

---

# ***Luminosity Optimization by Adjusting the $\beta$ -Function in the Interaction Point at LHC and RHIC.***

W. Wittmer

walter.wittmer@cern.ch

walter.wittmer@free.fr

CERN AB/ABP

&

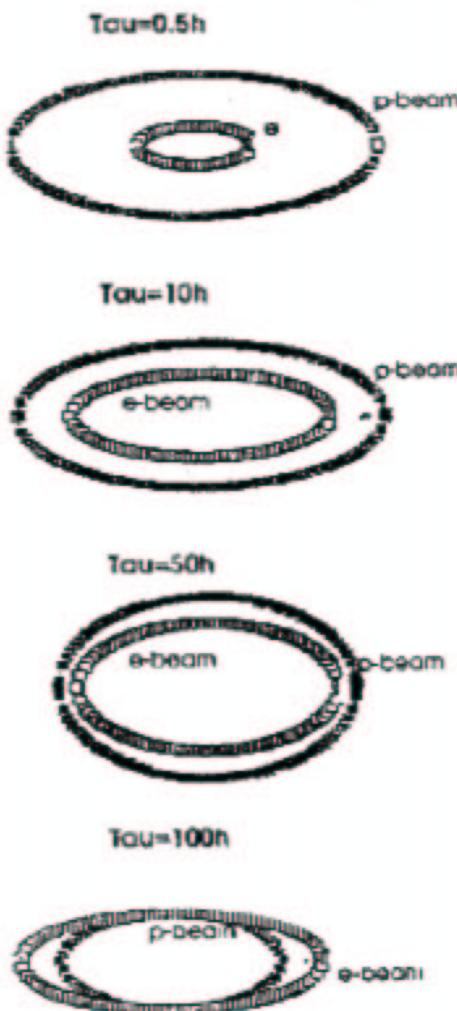
University of Technology Graz, Austria

# ***Table of Contents***

---

- Introduction
- Methods for Calculating Tuning Knobs
  - LHC
  - RHIC
- Performance tests
  - Operating Range
  - Field Errors
  - Beam-Beam Effects
- Experimental Test at RHIC
  - Correction Potential
  - Operative Range
  - $\beta^*$  Measurement

# Motivation - Beam Lifetime HERA, SPS



$\sigma_x^p$ ( $\mu m$ )	$\sigma_x^e$ ( $\mu m$ )	$\sigma_y^p$ ( $\mu m$ )	$\sigma_y^e$ ( $\mu m$ )	$\tau_p$ (hrs)
410	130	20	33	0.5
410	290	120	70	10
330	290	100	70	50
210	290	50	53	100
190	200	50	53	300

$$\tau = f \left( \frac{\sigma_1}{\sigma_2} \right)$$

$\frac{\sigma_1}{\sigma_2} \approx 1 \Rightarrow \left\{ \begin{array}{l} \text{poor life time due} \\ \text{to beam-beam} \end{array} \right.$

# **Luminosity Optimizing for LHC Ions**

- Initial beam (intensity) lifetime due to beam-beam interactions (non-exponential decay)

$$\tau_{NL} = \frac{k_b N_b}{n_{exp} L \sigma_{tot}} = \frac{22.4 \text{hour}}{n_{exp}}$$

for nominal  $L = 10^{27} \text{ cm}^{-2} \text{ s}^{-1}$   
with Pb-Pb

$n_{exp} \dots$  number of experiments illuminated

- But luminosity may be limited by experiment or quench limit

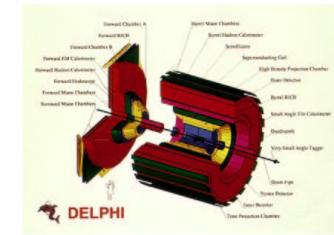
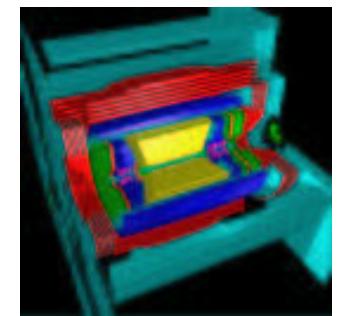
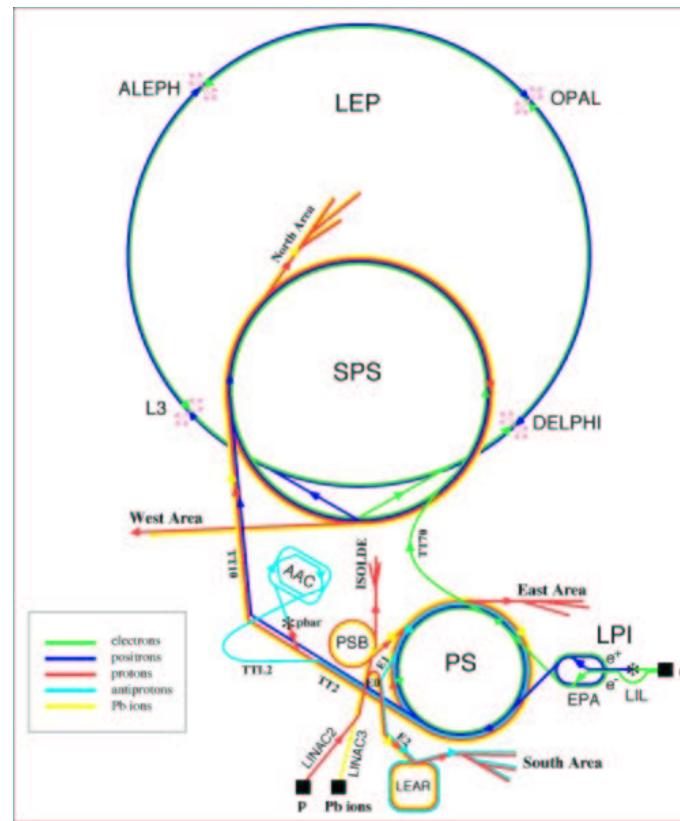
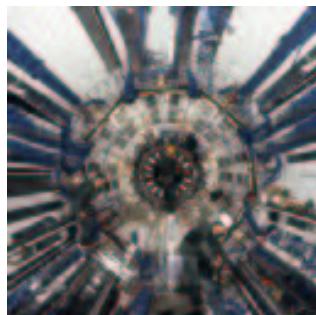
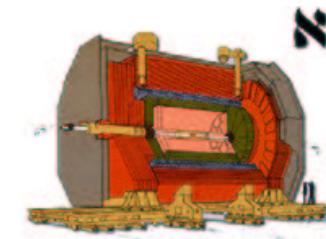
$$L = \frac{k_b N_b^2 f_0}{4\pi \sigma^{\star 2}} = \frac{k_b N_b^2 f_0}{4\pi \beta^{\star} \epsilon_n} \gamma$$

$\Rightarrow$  can have same luminosity by varying  $\beta^{\star} \propto N_b^2$

$\beta^{\star}$ -tuning during collision to maximise integrated luminosity - especially if  $N_b$  can be increased.

Presented by J.M.Jowett, LHC Performance Workshop, Chamonix,  
19/1/2004

# Luminosity in LEP Experiments



Each experiment requested at least the same luminosity as all the others.

## *Luminosity correction in LEP*

---

In LEP closest low  $\beta$  quadrupoles (QS0L and QS0R) were used for correction

- Both Beams Were Equal
- Main Error Source: QS0L QS0R
- Symmetric Lattice

Luminosity correction was easy in LEP

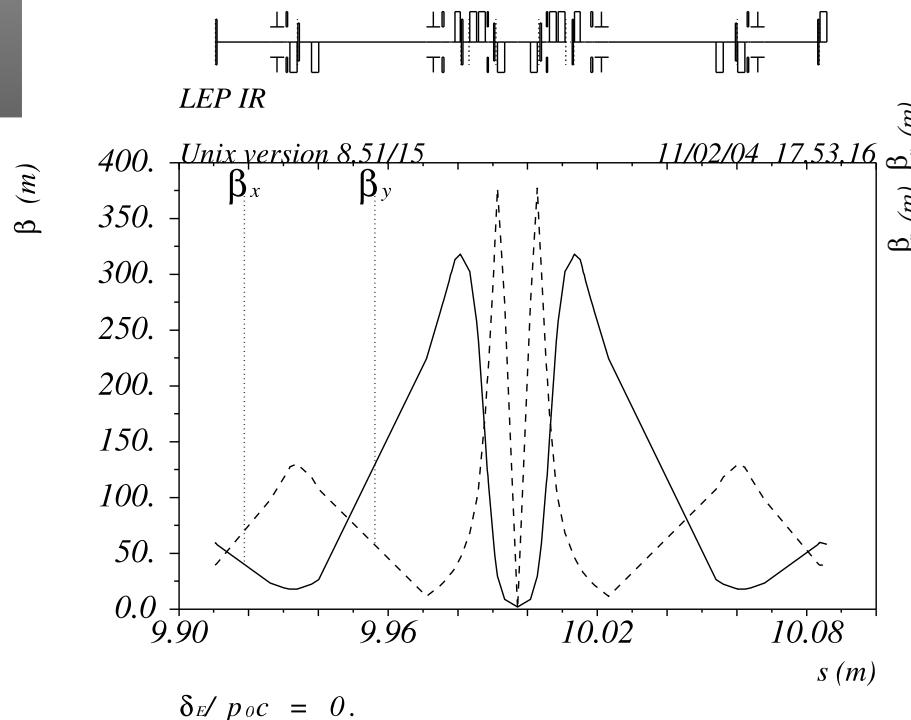
# *Comparison LEP and LHC-RHIC*

---

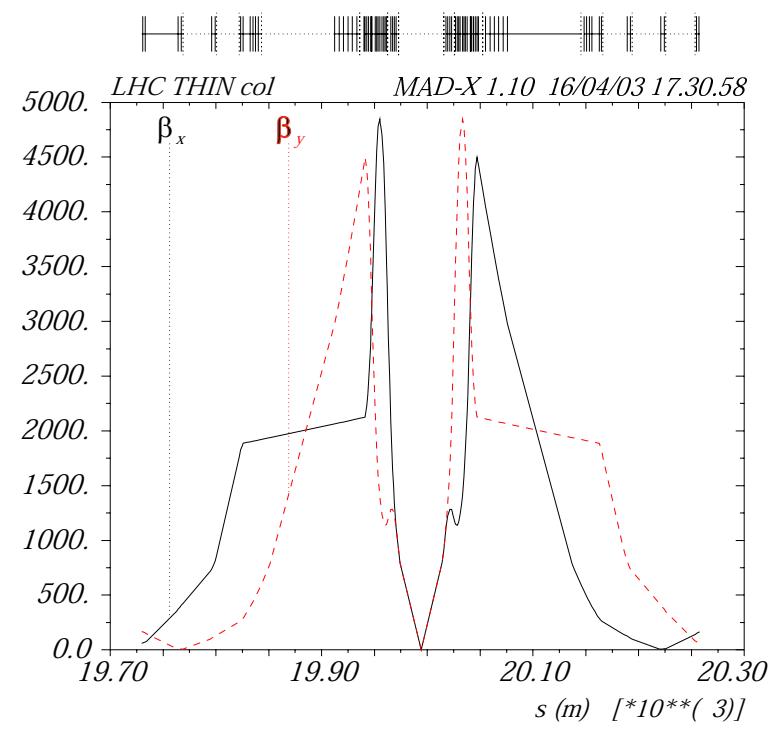
	LEP	LHC - RHIC
beam profile	flat	round
⇒ lattice	symmetric	asymmetric
⇒ correction	1 plane	2 planes
particles	$e^+e^-$	p p
⇒ beam pipe	common	mostly separated
⇒ errors	equal	unequal

# $\beta$ Function LEP LHC

LEP



LHC



# *Motivation - Luminosity correction in LHC-RHIC*

---

Issues of optimizing Luminosity in **LHC-RHIC**:

- triplet magnets cannot be used
- set of quads after separation have to be used
- beating from correction more difficult to avoid
- both planes have to be corrected orthogonally
- error source ???
- ...

If at all possible,  $\beta^*$  correction is extremely sophisticated!

# *Concept of Adjusting Luminosity and Beam Size*

---

$$\begin{aligned}\mathcal{L} &= \frac{N_1 N_2 f N_B}{4\pi \beta_x^* \beta_y^* \epsilon^2} \\ \sigma &= \sqrt{\beta^* \epsilon}\end{aligned}$$

$\mathcal{L}$  ... Luminosity

$N_{1,2}$  ... number of particles per bunch

$N_B$  ... number of bunches per beam

$f$  ... revolution frequency

$\beta_{x,y}$  ... Courant Snyder parameter

$\epsilon$  ... emittance

$\sigma$  ... rms beam size

# ***Principle of Adjusting $\beta^*$***

---

$$\Delta\beta^* = \frac{\beta^*}{2 \sin(2\pi Q)} \sum_{s=1}^n \beta(s) \Delta k(s) l \cos(2|\Delta\mu| - 2\pi Q)$$

$\Delta\beta^*$  ... change of  $\beta$  function at IP

$\beta^*$  ...  $\beta$  function at IP

$Q$  ... tune

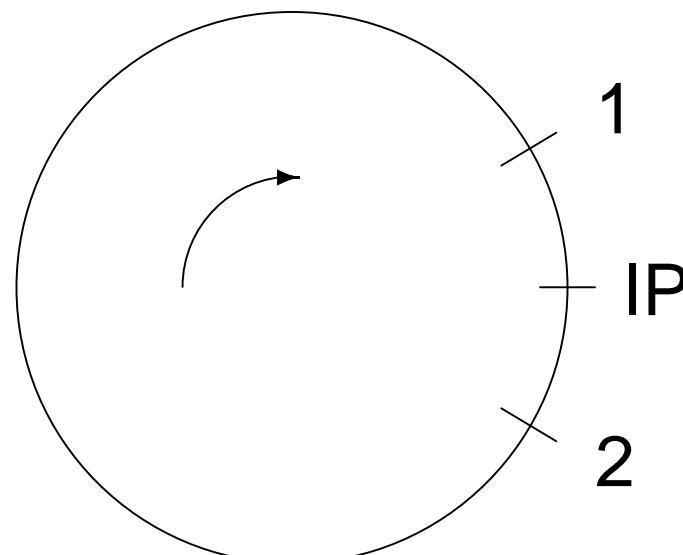
$\beta(s)$  ...  $\beta$  function at a quadrupole

$\Delta k(s)l$  ... change of integrated magnet strength

$\Delta\mu$  ... phase advance between quadrupole and IP

# ***Changes - Constraints***

$$M_{12} = M_{IP-2} \cdot M_{1-IP}$$



$$\left. \begin{array}{lcl} \beta_x^* & = & \beta_x^* + \Delta\beta_x^* \\ \beta_y^* & = & \text{const} \end{array} \right\} \text{at IP}$$

