Global and Local Compensation Of Long-Range Beam-Beam Interactions With Multipole Correctors

Jack J. Shi

Department of Physics & Astronomy , University of Kansas

Lihui Jin, Orithai Kheawpum

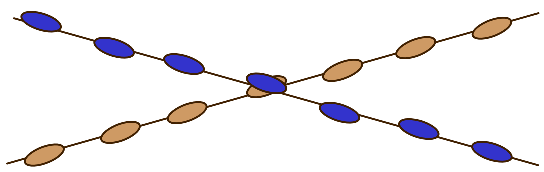
INTRODUCTION

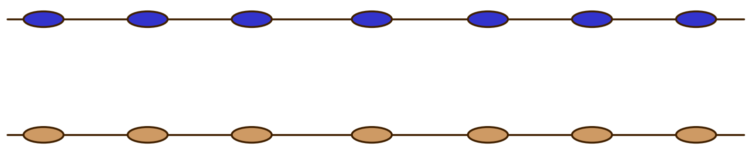
- In storage ring colliders, the long-range beam-beam effect could be a major factor that reduces the beam lifetime and limits the luminosity.

- For localized long-range beam-beam perturbations, the wire compensation could be an effective means to improve the dynamics of beams.

- In the case that many parasitic collisions are distributed around ring, no effective compensation scheme is currently available to compensate such non-localized long-range beam-beam perturbations.

Localized vs. Non-localized Beam-Beam Interactions





Parasitic collisions are only inside interaction regions

⇓

Localized long-range beam-beam perturbations

Parasitic collisions are distributed around ring

⇓

Non-localized long-range beam-beam perturbations

SOLUTION for a compensation of localized or non-localized long-range beam-beam perturbations:

Global and Local compensation of long-range beam-beam interactions by using multipole correctors based on a minimization of nonlinearities in one-turn or/and sectional maps of a storage ring collider.

Two Possibilities For Obtaining One-Turn or Sectional Maps:

1. design lattice and measured field errors
2. measurement of maps with beam-dynamics experiment

$\Longrightarrow$ Beam-Based Compensation

PRINCIPLE OF THE COMPENSATION WITH MAPS

Without Beam-Beam Interactions:

Dynamics of beams can be described by a one-turn map or a groups of sectional maps that contains all global information of nonlinearities in the system. By minimizing nonlinear terms of the maps order-by-order with a few groups of multipole correctors, the nonlinearity of the system can be reduced locally or globally.

With Long-Range Beam-Beam Interactions:

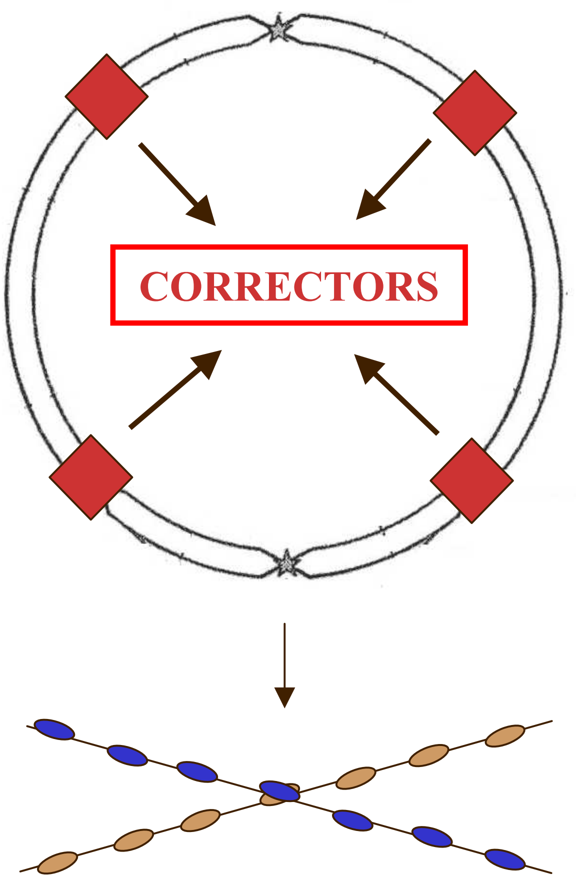
In general, the beam separation at parasitic collisions is much larger than the beam size. In the phase-space region occupied by beams, the long-range beam-beam interaction can be expanded into a Taylor series at the beam separation and included into the maps for the compensation of the nonlinearity of the system.

TESTING MODEL: LHC Collision Lattice

- Two IPs (IP1 and IP5)
- 15 parasitic crossings on each side of an IP
- All multipole field errors up to 10th order in IRs, mixed configuration for insertion quads (MQX)
- Crossing angle = 300 μrad with vertical crossing at IP1 and horizontal crossing at IP5
- $(\nu_x, \nu_y) = (0.31, 0.32)$

