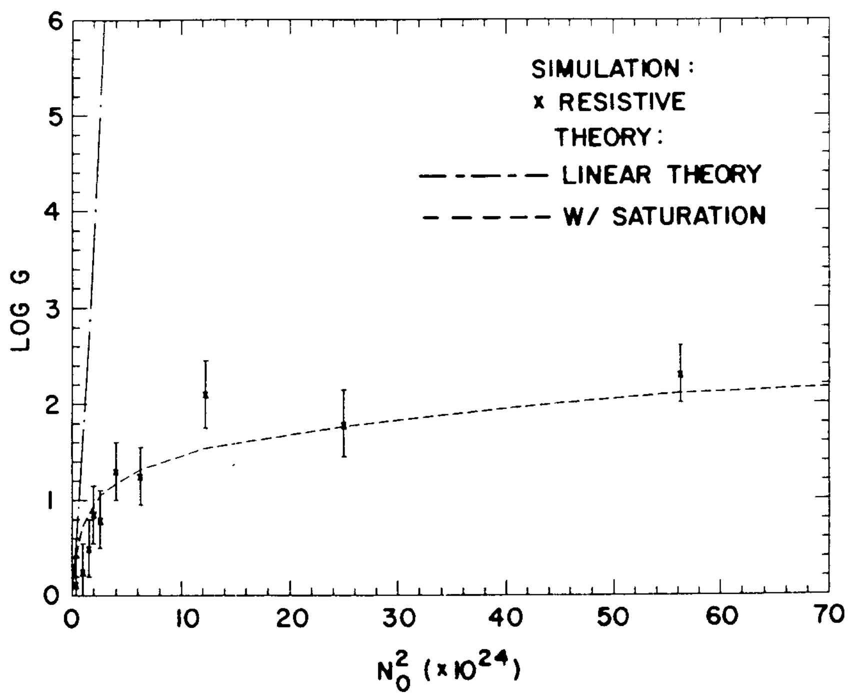


Figure 5.8: A comparison of the growth of microwave instabilities due to broad-band resistive impedances.

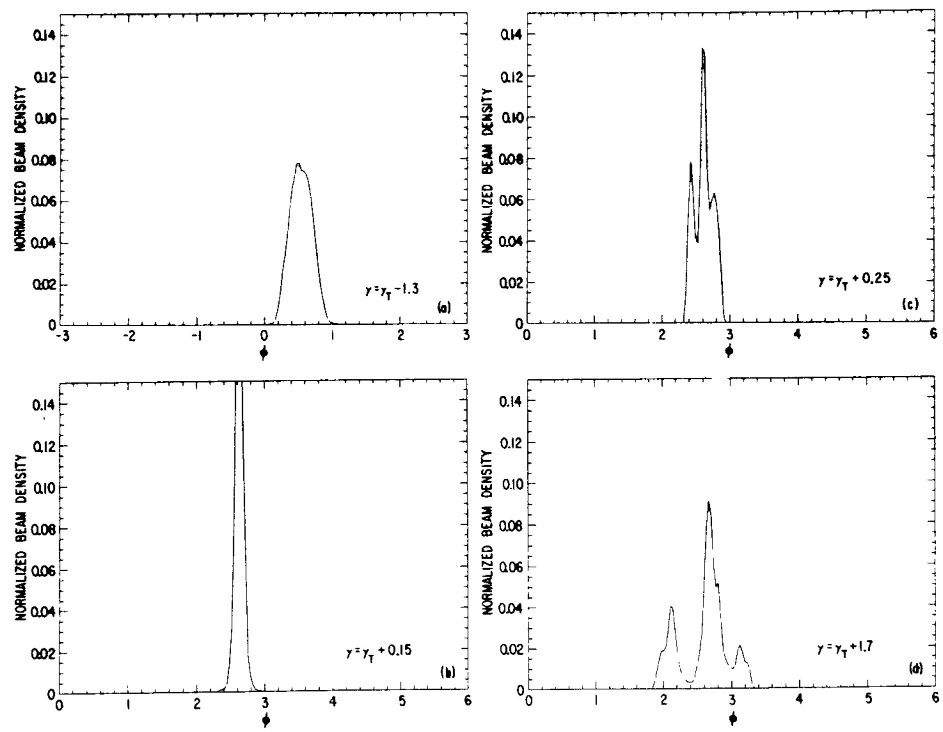


Figure 5.5: Development of microwave instabilities at transition.