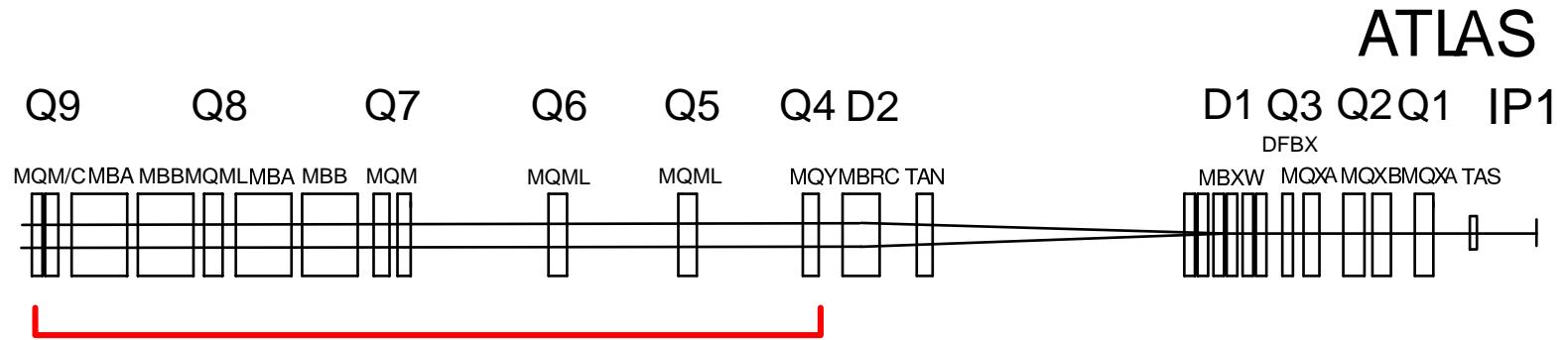
or

$$\left. \begin{array}{lcl} \beta_x^* & = & \text{const} \\ \beta_y^* & = & \beta_y^* + \Delta\beta_y^* \end{array} \right\} \text{at IP}$$

$$\left. \begin{array}{lcl} \beta \\ D \end{array} \right\} = \text{const at } 1,2$$

$$\left. \begin{array}{lcl} \alpha \\ D \end{array} \right\} = \text{const at IP}$$

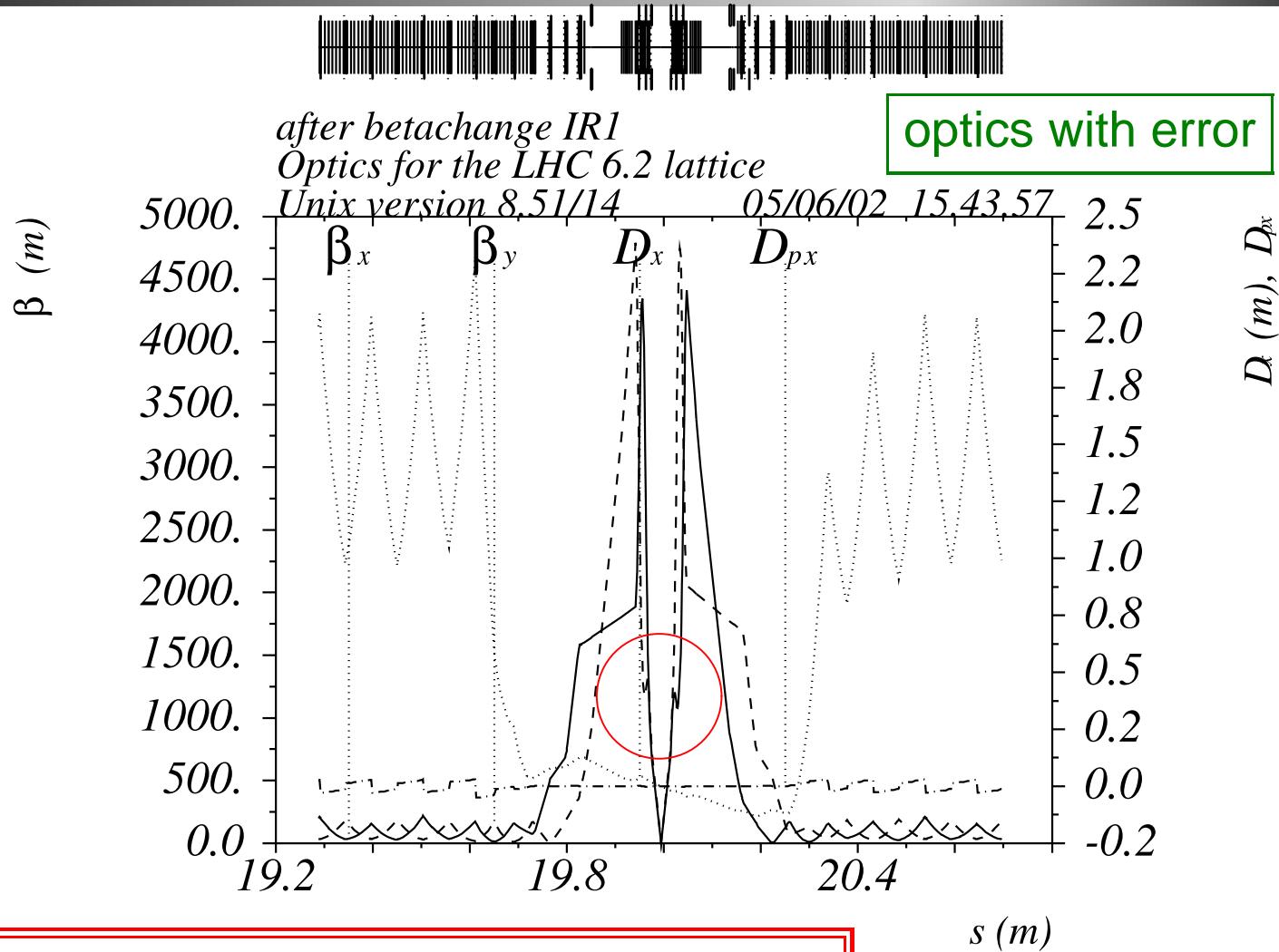
# $\beta^*$ Tuning Knobs for LHC



$$m \cdot \begin{pmatrix} \Delta K_1 \\ \Delta K_2 \\ \Delta K_3 \\ \vdots \\ \Delta K_n \end{pmatrix} \longrightarrow \Delta \beta_{x,y}$$

$$m = 1 \iff \Delta \beta_{x,y} = 1m$$

# *Concept of non Local Correction using Tuning Knobs*



[\*10\*\*(\* 3)]

## *Goal of $\beta^*$ Tuning Knobs*

---

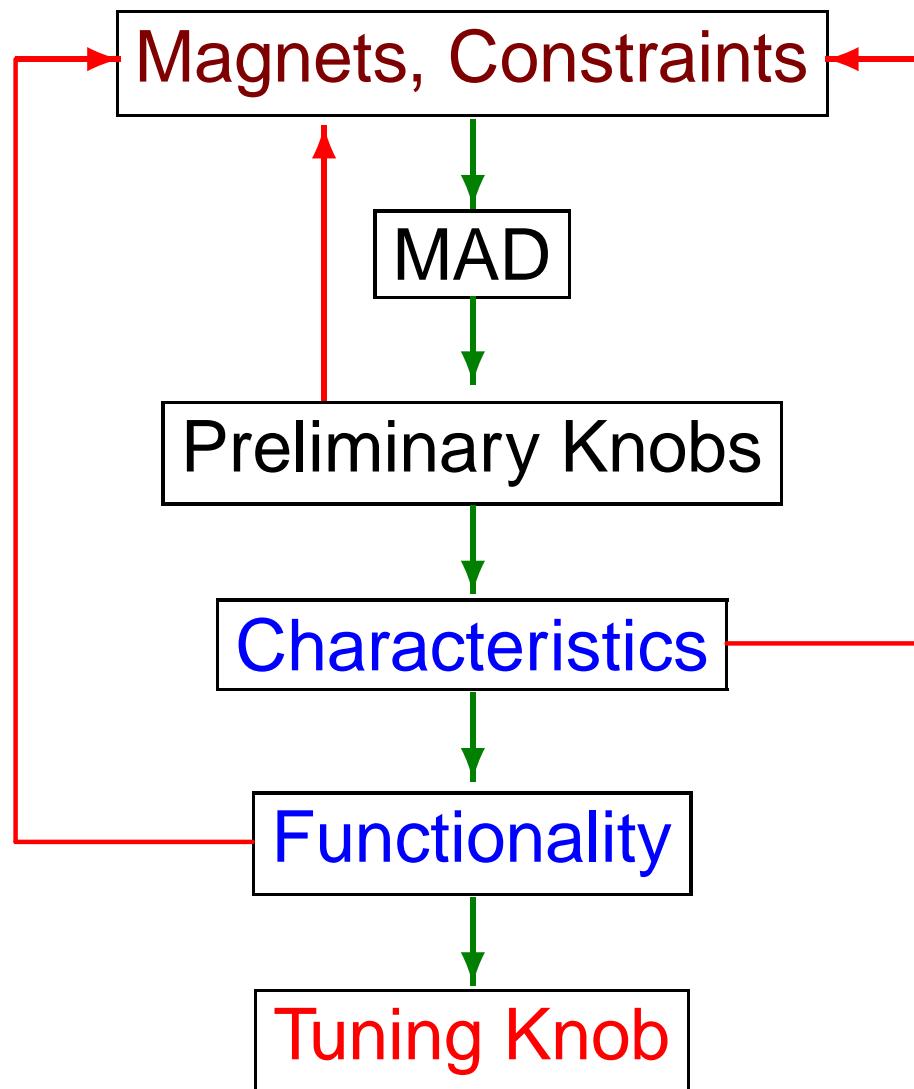
- Change  $\beta$  function at the IP.
- Confine changes to selected area (IR).
- Minimize perturbation to other optics functions.
  - Constraints
  - Observables

# *Methods of Calculating $\beta^*$ Tuning Knobs*

---

- MAD Matching (empirical)
  - direct
  - linear fit  $\Delta K = f(\beta)$
- Response Matrix (analytical)

# **MAD MATCHING**



# Magnets - Constraints for MAD

**MATCH**

	Magnets	Constraints
available	<b>LHC</b> { KQ4-KQ13 R+L  <b>RHIC</b> { K(1-3)A8 K(4-6)A8(I,O) K(4,56)M8, K7A8 K(D,F)AA8, KFBA8	$\alpha, \beta, D, D', X, (pX), Y, (pY)$ at IP1 and IP5 or { $\alpha, \beta, D, D', X, (pX), Y, (pY)$ at IP1 $\beta, D, X, Y,$ at I, II with (I) $\frac{\Delta\mu = \frac{\pi}{4}(2k+1)}{k=0,1\dots}$ (II) and $Q, Q'$
finally used <b>LHC</b>	KQ4 - KQ9 (KQ13) R+L	$\alpha, \beta, D$ at IP1 and IP5
finally used <b>RHIC</b>	KQ4-KQ6, L+R K7A8, KFBA8 KDAA8, KFAA8	$\alpha, \beta, (D)$ at IP8 and IP6 $Q$

**MAD matching: no. magnets = no. constraints (12,10)**

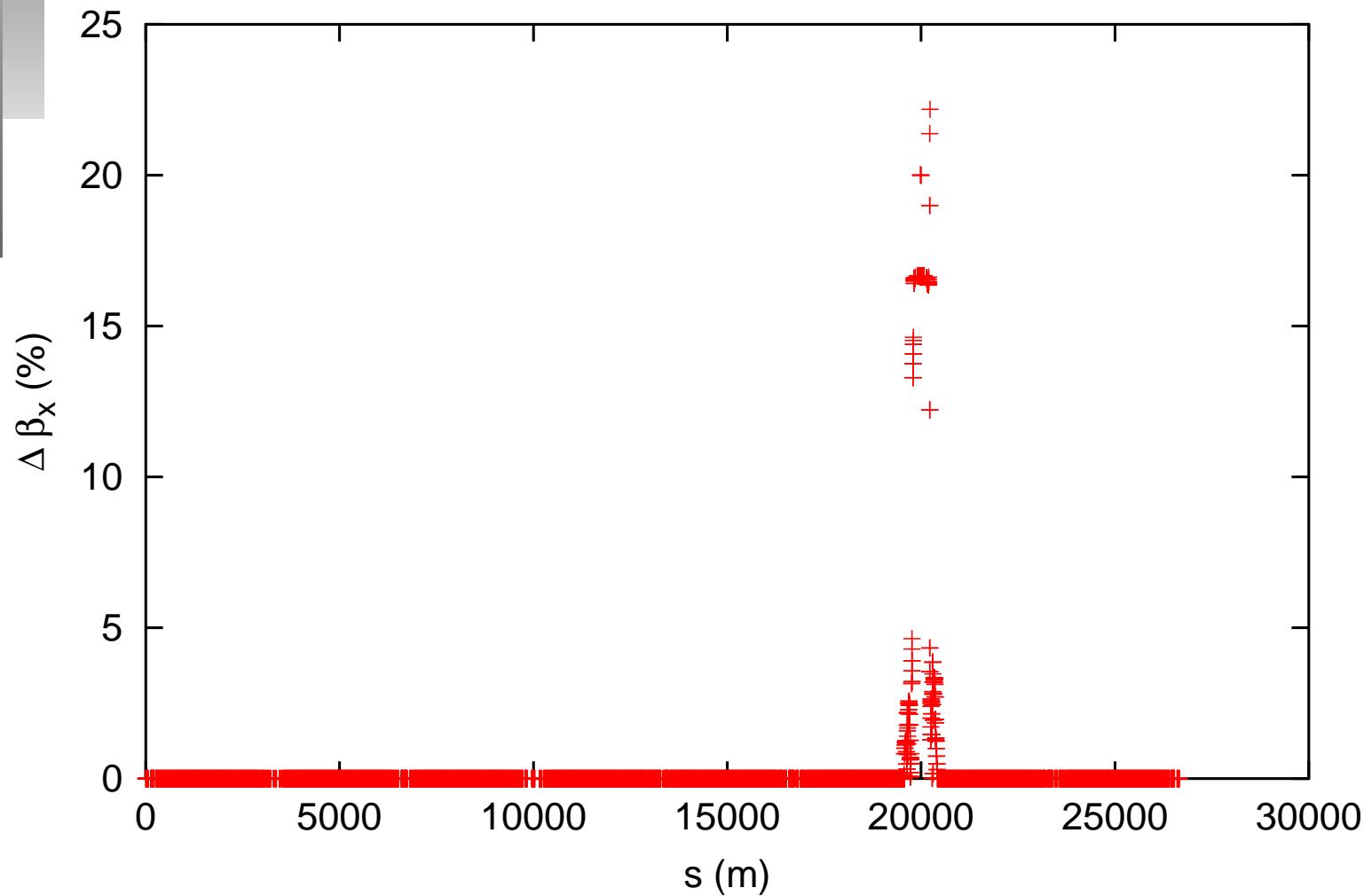
# *Allowed Variation of Constraints & Observables.*

operative range of knobs  $\Rightarrow$  10 % of nominal value ( $\beta^* = 0.5m$ ).