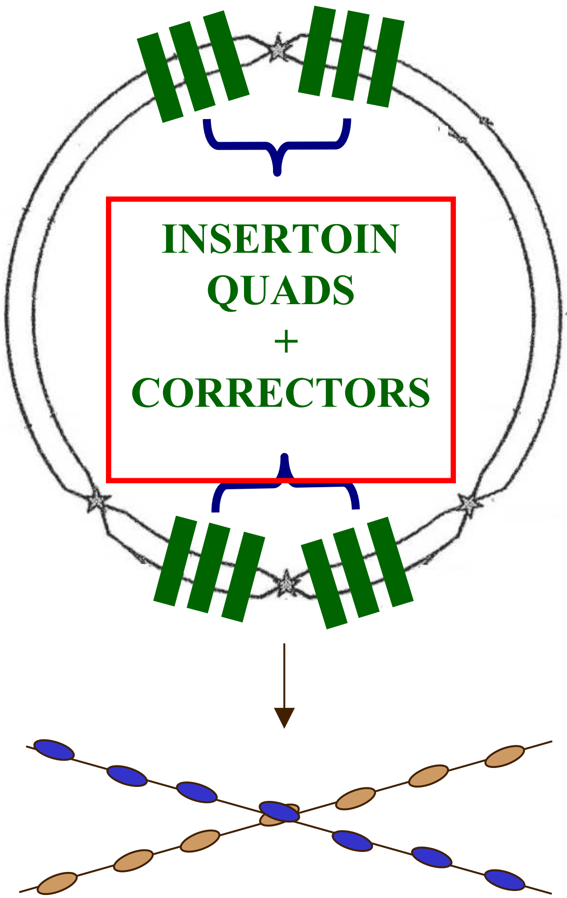
Layout of the Correctors in the Study Model

4 Groups of Correctors in Arcs For the Global Compensation



Scheme 1

4 Groups of Correctors in IRs For the Local Compensation



Scheme 2

FORMULAS FOR THE MINIMIZATION OF NONLINEARITIES

Long-range beam-beam kick,

$$\Delta\vec{p} = G_0 \frac{\vec{r}+\vec{r}_0}{|\vec{r}+\vec{r}_0|^2}\left(1 - e^{-\frac{|\vec{r}+\vec{r}_0|^2}{2\sigma^2}}\right)$$

The Taylor expansion of $\Delta\vec{p}$,

$$\Delta\vec{p} = G_0 \vec{F}_1 F_2 F_3$$

where

$$\vec{F}_1 = \vec{r}+\vec{r}_0$$

$$F_2 = \frac{1}{|\vec{r}+\vec{r}_0|^2} = \sum_{n=0}^{\infty} \frac{(-1)^n}{r_0^{2(n+1)}}\left(|\vec{r}+\vec{r}_0|^2 - r_0^2\right)^n$$

$$F_3 = 1 - e^{-\frac{|\vec{r}+\vec{r}_0|^2}{2\sigma^2}} = 1 - e^{-\frac{r_0^2}{2\sigma^2}} \sum_{n=0}^{\infty} \frac{(-1)^n}{2^n \sigma^{2n} n!}\left(|\vec{r}+\vec{r}_0|^2 - r_0^2\right)^n$$

4-dimensional one-turn map:

$$M_0 = R_0\, e^{:H_3:} e^{:H_4:} \cdots e^{:H_n:} \cdots.$$

where

$$H_n = \sum_{i+j+k+l=n} u_{ijkl}^{(n)}\, x^i\, p_x^j\, y^k\, p_y^l$$

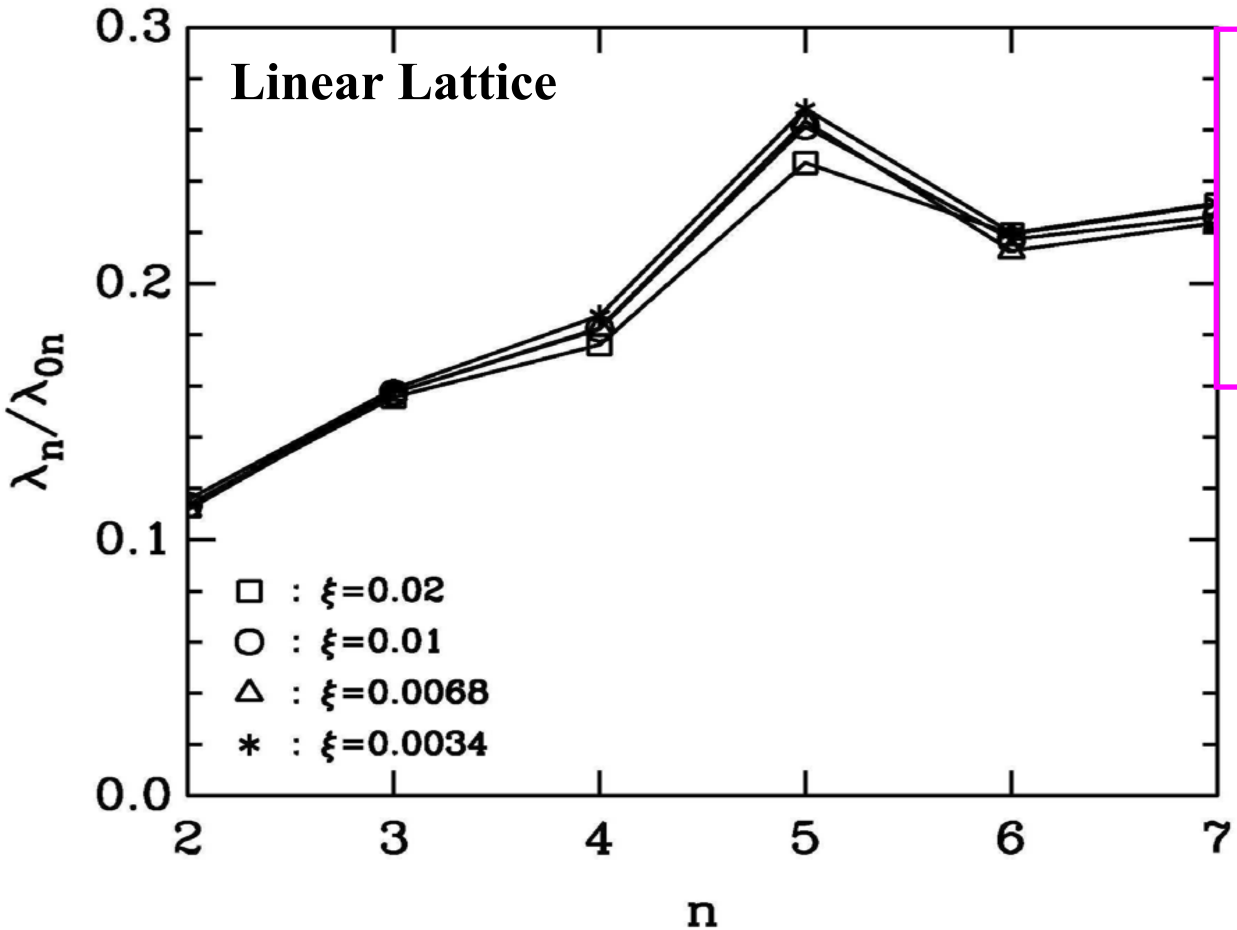
Postulation:

The nth-order nonlinearity in a one-turn map can be characterized by the magnitude of its nth-order coefficients defined by

$$\lambda_n = \left\{ \sum_{i+j+k+l=n+1} \left[u_{ijkl}^{(n+1)}\right]^2 \right\}^{1/2} \quad \text{for } n \geq 2.$$

With a few multipole correctors, $\{\lambda_n \mid n \geq 2\}$ can be minimized order-by-order and, consequently, the nonlinearity of the system including long-range beam-beam interactions and multipole field errors will be reduced.

Magnitude of Map Coefficients v.s. Order of Compensation



Correctors In Arcs

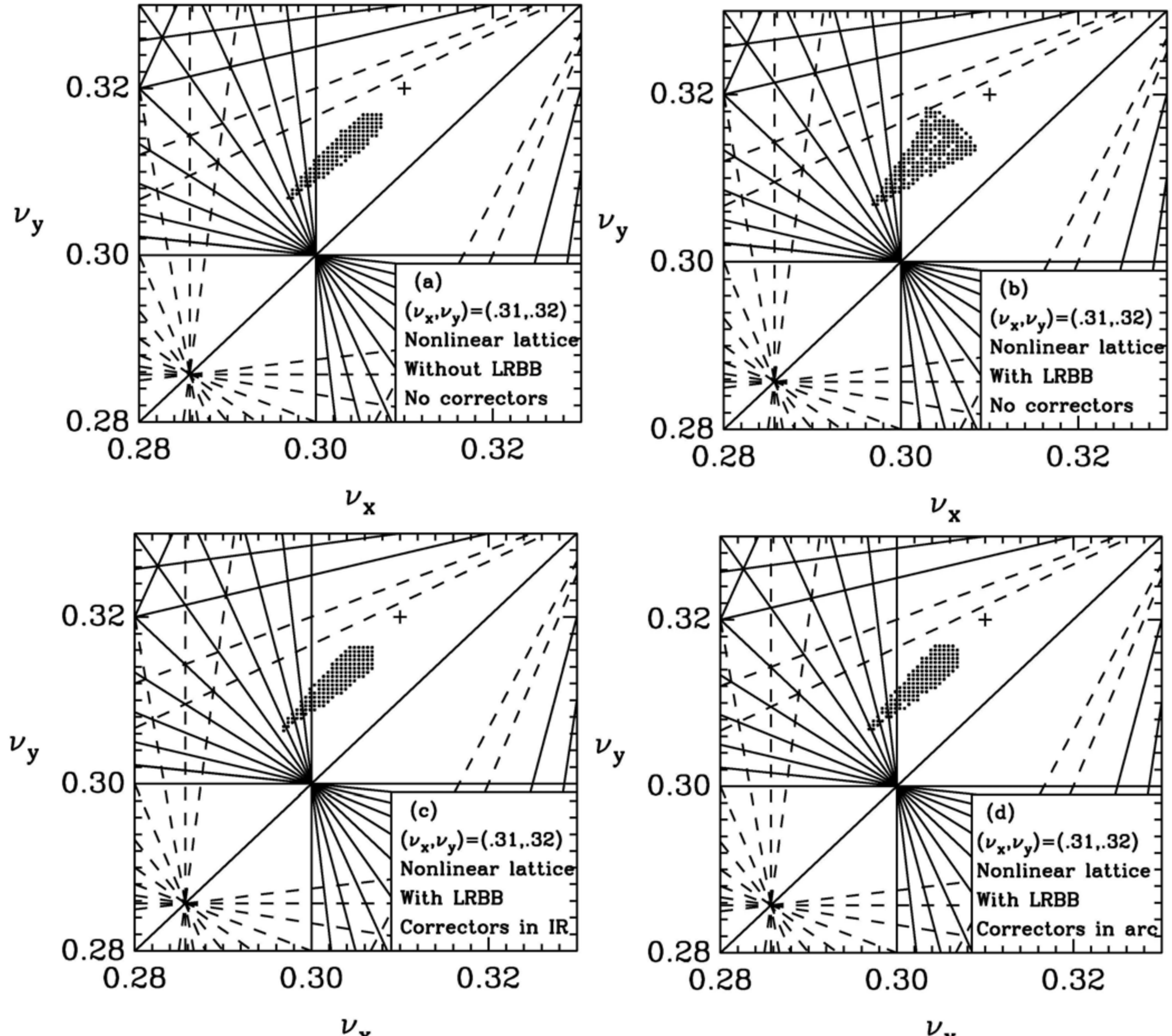
λ_n : **With Compensation**

λ_{0n} : **No Compensation**

- **Percentage reduction of λ_n is independent of ξ .**
- **Reduction rate of λ_n decreases when n>5 due to the difficulty of optimization in a large dimensional parameter space.**

Tune Spread With/Without The Multipole Compensation

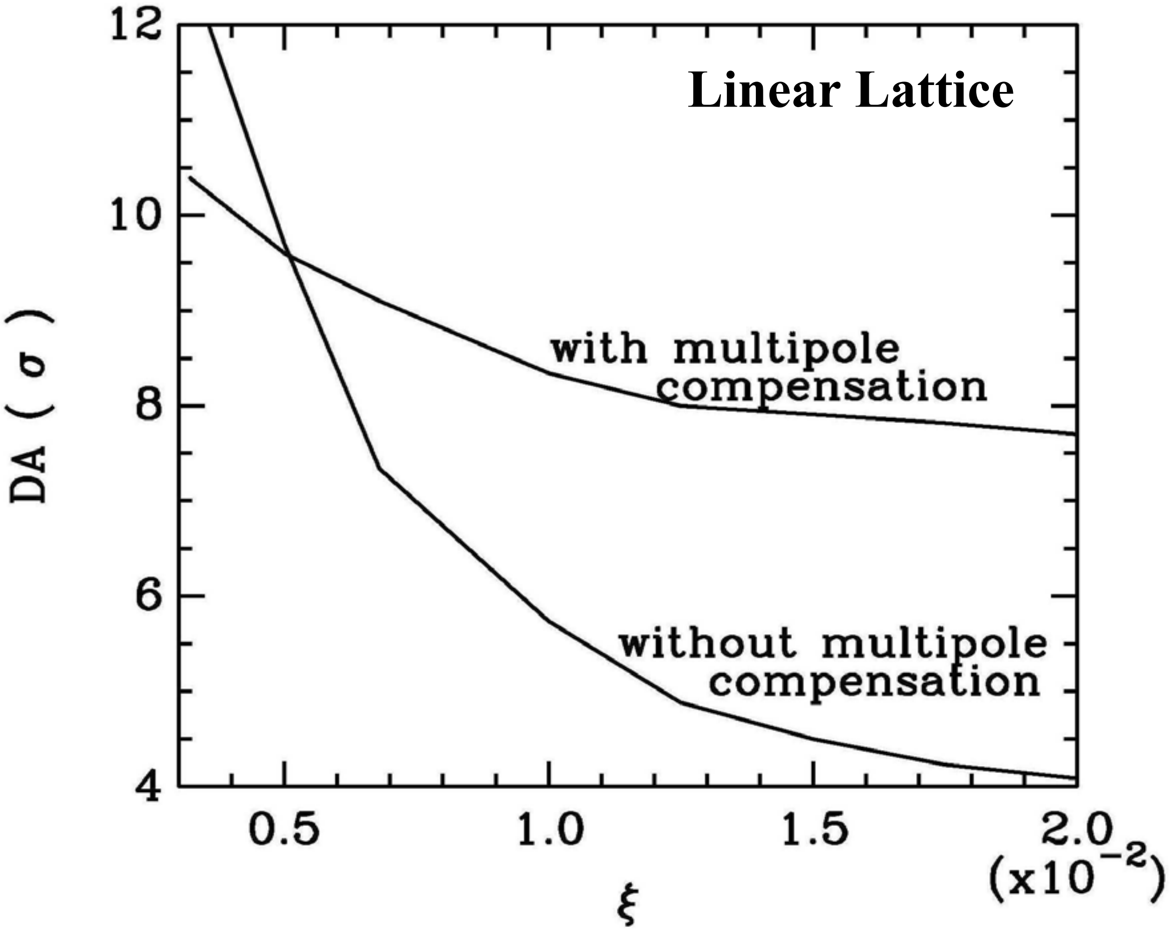
Both global & local multipole compensation eliminate the tune spread due to long-range beam-beam interactions.