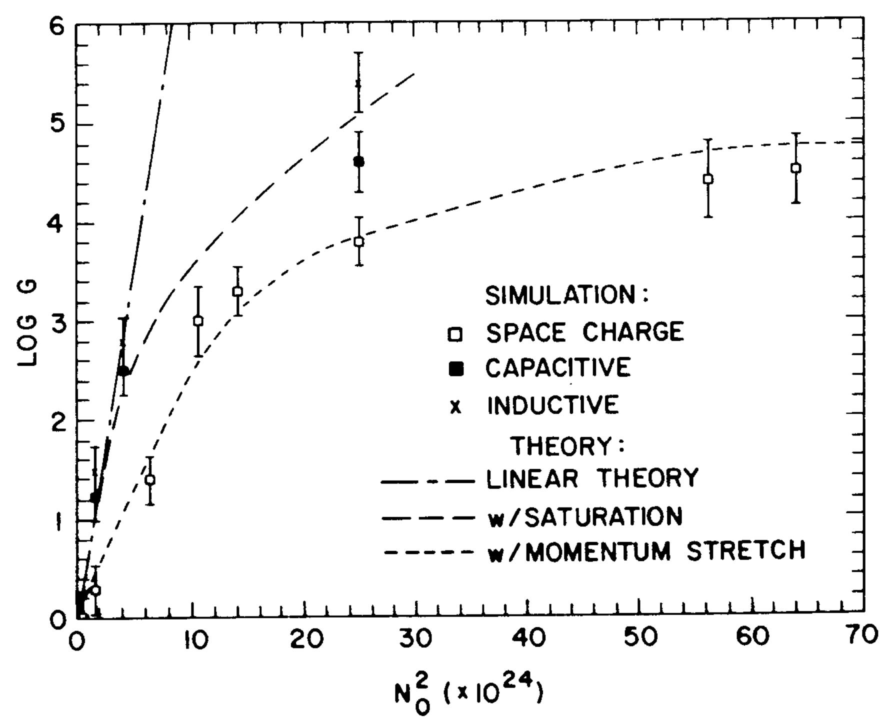


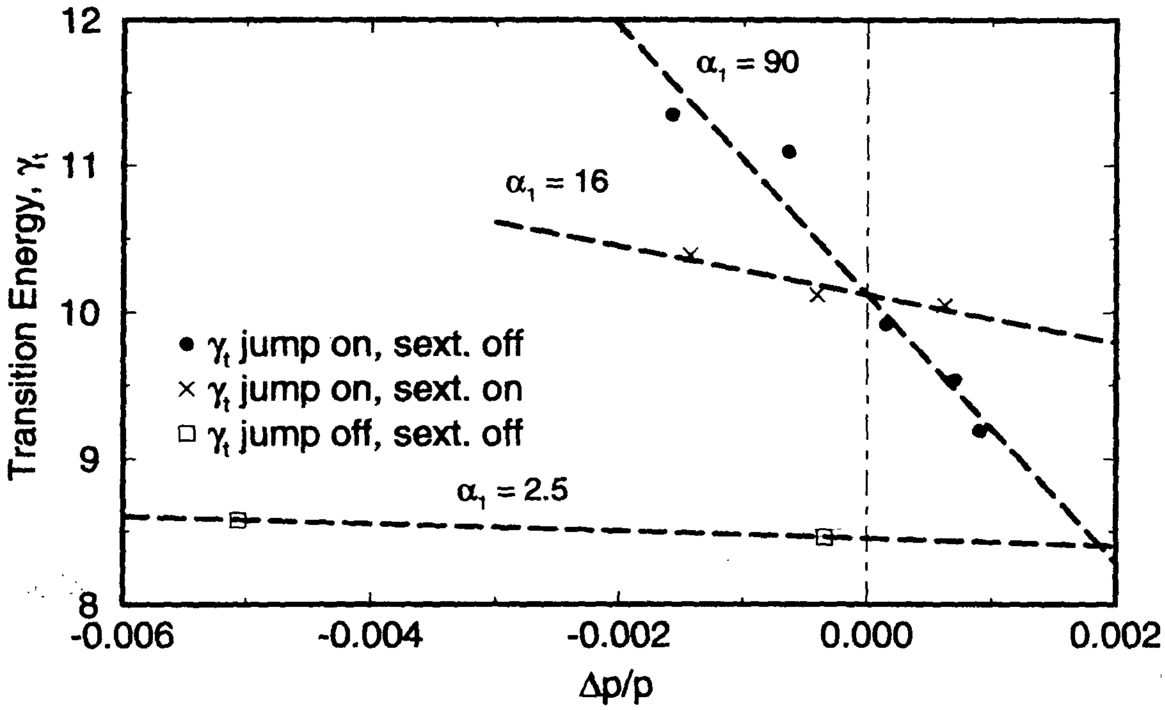
Figure 5.6: A comparison of the growth of microwave instabilities due to space charge and reactive impedances.

Transition jump

- transition jump is the only ways for RHIC to cross transition without loss
- first-order jump scheme developed using pulsed quadrupole correctors

Peggs, Tepikian, Trbojevic

- without further enhancing nonlinear factor α_1



IV. Summary

- Intra-beam scattering is the leading mechanism of luminosity degradation, emittance growth and beam loss for RHIC. The effect can be compensated by larger RF voltage, quicker re-filling, and ultimately beam cooling

- Transition crossing in a slow-ramping ring is complicated by chromatic nonlinear effect, beam self-field mismatch, and microwave instability. The effect can be compensated by a first-order transition jump

- Theoretical predictions are to be confirmed by real machine performance in the coming year