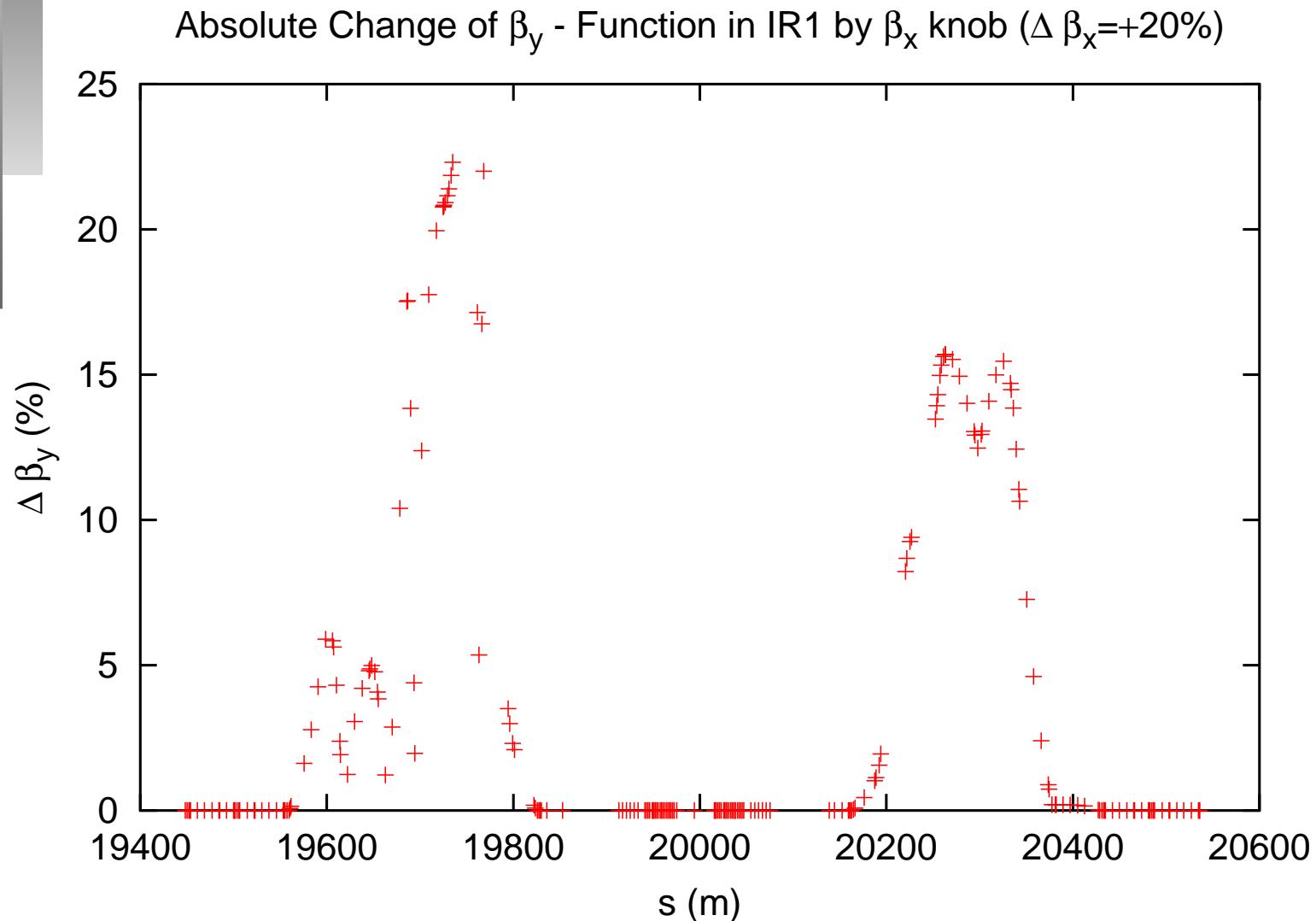
constraint observable	allowed variation
$ \alpha^* $	$< 0.1$
$\beta_{\perp}^*$	$< 1\% \text{ of } \Delta\beta^*$
$ D^* $	$< 3 \cdot 10^{-2} m$
$X^*, Y^*$	$< 16.0 \cdot 10^{-6} m$
$pX^*, pY^*$	$< 10\%$
$Q$	$< 0.025(0.1)$

# ***Global Constraints behavior $\beta_x$***

Absolute Change of  $\beta_x$  - Function in LHC by  $\beta_x$  knob ( $\Delta \beta_x = +20\%$ )