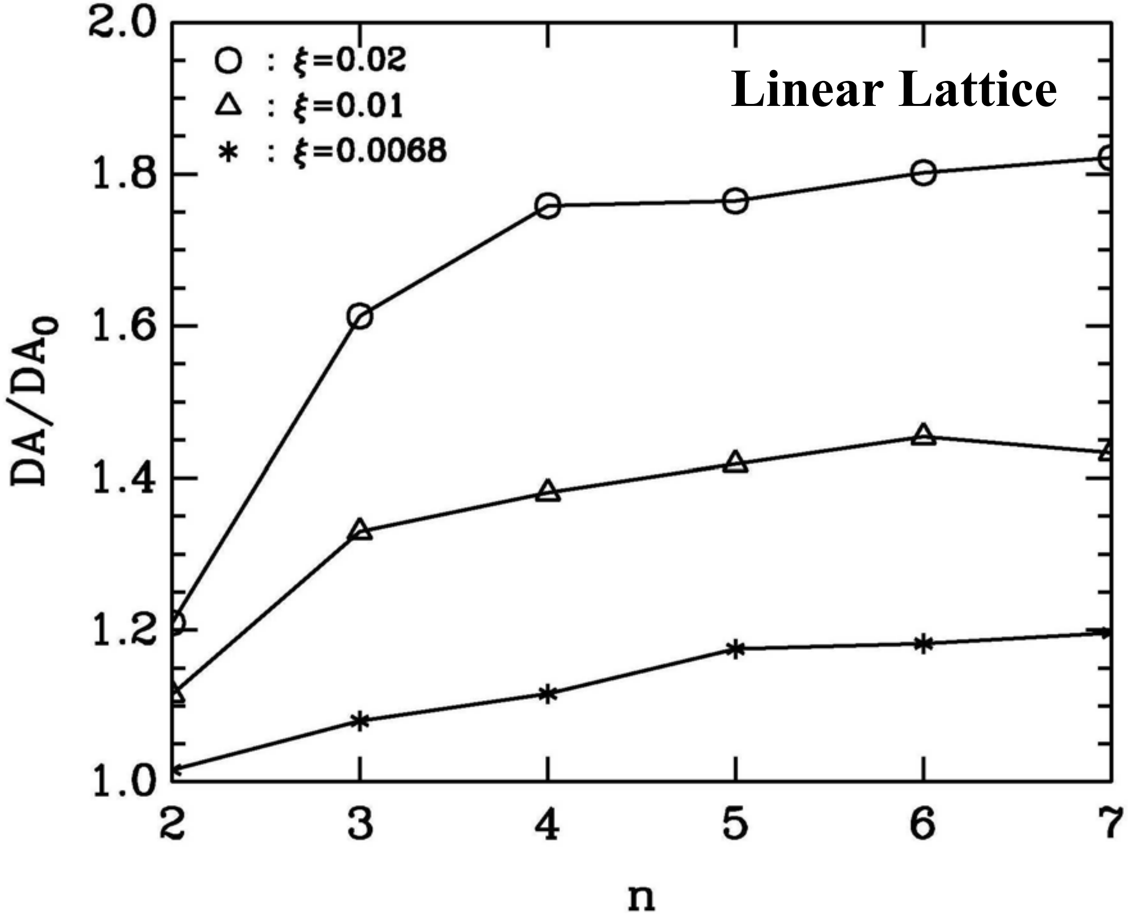
Dynamic Aperture v.s. Beam-Beam Parameter

- The reduction of DA over ξ is much slower after the compensation.

- When the original DA is larger than 9σ, that is the average beam separation at parasitic collisions, the compensation is no longer effective in the increase of DA because the Taylor expansion of LRBB is no longer valid in that region but it still improves the linearity of the phase-space region relevant to the beams.