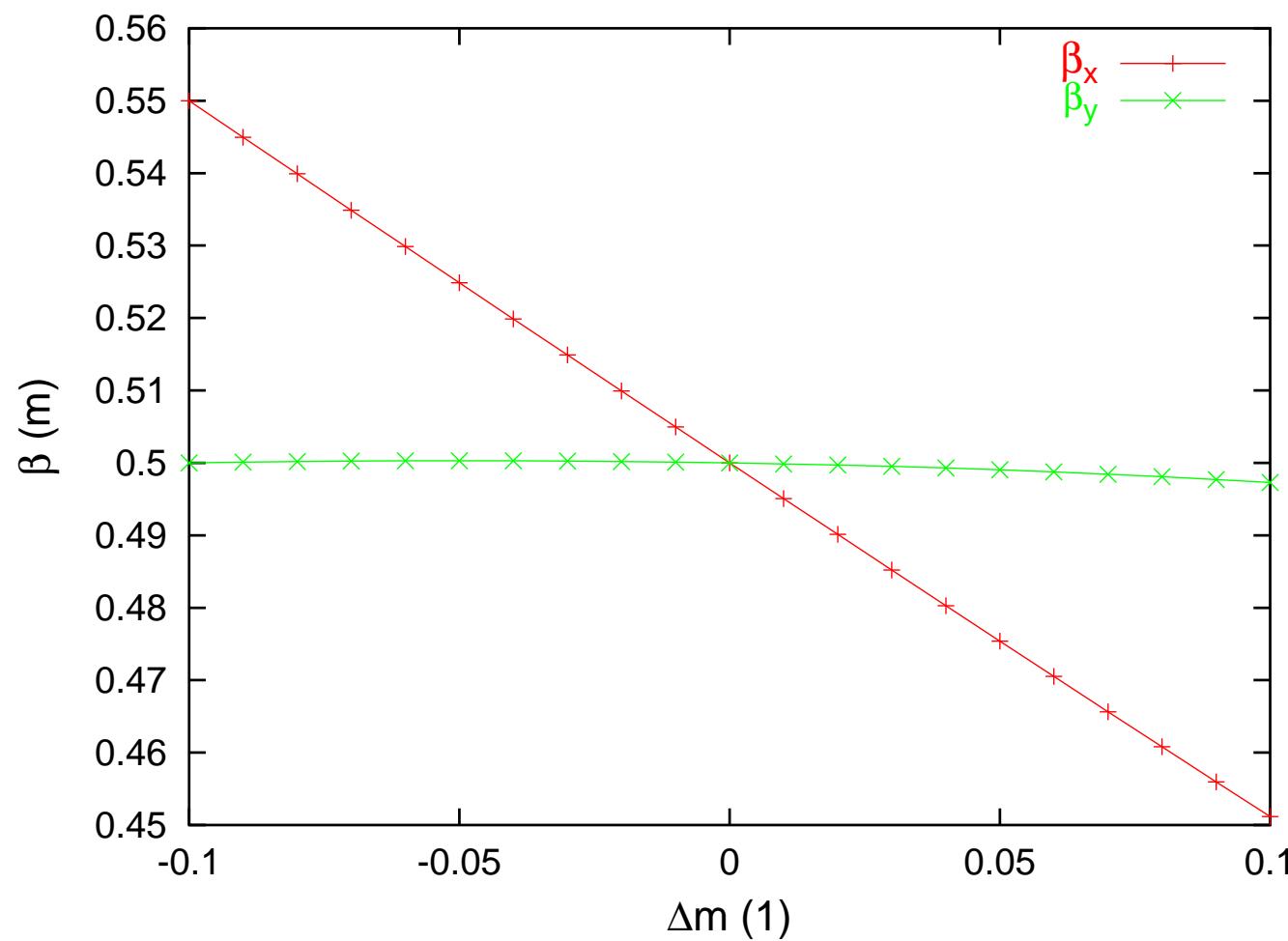


# *IR1 Constraints behavior $\beta_y$*

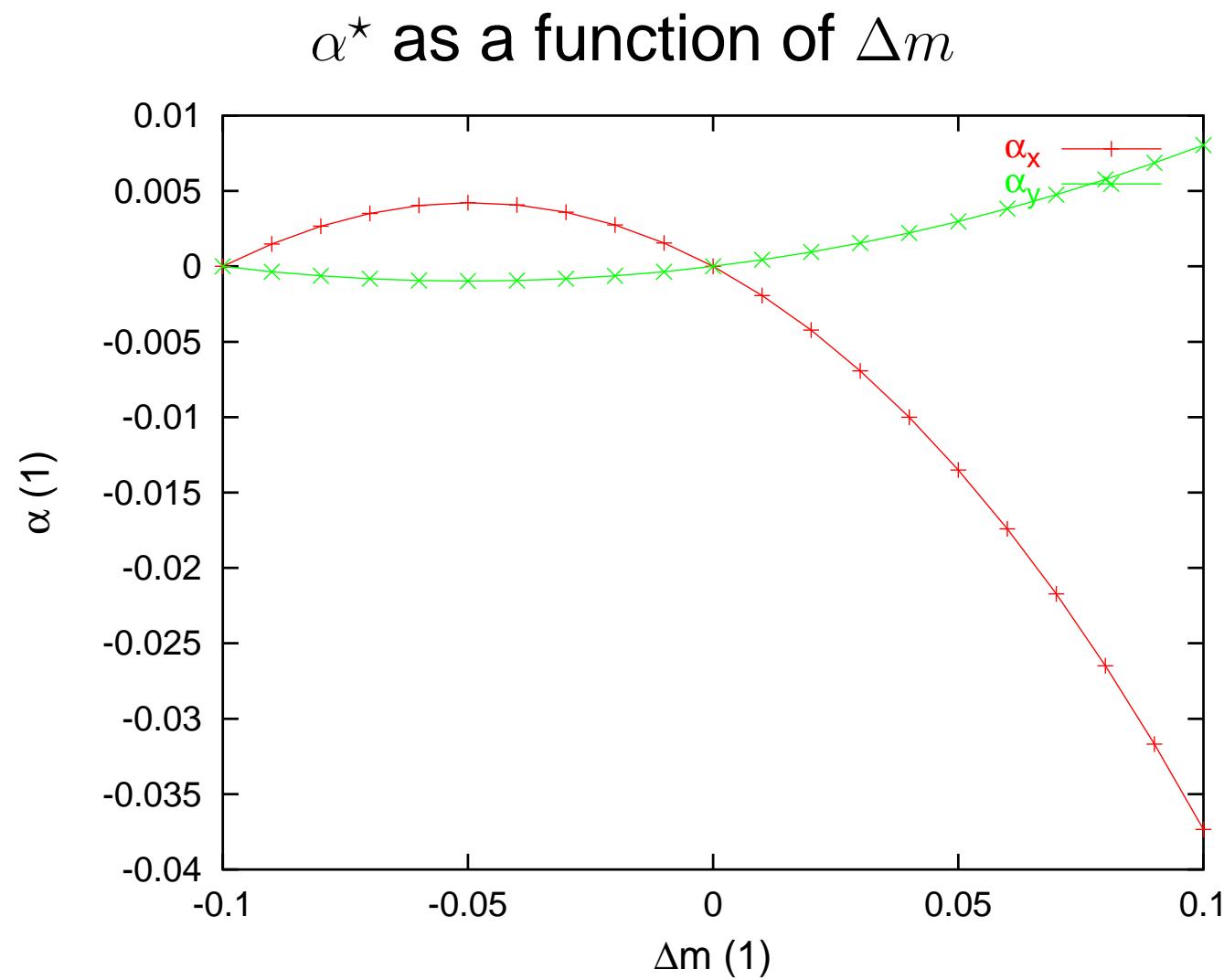


# *Result with Direct Method*

$\beta^*$  as a function of  $\Delta m$



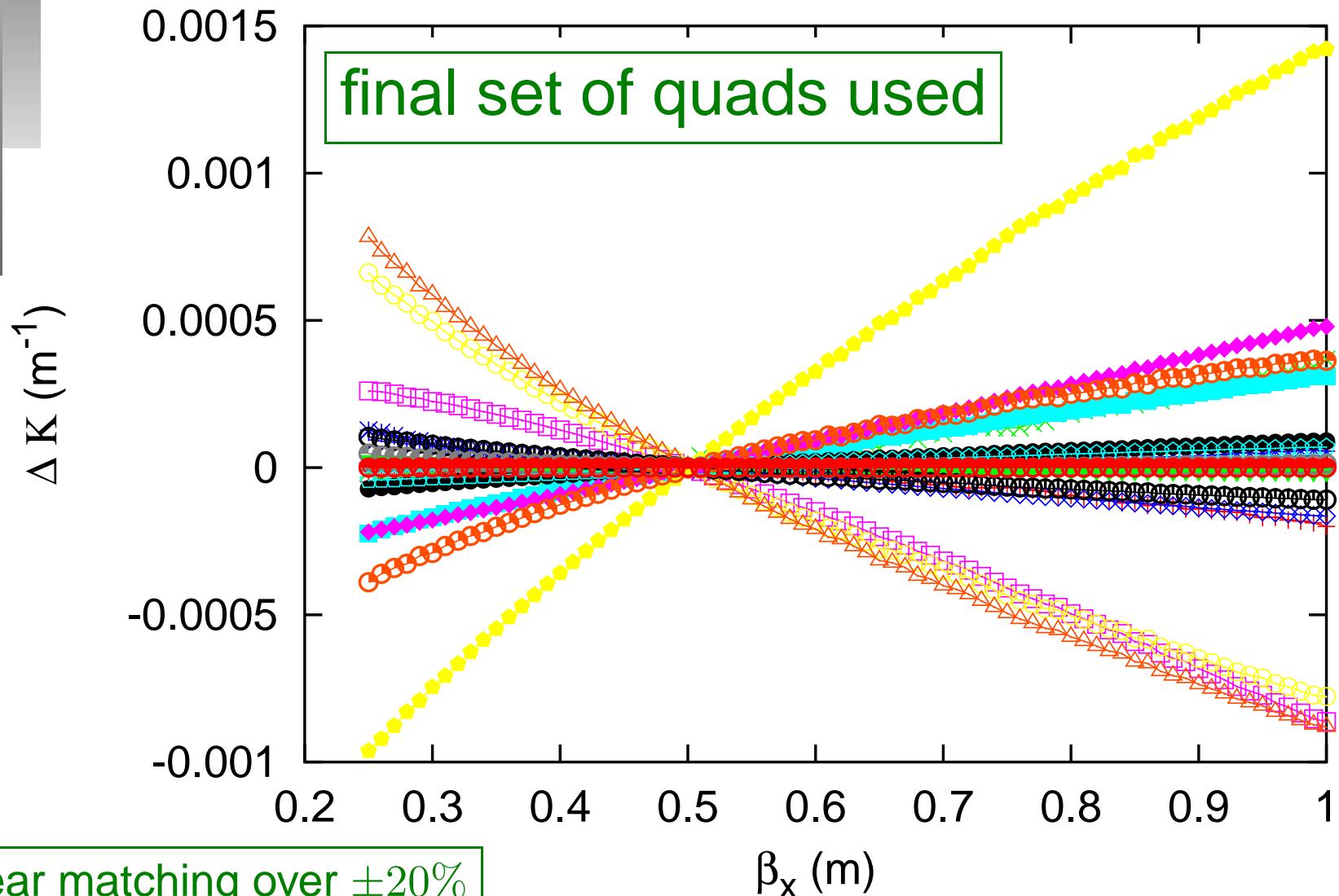
# *Result with Direct Method*



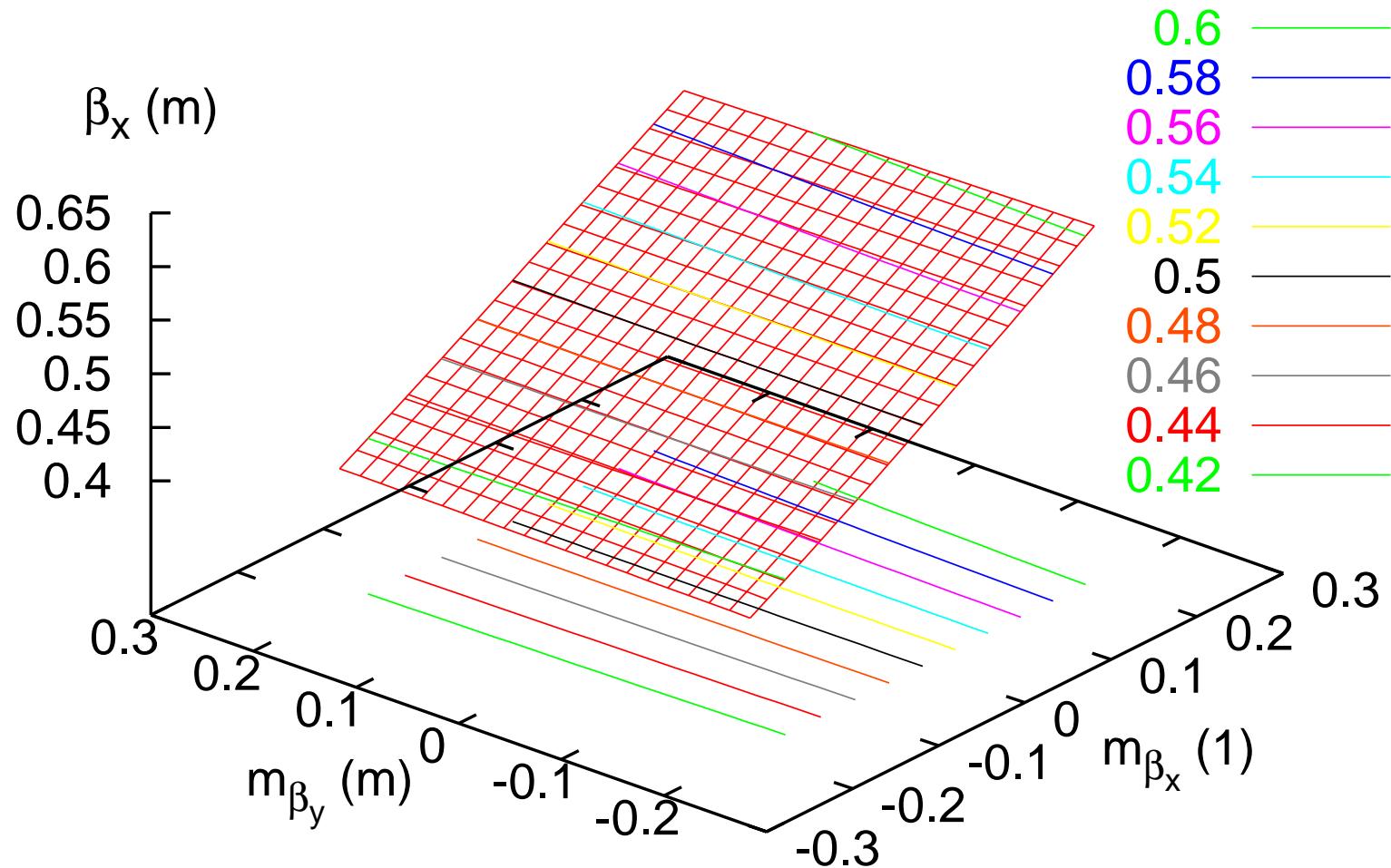
# Matching Results with MAD -

$$\Delta K = f(\beta)$$

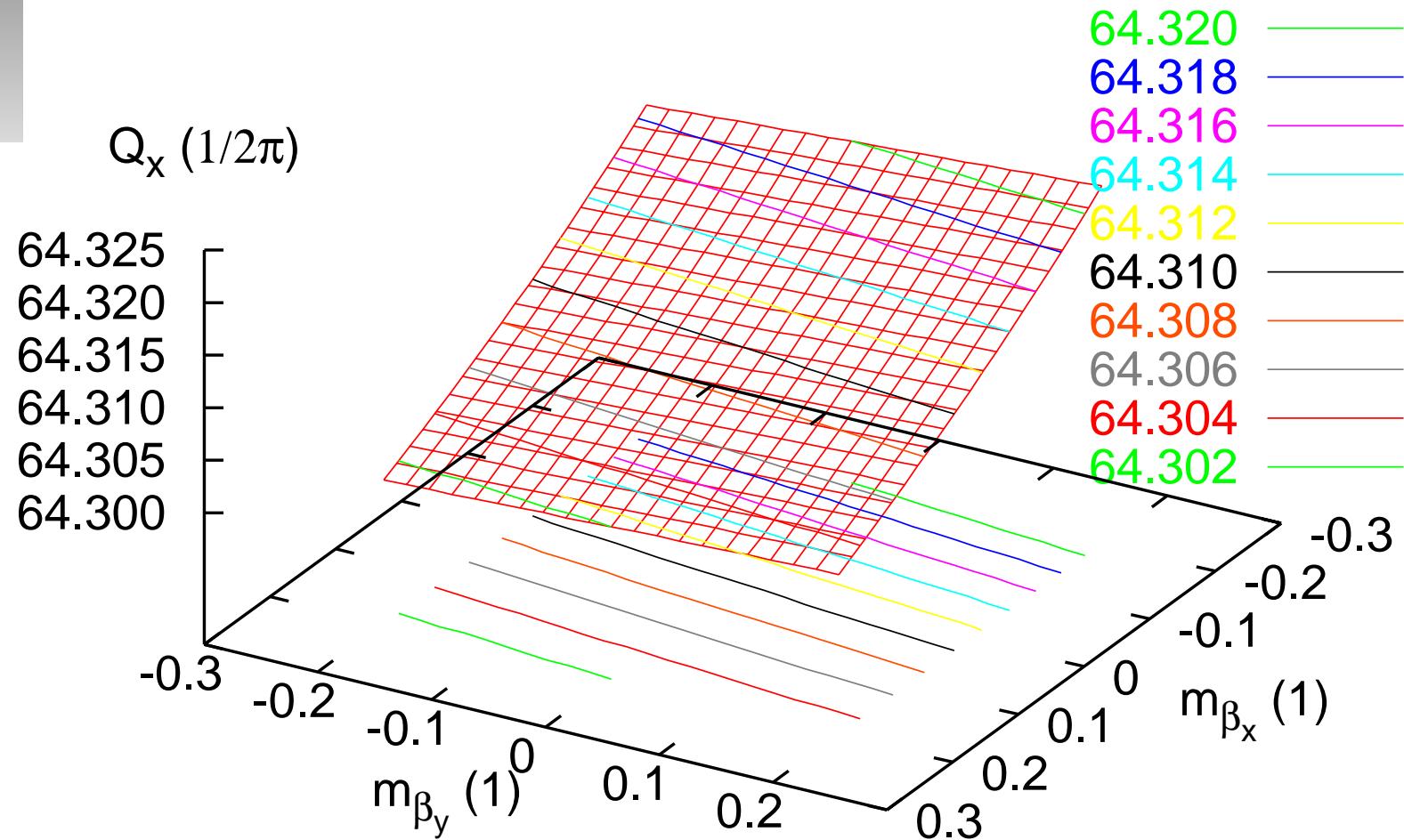
Results for  $\Delta K = f(\beta_x)$  in IP1



**Characteristic:**  $\beta_x = f(m_{\beta_x}, m_{\beta_y})$



**Characteristic:**  $Q_x = f(m_{\beta_x}, m_{\beta_y})$



# Summary Characteristic

---

$K_{\beta_x}$ (m)	$K_{\beta_y}$ (m)	$\beta_x$ (m)	$\beta_y$ (m)	$\alpha_x$ (1)	$\alpha_y$ (1)	$D_x$ (m)	$D_y$ (m)	$Dp_x$ (1)
-0.1	-0.1	4.00e-01	4.06e-01	3.16e-02	-3.45e-02	6.14e-03	6.72e-03	3.20e-02
0.1	-0.1	6.05e-01	4.10e-01	2.15e-02	-1.76e-02	1.50e-02	6.53e-03	3.23e-02
-0.0	0.0	5.00e-01	5.00e-01	2.71e-09	-4.20e-09	1.03e-02	6.73e-03	3.11e-02
-0.1	0.1	4.05e-01	6.14e-01	2.20e-02	-2.35e-02	6.19e-03	7.29e-03	2.86e-02
0.1	0.1	5.97e-01	6.05e-01	2.42e-02	-3.61e-02	1.46e-02	7.30e-03	2.91e-02

$K_{\beta_x}$ (m)	$K_{\beta_y}$ (m)	$Dp_y$ (1)	X (m)	Y (m)	pX (1)	pY (1)	$Q_x$ ( $1/2\pi$ )	$Q_y$ ( $1/2\pi$ )
-0.1	-0.1	2.14e-02	5.25e-13	3.56e-08	-5.80e-13	1.50e-04	64.3187	59.3240
0.1	-0.1	2.15e-02	8.55e-13	1.56e-07	-3.59e-13	1.50e-04	64.3012	59.3252
-0.0	0.0	1.92e-02	5.38e-13	-1.53e-10	-7.02e-13	1.50e-04	64.3100	59.3200
-0.1	0.1	1.72e-02	4.92e-13	-1.55e-07	-1.90e-13	1.49e-04	64.3215	59.3158
0.1	0.1	1.79e-02	5.29e-13	-3.74e-08	-8.30e-13	1.50e-04	64.3027	59.3140

$K_{\beta_x}$ (m)	$K_{\beta_y}$ (m)	$Bmag_x$ (1)	$\beta_y beat_{max}$ (%)	$Bmag_y$ (1)	$\beta_y beat_{max}$ (%)
-0.1	-0.1	1.000101	0.014218	1.000035	0.008416
0.1	-0.1	1.000064	0.011279	1.000005	0.003278
-0.0	0.0	1	0	1	0
-0.1	0.1	1.000049	0.009932	1.000004	0.002981
0.1	0.1	1.000127	0.015964	1.000073	0.012092

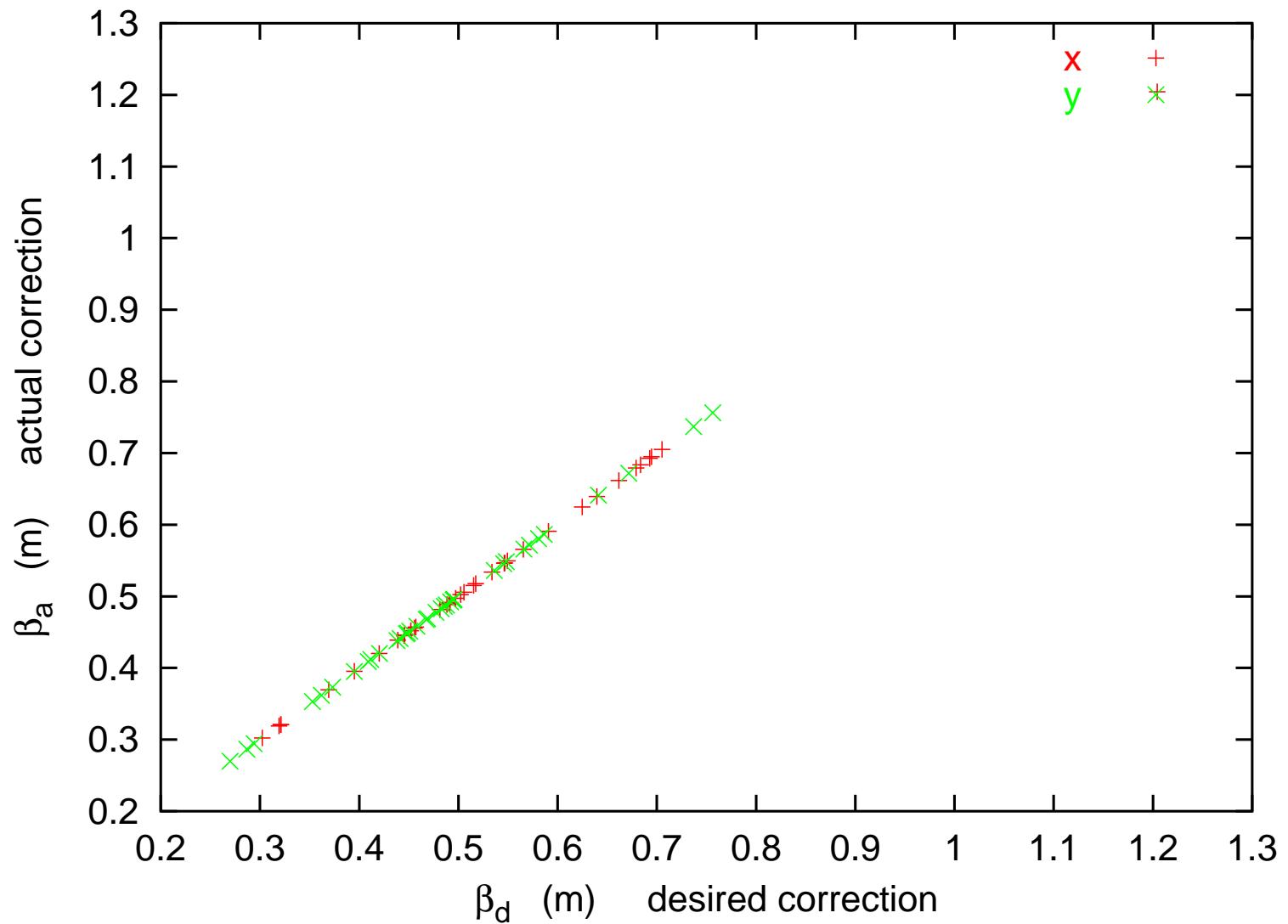
# *Functionality: Error Test*

## Main Area of Application

error source	field error type
main arc quadrupoles	MQ
triplet quadrupoles	MQX

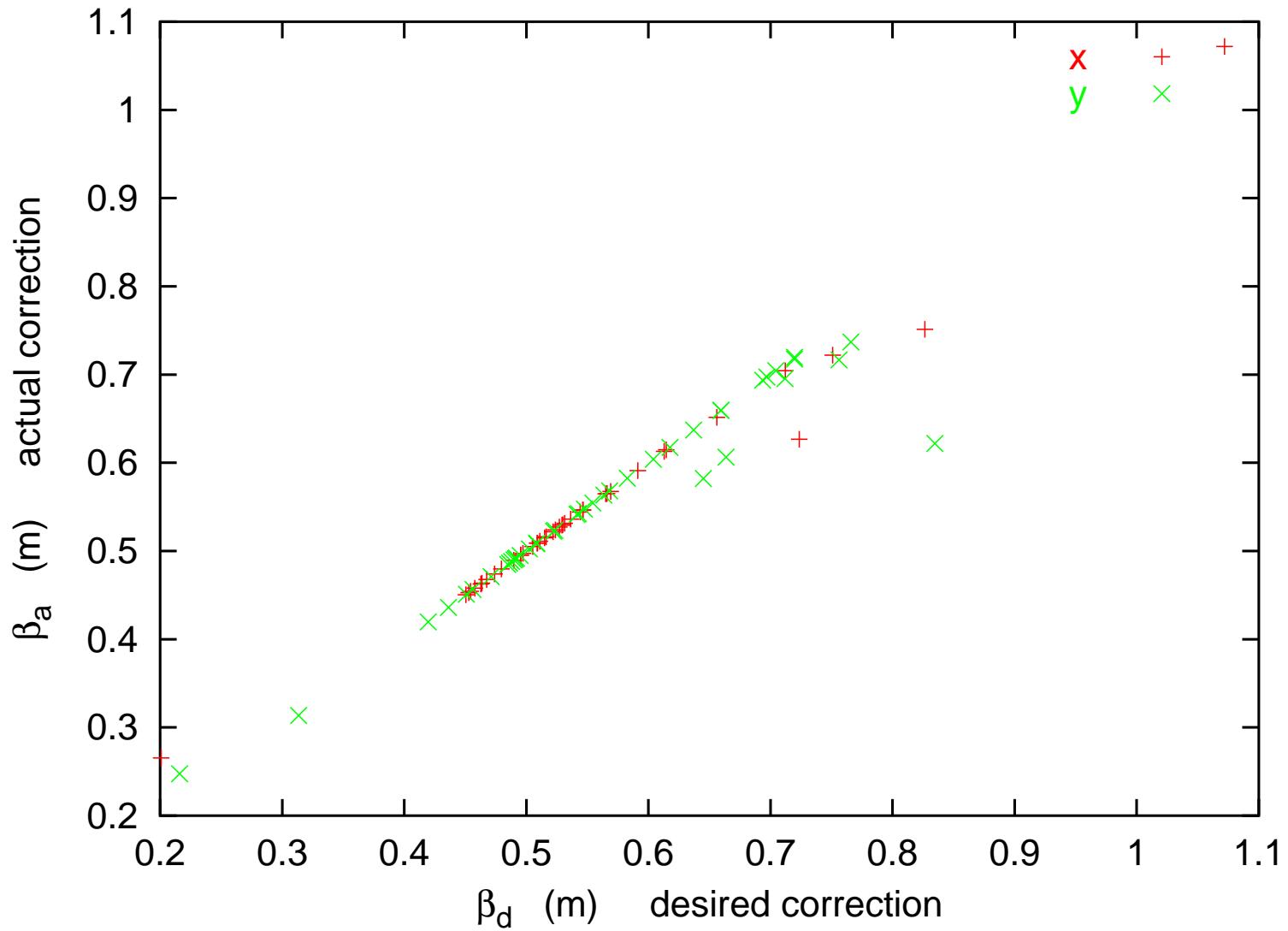
tested were: b1 - b12, a2 - a12

# Evaluate Results MQ



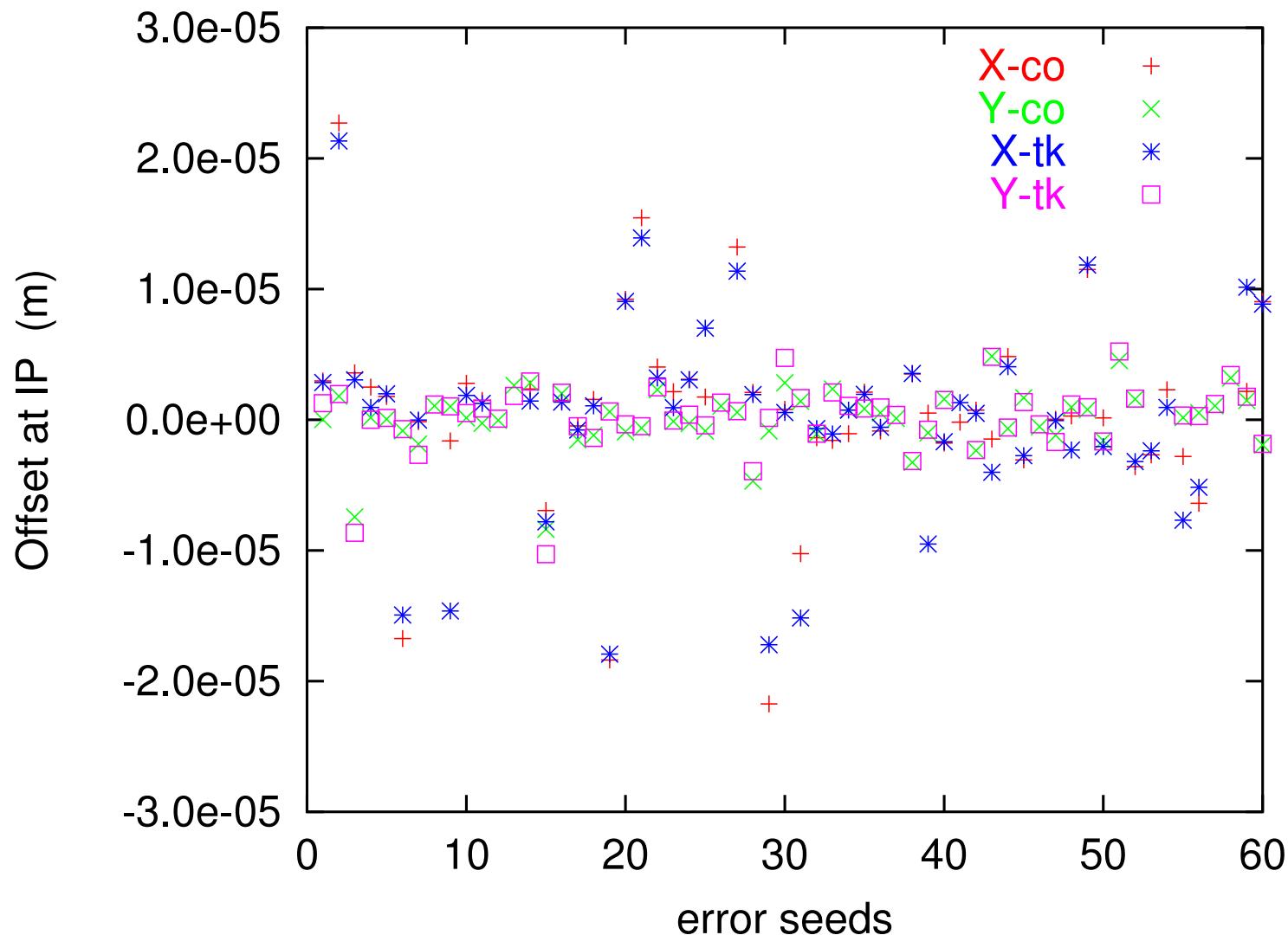
all error seeds corrected

# Evaluate Results MQX



error seeds with  $\Delta\beta < \pm 20\%$  corrected

# Evaluate Results *Steering Effect of Knobs*



All steering errors caused by knobs  $< 16\mu\text{m}$ .

# ***Monitoring Critical Points***

---

Monitoring changes of constraints created by applying knobs for correction in IP1:

- Experiments IP2, IP5, IP8
- acceleration cavities in IR4
- cleaning insertions IP3, IP7

$$|\Delta\beta_{max}| < 2(\%)$$

$$|\Delta D_{max}| < 0.7(m)$$

## *Summary MAD Match*

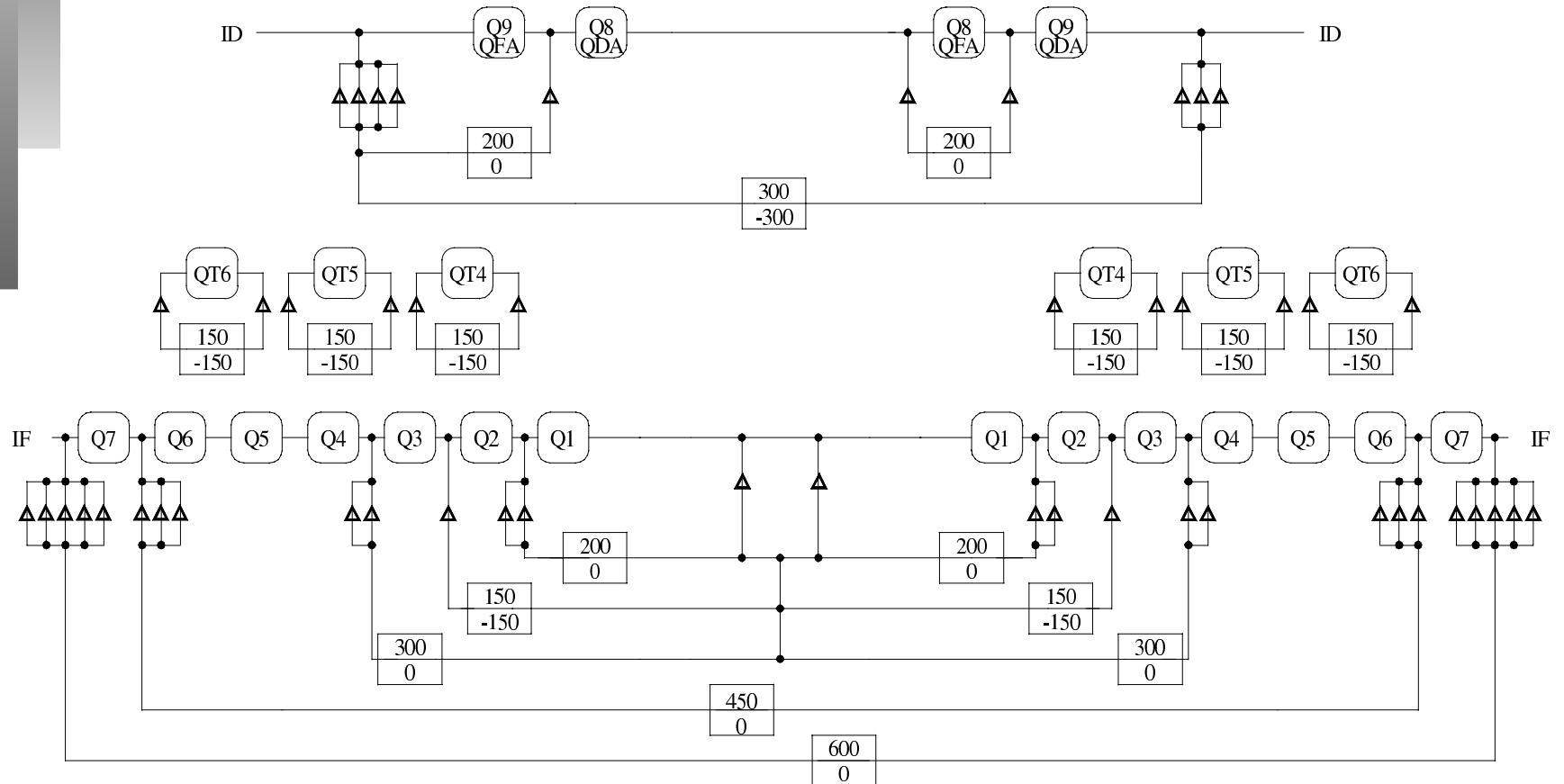
---

- **Characteristics:** Knobs achieve desired changes in a range of  $\pm 20\%$  of nominal value. The perturbations of other optics function lie within the specification.
- **Correction Efficiency:** All errors coming from the arc can be corrected. Errors from triplet magnets can be corrected within a range of  $\pm 20\%$ .

# *Comparison RHIC - LHC*

item	accelerators	
	RHIC	LHC
lattice	asymmetric	
beam pipe	mostly separated	
beam pipe triplet	separate	common
Xing	no	yes
power supply	nested	separate