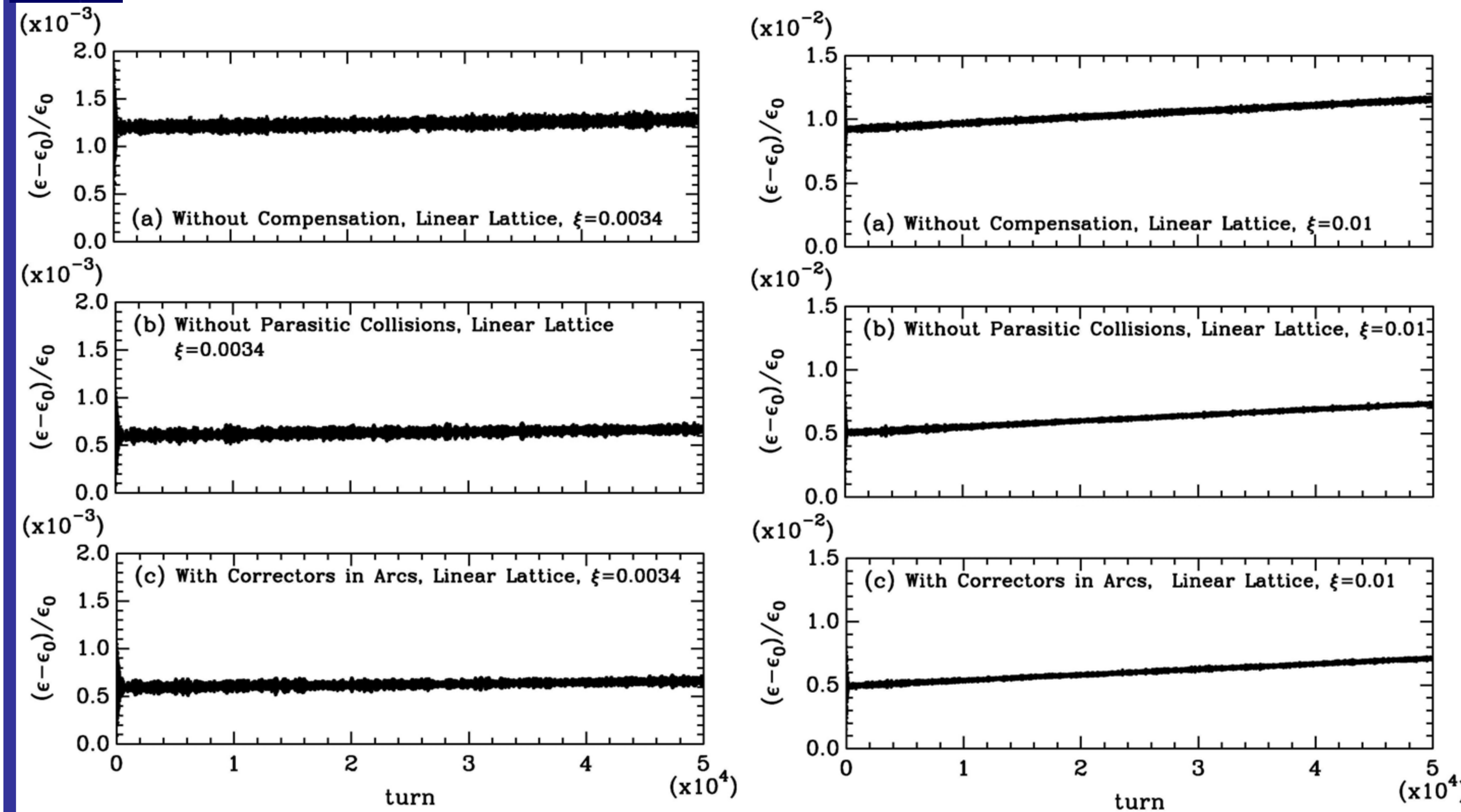


Dynamic Aperture v.s. Order of the Compensation



Improvement of DA becomes less pronounced when n>4. This indicates that the 3rd-/4th-order nonlinearity dominate long-range beam-beam interaction.