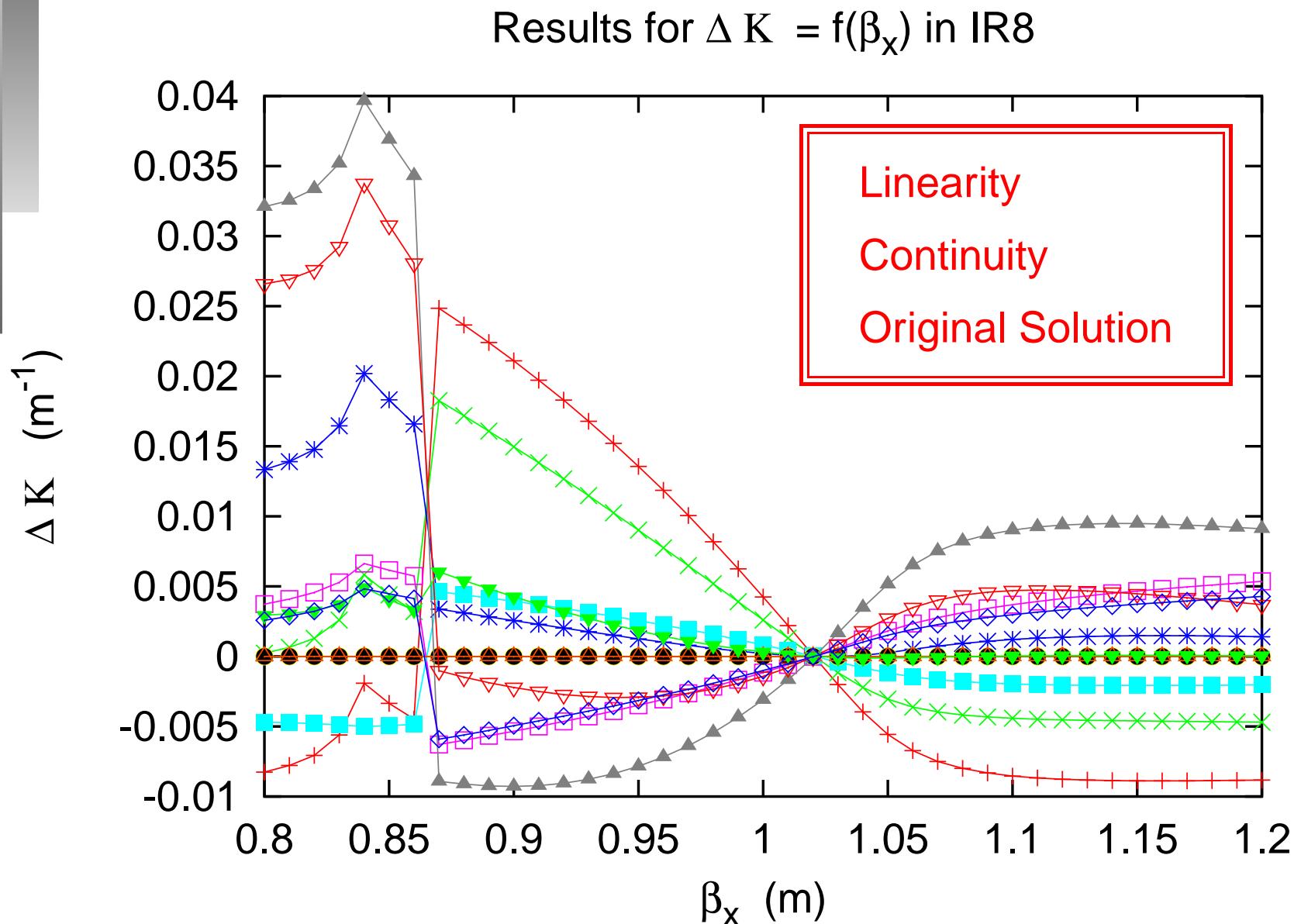
# RHIC - Nested Power Supply System



PENETRATION SYMBOLS

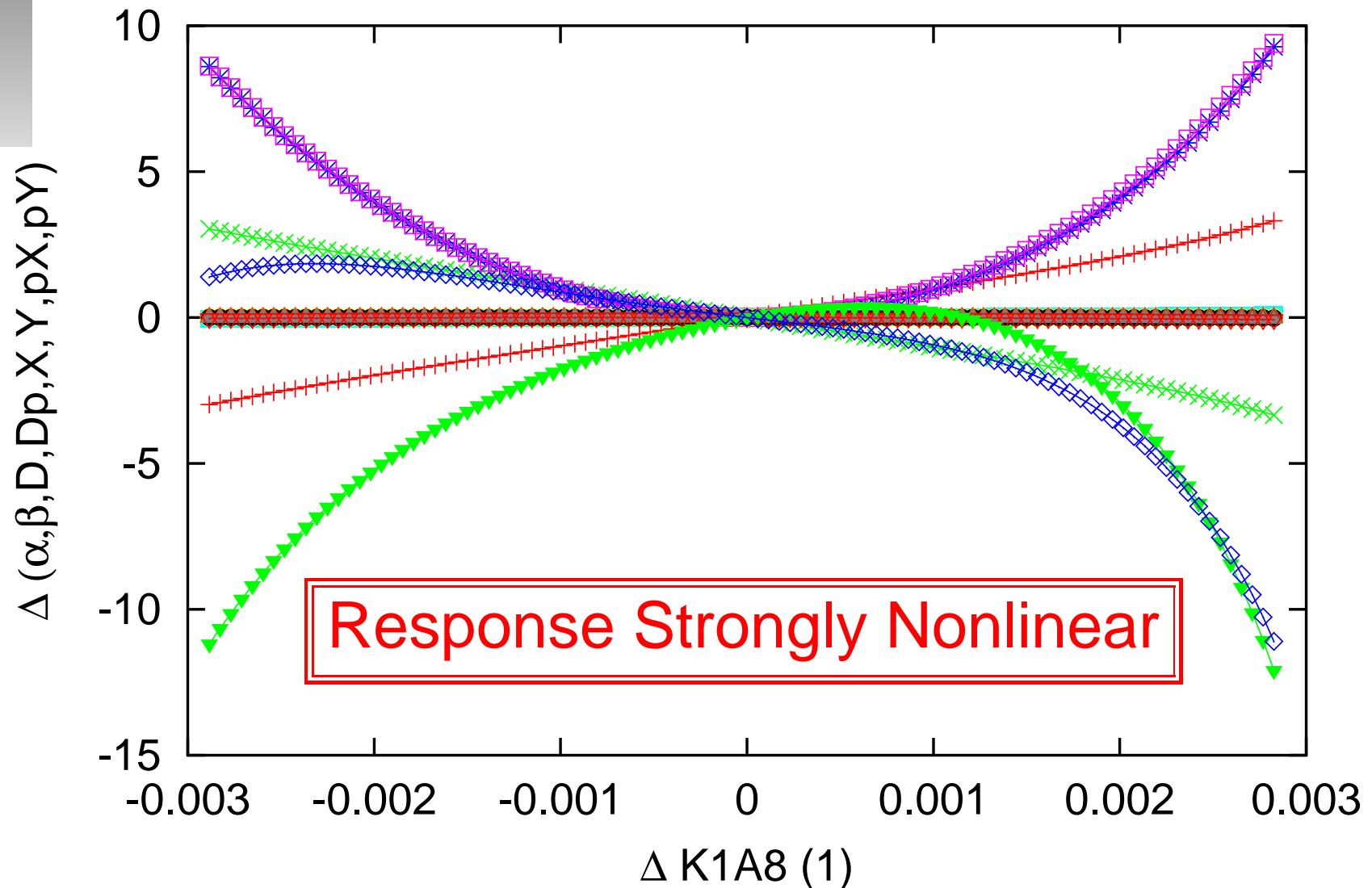
6600	2000	150

# RHIC - MAD Matching Results



# $\Delta K_{Quad}$ Response

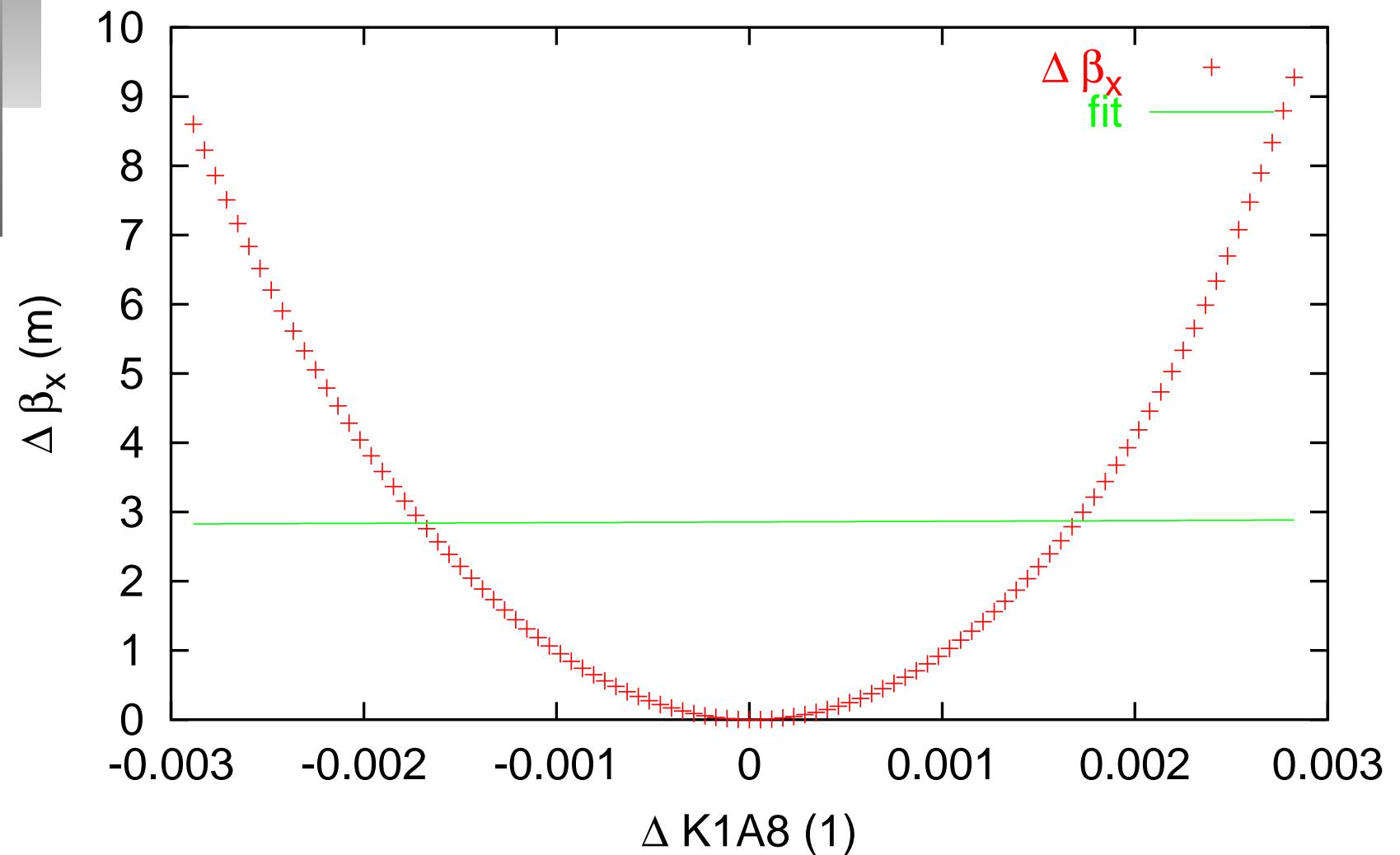
Results for Parameters  $(\alpha, \beta, D, Dp, X, Y, pX, pY) = f(\Delta K1A8)$  in IR1



# *Response Matrix Analysis - Magnet*

## *Category 1*

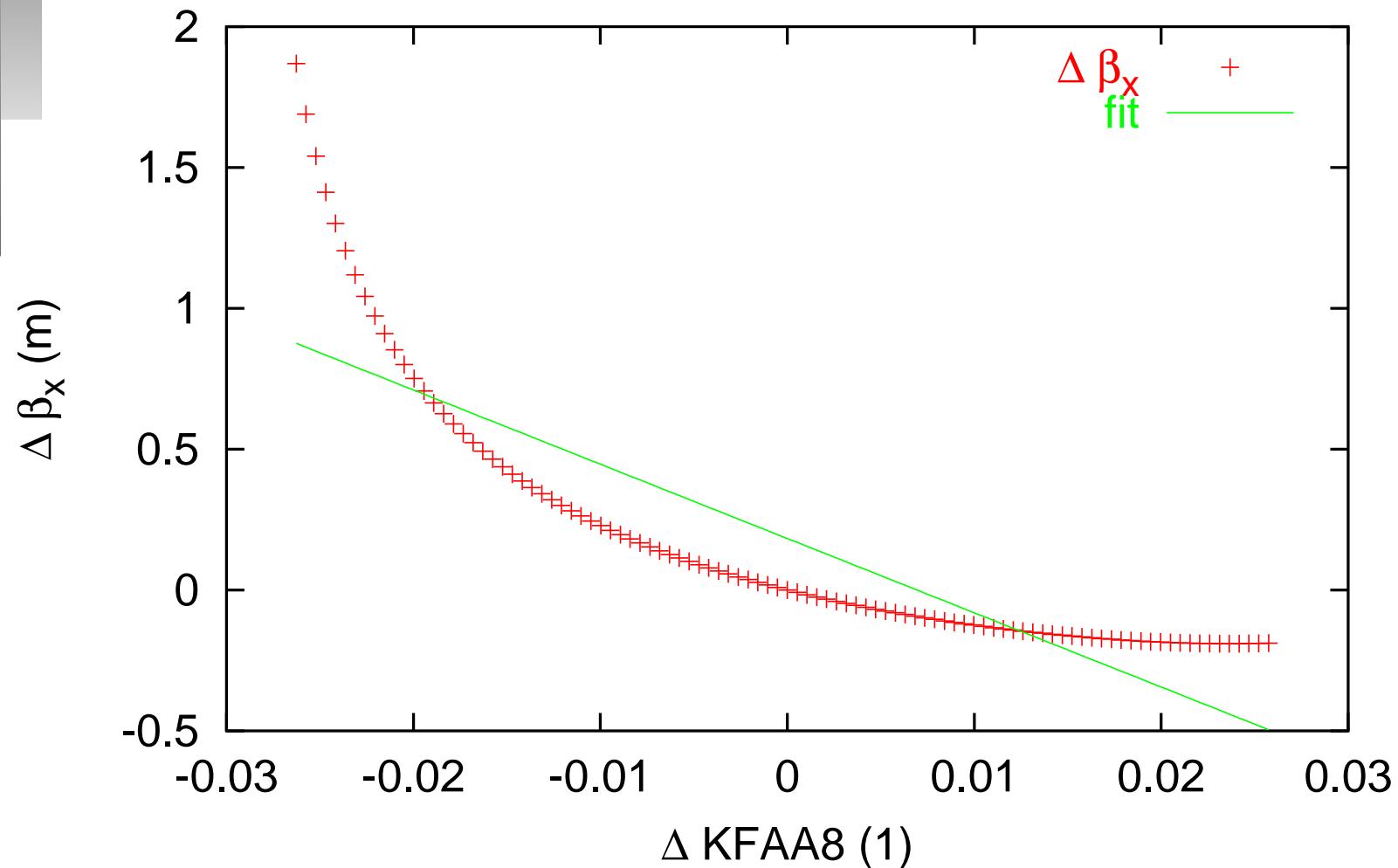
Results for  $\Delta \beta_x = f(\Delta K1A8)$  in IR8. Plot results for linear fit:  $f(x) = k^*x + b$   
 $k = 9.74396$ .  $b = 2.85552$



# *Response Matrix Analysis - Magnet*

## *Category 2*

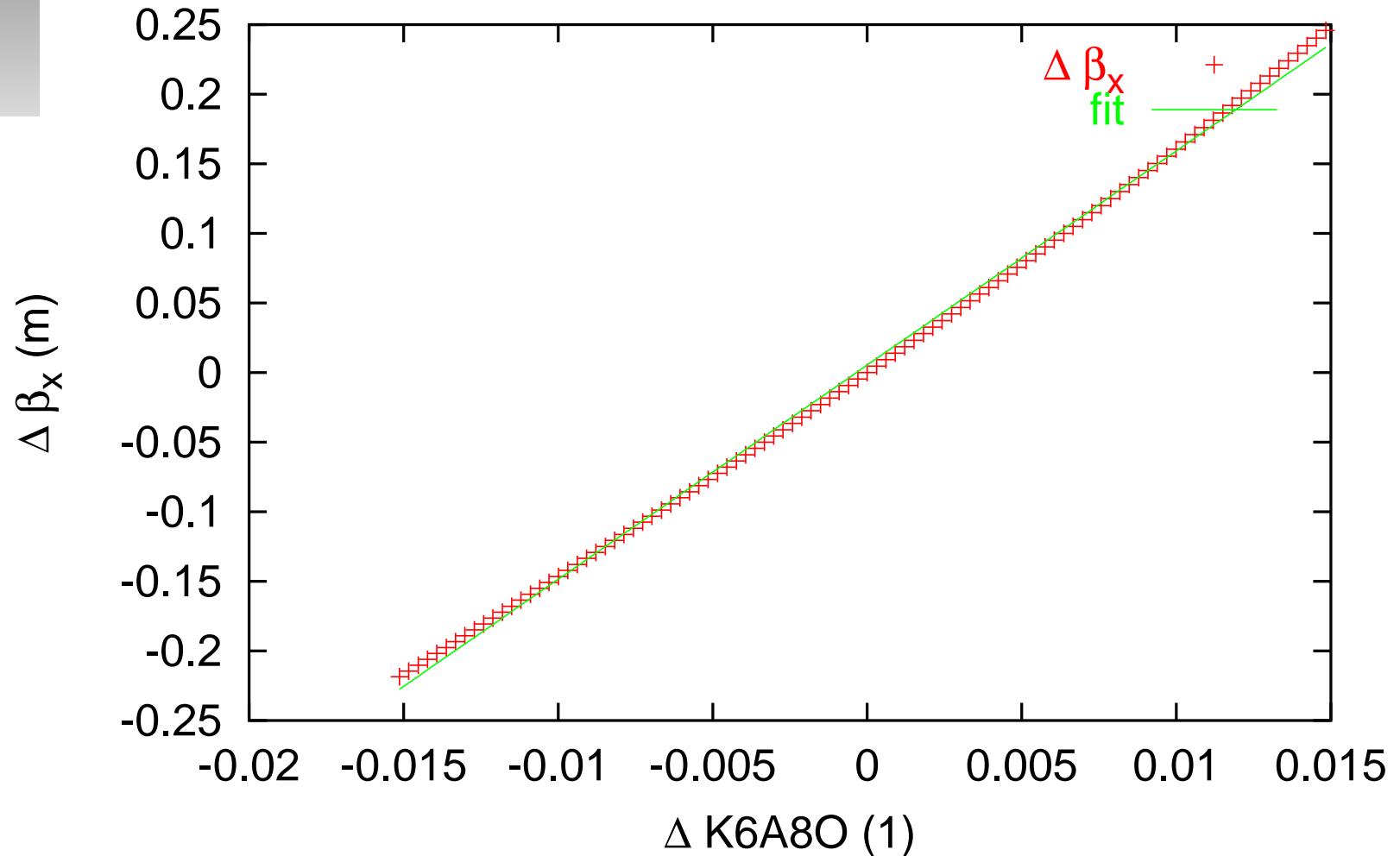
Results for  $\Delta \beta_x = f(\Delta KFAA8)$  in IR8. Plot results for linear fit:  $f(x) = k^*x + b$   
 $k = -26.3685$ .  $b = 0.182956$



# *Response Matrix Analysis - Magnet*

## *Category 3*

Results for  $\Delta \beta_x = f(\Delta K6A8O)$  in IR8. Plot results for linear fit:  $f(x) = k*x + b$   
 $k = 15.391$ .  $b = 0.00538547$

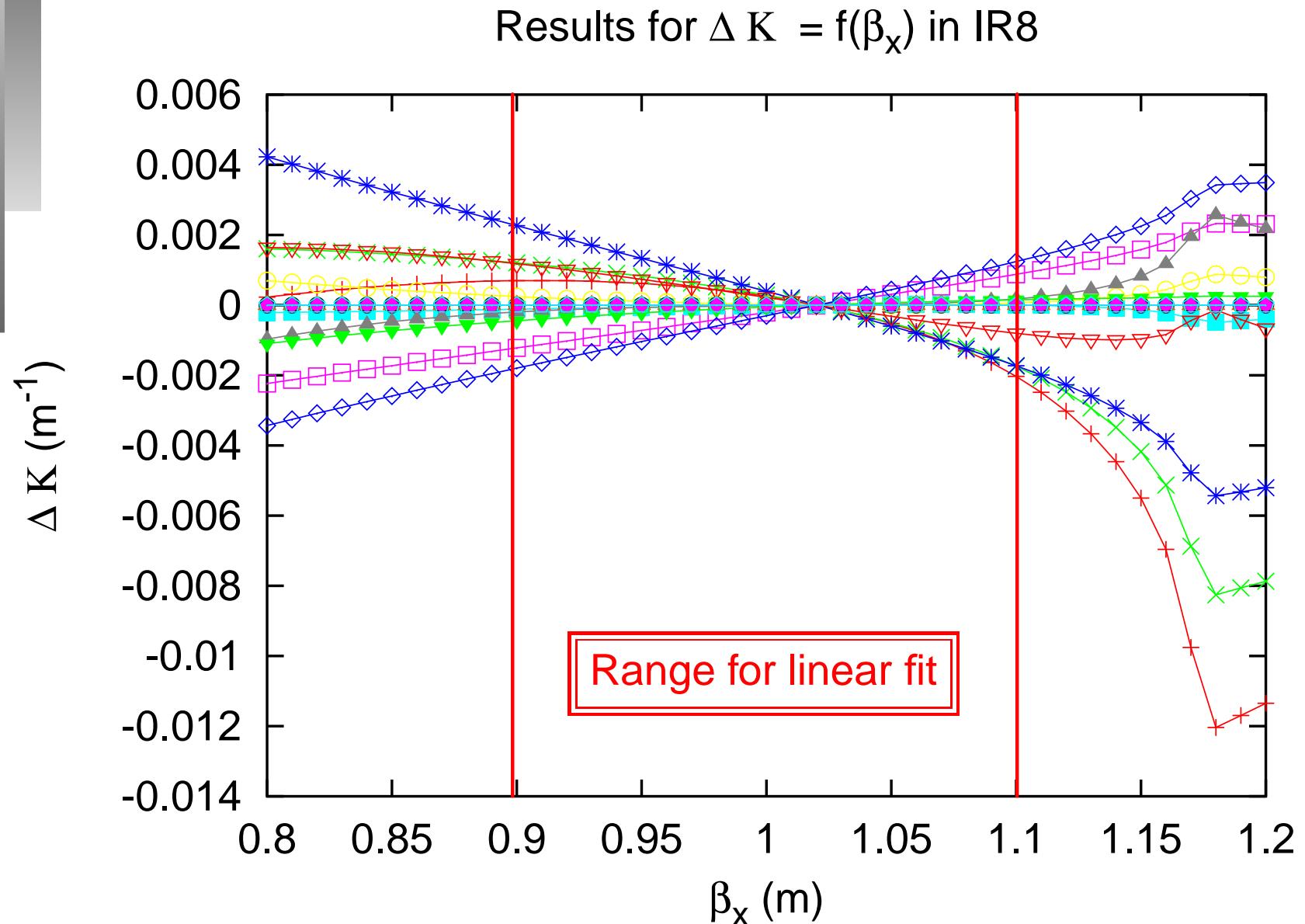


# *Result from Response Analysis*

---

name	cat	usable range	name	cat	usable range
K1A8	1	-	K5A8O	3	$\pm 50\%$
K2A8	1	-	K6A8I	3	$\pm 50\%$
K3A8	1	-	K6A8O	3	$\pm 50\%$
K4A8I	3	$\pm 50\%$	K7A8	2	$\pm 5\%$
K4A8O	3	$\pm 50\%$	KDAA8	2	$\pm 5\%$
K4M8	1	-	KFAA8	2	$\pm 5\%$
K56M8	1	-	KFBA8	2	$\pm 5\%$
K5A8I	3	$\pm 50\%$			

# **Result $\Delta K_{Quad}$ Response Combined with MAD MATCHING**



# *Benefit of Response Matrix Analysis*

---

- learn about the behavior of parameters  $\Rightarrow$  choice of constraints.
- try inversion to solve the problem but remember **non linear problem**

# **Response Matrix**

$$\mathbf{R} = \begin{pmatrix} \frac{\partial \alpha_x}{\partial K_{K1A8}} & \frac{\partial \alpha_x}{\partial K_{K2A8}} & \frac{\partial \alpha_x}{\partial K_{K3A8}} & \cdots & \cdots & \cdots & \frac{\partial \alpha_x}{\partial K_{KDAA8}} \\ \frac{\partial \alpha_y}{\partial K_{K1A8}} & \frac{\partial \alpha_y}{\partial K_{K2A8}} & \frac{\partial \alpha_y}{\partial K_{K3A8}} & \cdots & \cdots & \cdots & \frac{\partial \alpha_y}{\partial K_{KDAA8}} \\ \frac{\partial \beta_x}{\partial K_{K1A8}} & \frac{\partial \beta_x}{\partial K_{K2A8}} & \frac{\partial \beta_x}{\partial K_{K3A8}} & & & & \frac{\partial \beta_x}{\partial K_{KDAA8}} \\ \vdots & \vdots & \ddots & & & & \vdots \\ \vdots & \ddots & \ddots & & & & \vdots \\ \vdots & & & & & & \vdots \\ \frac{\partial dQ_2}{\partial K_{K1A8}} & \frac{\partial dQ_2}{\partial K_{K2A8}} & \frac{\partial dQ_2}{\partial K_{K3A8}} & \cdots & \cdots & \cdots & \frac{\partial dQ_2}{\partial K_{KDAA8}} \end{pmatrix}$$

$$\vec{\Delta K} = \mathbf{R}^{-1} \vec{\Delta P}$$

# *Methods of Matrix Inversion*

---

- Direct Inversion (numerical)
  - condition of matrix
  - dimensions of matrix
- Singular Value Decomposition

# **SVD Singular Value Decomposition**

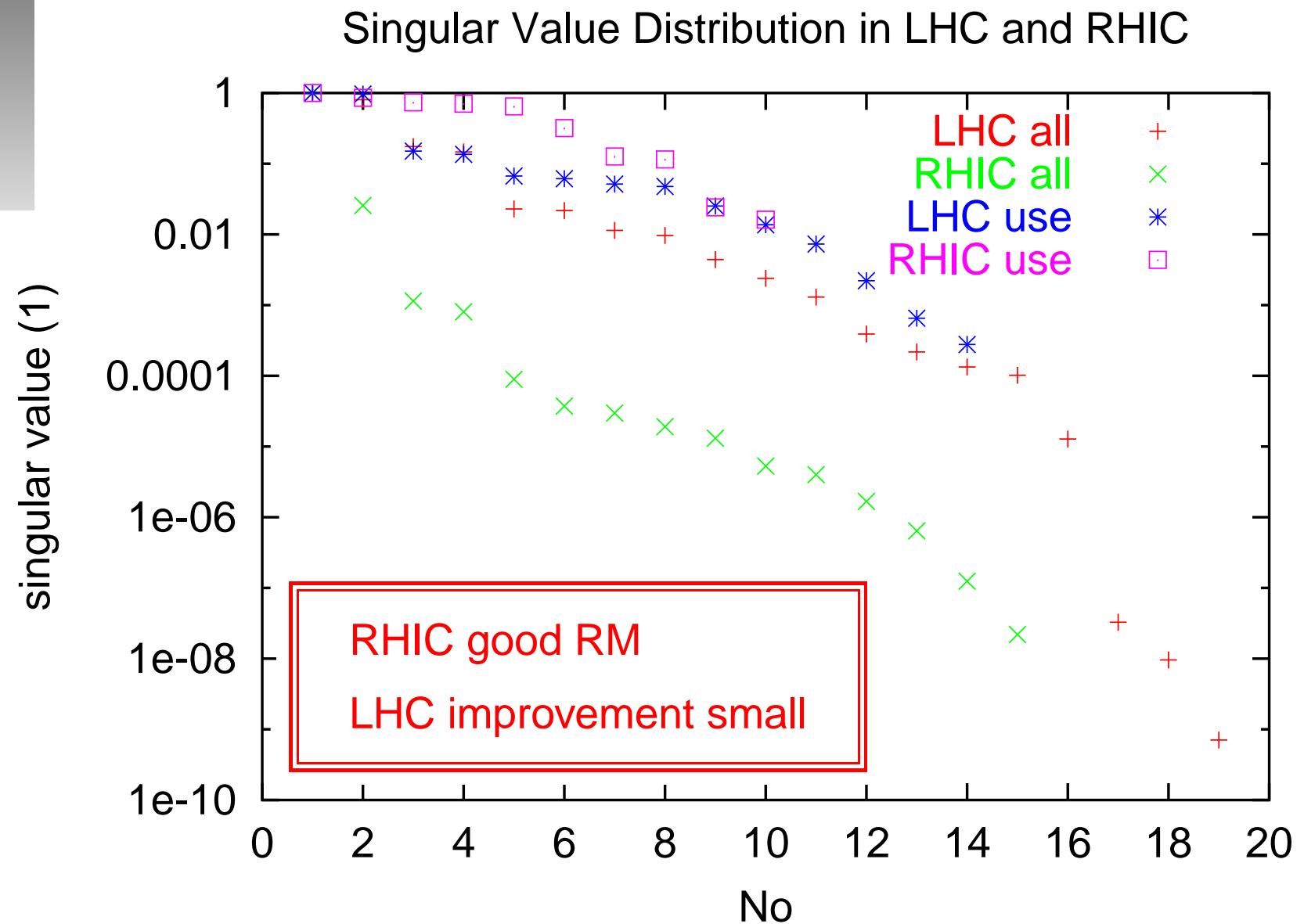
---

$$\mathbf{A} = \mathbf{U}\mathbf{S}\mathbf{V}^T$$

$$\left( \begin{array}{c} \mathbf{A} \end{array} \right) = \left( \begin{array}{c} \mathbf{U} \end{array} \right) \cdot \left( \begin{array}{cccc} s_1 & & & \\ & s_2 & & \\ & & \ddots & \\ & & & s_m \end{array} \right) \cdot \left( \begin{array}{c} \mathbf{V}^T \end{array} \right)$$

$$\mathbf{A}^{-1} = \mathbf{V}\mathbf{S}^{-1}\mathbf{U}^T$$

# *Singular Value Distribution*



# *Approach to Solve Problem of Non Linearity*

---

**Problem:** Linear methods are used to solve a non linear problem.

**Possible Solution:** By minimizing the  $\vec{\Delta K}$  vector the error introduced by non linearity is minimized.

**Drawback:** This has to be achieved at the cost of the constraints.

**Advantage:** The overall performance can improve significantly.

# ***Summary LHC***

---

operative range of knobs:  $\pm 0.1m$  at  $\beta^* = 0.5m$

$$\begin{aligned} |\alpha_{max}^*| &= 3.1 \cdot 10^{-3} \\ \Delta\beta_{\perp max}^* &= 0.34\% \text{ of } \Delta\beta^* \\ |D_{max}^*| &= 1.5 \cdot 10^{-2} m \\ X_{max}^* &= 8.5 \cdot 10^{-13} m \\ Y_{max}^* &= 1.6 \cdot 10^{-7} m \\ \Delta p Y_{max}^* &= 0.2\% \\ \Delta Q_{max} &= 0.01 \\ \beta_{max}^{beat} &= 1.6\% \\ D_{max}^{beat} &= 0.7 m \end{aligned}$$

# *Summary RHIC*

---

operative range of knobs:  $\pm 0.2m$  at  $\beta^* = 1m$

$$\begin{aligned} |\alpha_{max}^*| &= 1.6 \cdot 10^{-2} \\ \Delta\beta_{\perp max}^* &= 0.17\% \text{ of } \Delta\beta^* \\ |D_{max}^*| &= 1.9 \cdot 10^{-2} m \\ \Delta Q_{max} &= 0.003 \end{aligned}$$