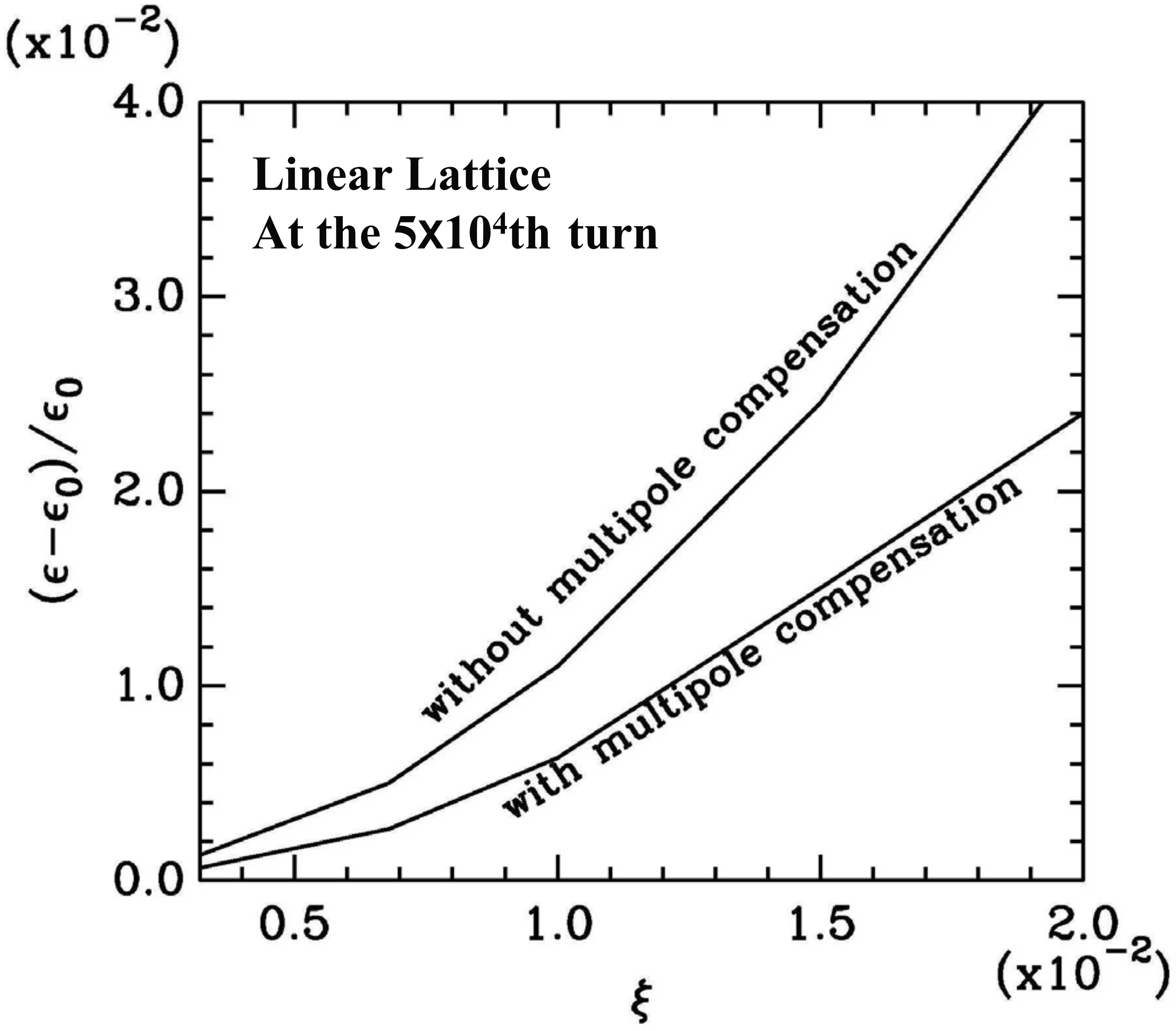
Emittance Growth in Linear Lattice



The multipole compensation eliminates the effect of long-range beam-beam interactions on the emittance growth.

Emittance Growth v.s. Beam-beam Parameter

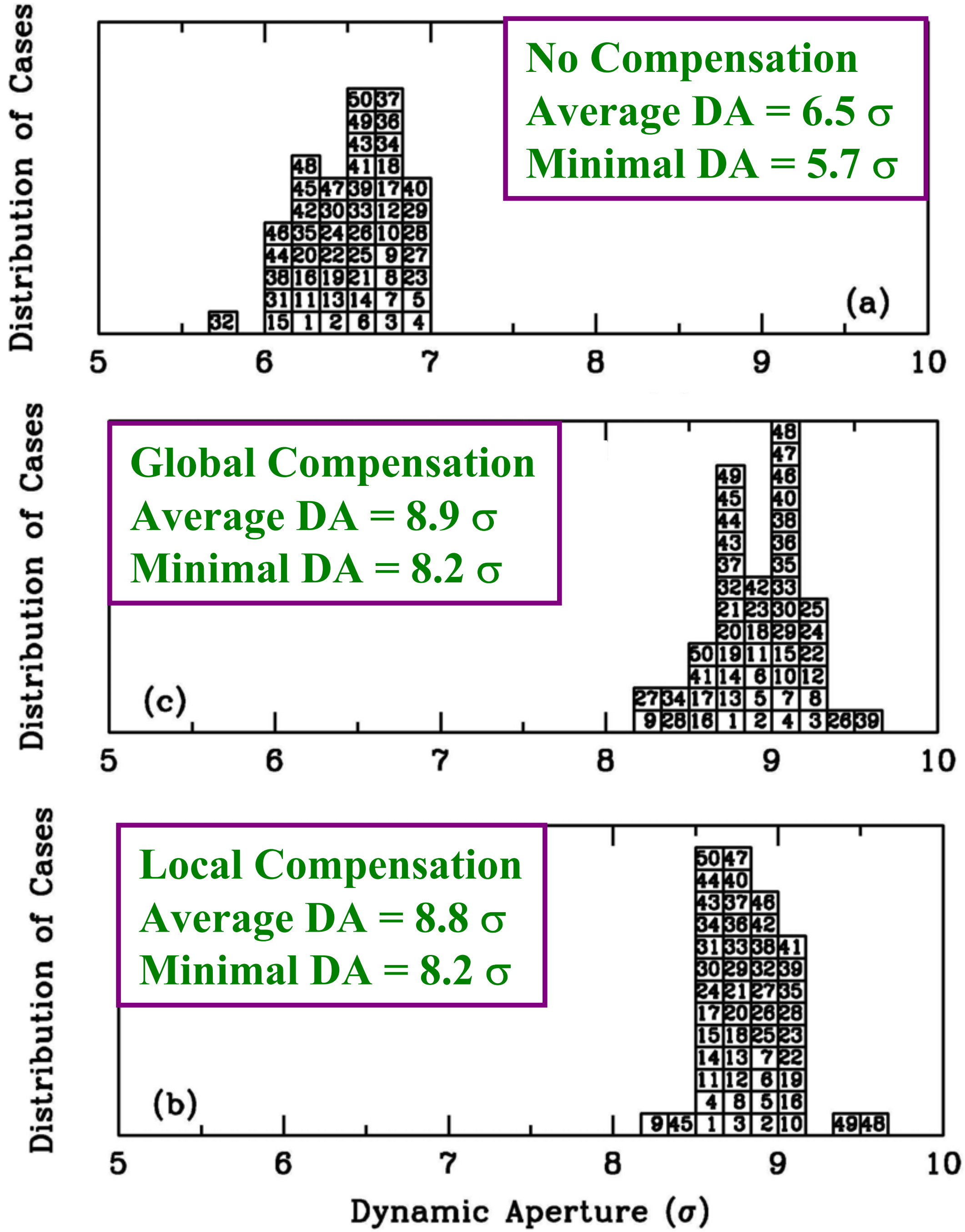
The compensation improves the linearity of phase-space region near the closed orbit even when the original dynamic aperture is larger than 9 σ .