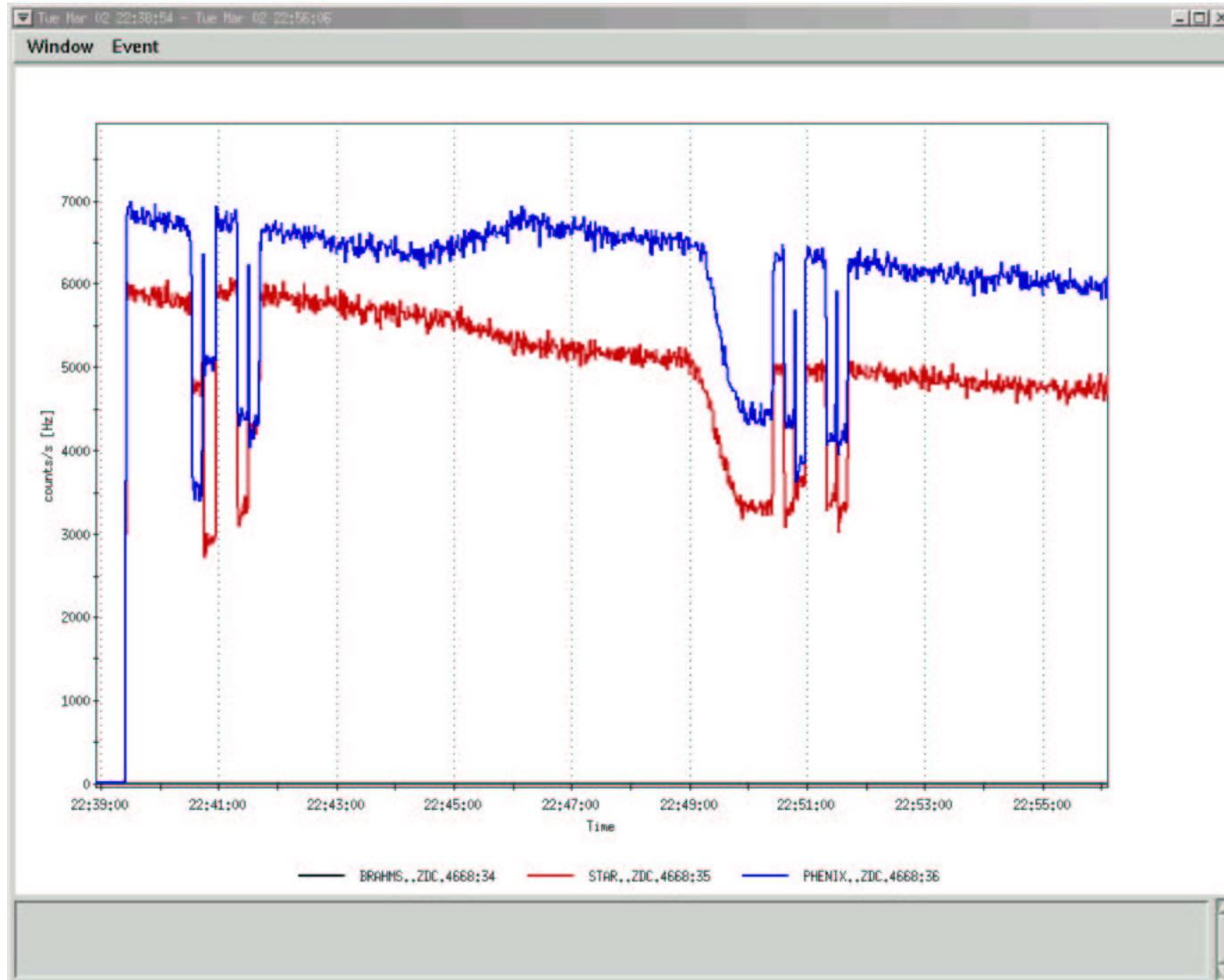
# **RHIC Beam Experiment**

---

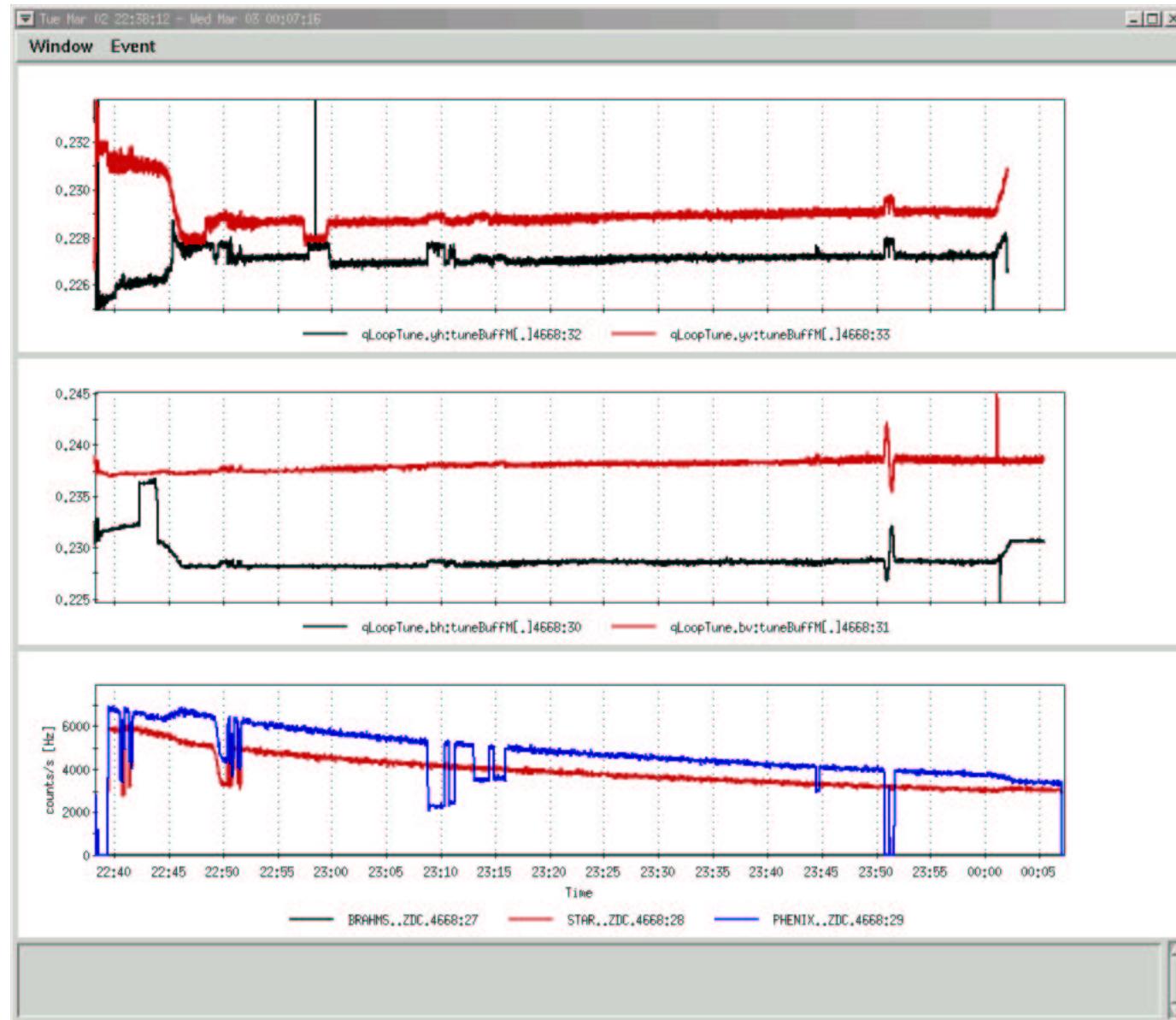
Response Matrix approach based on  
Online Model Johannes

Goal : Squeeze  $\beta^*$  as much as possible

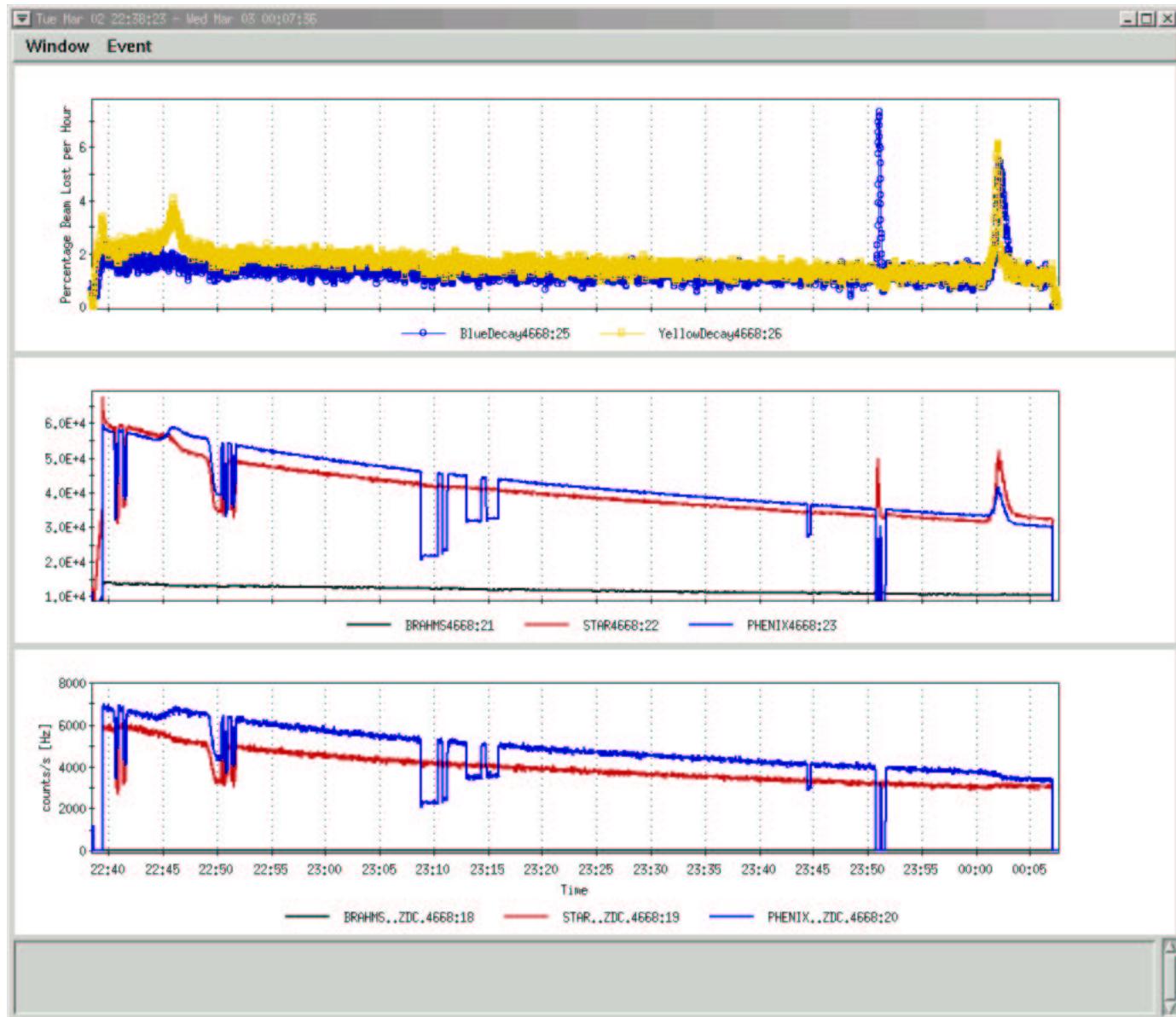
# Beam Experiment Results *RHIC* - Rates



# Beam Experiment Results RHIC - Rates and Tunes

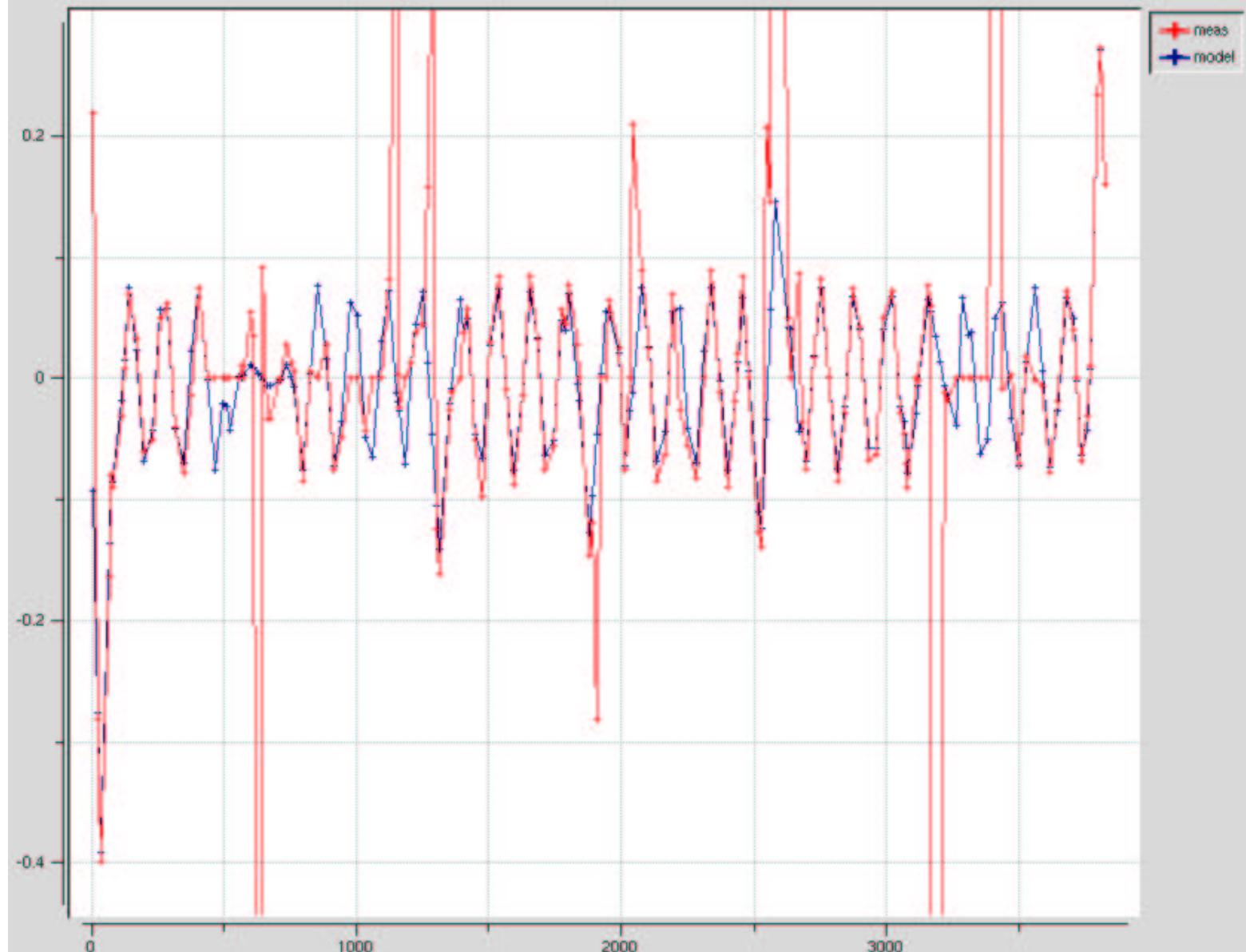


# Beam Experiment Results RHIC - Rates, Background and Tunes



# *Beam Experiment Results RHIC -*

## *Dispersion Measured and Model*



# Preliminary Summary RHIC Beam Experiment

---

operative range of knobs:  $m > -0.12$  at  $\beta^* \approx 1m$

$$\Delta\beta^* = -10\% ?$$

$$|\Delta D_{max}^*| = ?$$

$$\mathcal{L}_{Phoenix} \approx +10\% ?$$

$$\mathcal{L}_{Star} \approx -2\% ?$$

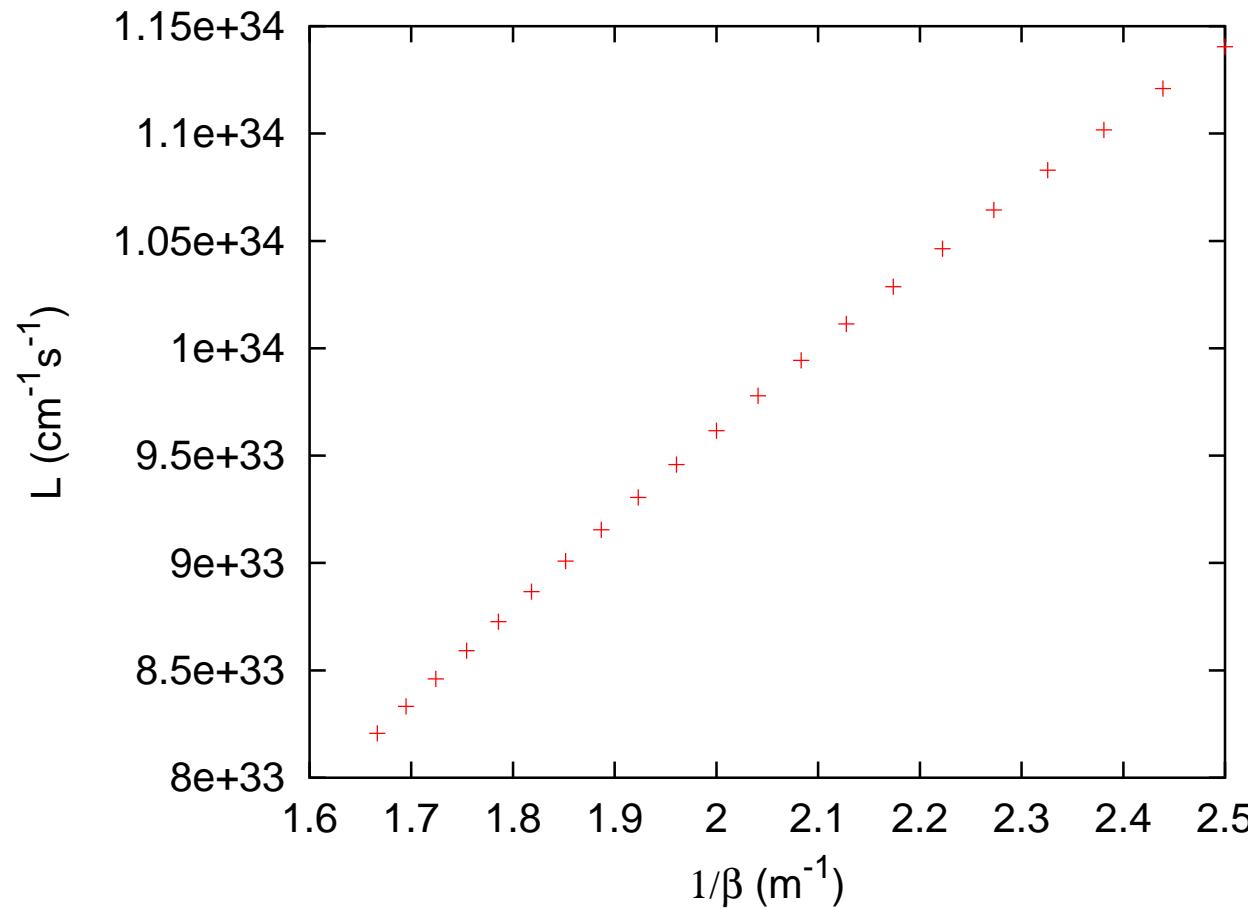
$$\Delta Q_{max} = 0.003$$

## *Further covered Topics*

---

- $\beta^*$  Measurement Utilizing Tuning Knobs
- Influence of the Knobs on
  - Closed Orbit
  - Crossing Angle
  - Beam-Beam (MAD)
  - Parasitic Beam-Beam (Train)

# Increase LHC Luminosity with Tuning Knobs



Long range and head on beam-beam effects do not prevent luminosity gains when using tuning knobs to squeeze the  $\beta$  functions at the IP.

# References

---

'Luminosity Optimization by Adjusting LHC  $\beta^*$  at Collision.' CERN Proceedings Nanobeam 2002 Lausanne, Switzerland, 2-6 September 2002, p 199-205, see also the nanobeam web page:  
<http://icfa-nanobeam.web.cern.ch/icfa-nanobeam/paper/wittmer.pdf>

W. Wittmer, A. Verdier, F. Zimmermann 'Correcting the LHC  $\beta^*$  at Collision' CERN-LHC-Project-Report-647, see also  
[http://warrior.lbl.gov:7778/pacfiles/papers/WEDNESDAY/AM\\_POSTER/WPAB083/WPAB083.PDF](http://warrior.lbl.gov:7778/pacfiles/papers/WEDNESDAY/AM_POSTER/WPAB083/WPAB083.PDF)

Adjusting the  $\beta$ -functions in PHENIX of RHIC. to be published