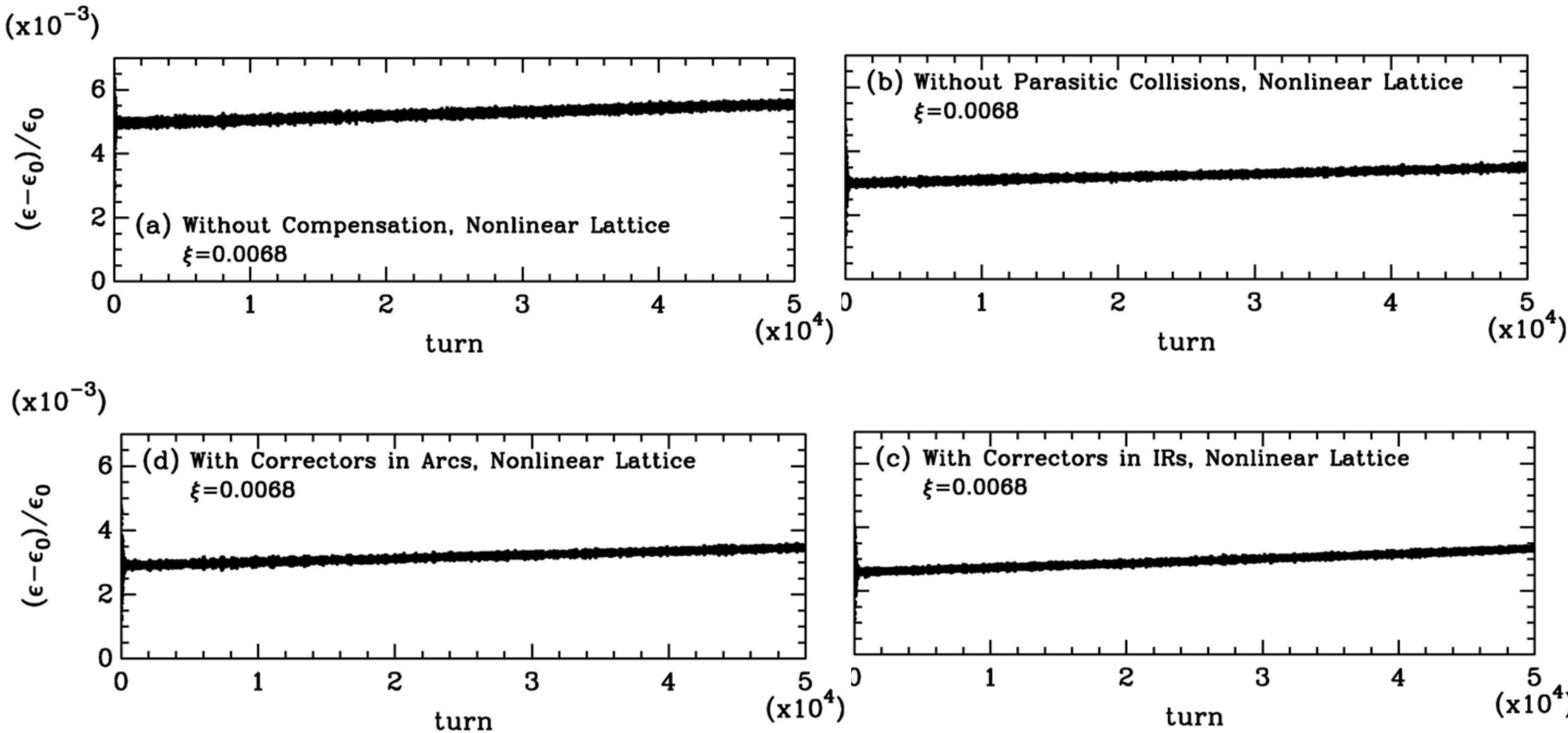
Compensation in Nonlinear Lattice

50 samples of LHC collision lattice without or with the compensation of long-range beam-beam interactions. Head-on and long-range beam-beam interactions in IP1 and IP5 are included. The number in each block identifies each random sample.

$\xi = 0.0068$



Emittance Growth in Nonlinear Lattice



Due to a simultaneous compensation of nonlinear field errors in lattice and long-range beam-beam interactions, the emittance growth after the multipole compensation is smaller than that without long-range beam-beam interactions and the compensation.

CONCLUSIONS

- With a few groups of correctors, nonlinear terms in a one-turn/sectional map including long-range beam-beam interactions can be minimized order-by-order and, consequently, the linearity of the phase-space region occupied by beams can be improved.

- The unique features of this compensation scheme include:

  (a) long-range beam-beam effects due to localized or/and non-localized parasitic collisions can be controlled;

  (b) the overall nonlinearity in a system including long-range beam-beam interactions and nonlinear field errors in lattice can treated systematically with a same group of correctors.

- If a one-turn or sectional map can be measured in beam-dynamics experiments “accurately”, this compensation scheme can be used for a beam-based compensation of nonlinearities.