Nonlinear, Skew Chromaticity, RF Phase Modulation

- Nonlinear chromaticity
  - Measurement
  - Correction using octupoles
- Skew chromaticity
  - Measurement
- RF phase modulation
- Summary

Nonlinear Chromaticity

$$\nu = \nu_0 + \xi_1 \delta + \xi_2 \delta^2 + \xi_3 \delta^3 + \cdots$$

$$\delta \equiv \frac{\Delta p}{p}$$

- Change δ by changing the frequency of the RF cavities
- Measure the tunes using the PLL tune-meter for each δ
- Calculate the chromaticities, $\xi_{1,2,3,\ldots}$, using linear regression

S. Tepikian, V. Ptitsyn, M. Brennan, P. Cameron, A. Della Penna

Nonlinear Chromaticity

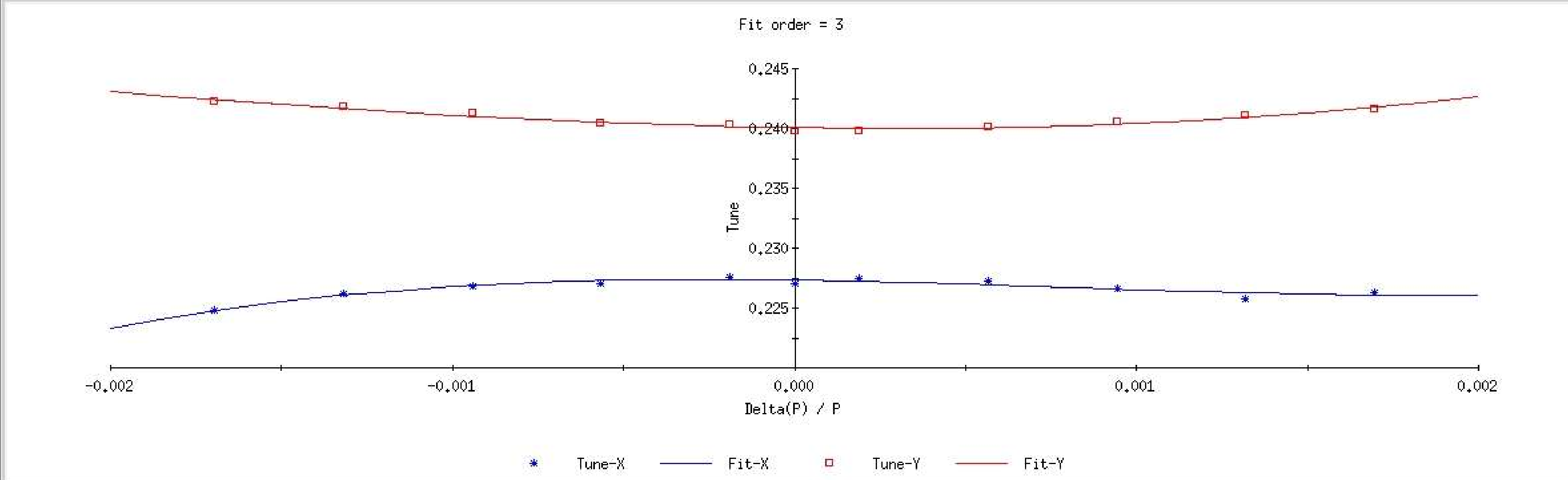
- Radial steering models
  - Stepped
    - 5 and 10 steps
  - Sine function
    - 0.4mm amplitude at 1Hz
    - 1mm amplitude at 0.4Hz
    - 2mm amplitude at 0.2Hz

Nonlinear Chromaticity

- Separate the tunes to reduce coupling
- Adjust the linear chromaticity to zero with the sextupoles
- Compare radial steering shift to the BPM's
- Don't exceed the radial aperture

Analysis of Data -- Yellow ring -- Wed Apr 23, 2003 05:45:28

Tune vs. Momentum



Row	Delta(p)/p	PLL X	+- Sigma	Tune X #1	Tune X #2	Select	PLL Y	+- Sigma	Tune Y #1	Tune Y #2	Select
1	-0.00169597	0.22475	6.17012e-05	0.224609	0.396484	PLL	0.242233	0.000108311	0.196289	0.279297	PLL
2	-0.00131928	0.226145	5.18518e-05	0.225586	0.238281	PLL	0.241831	9.54268e-05	0.270508	0.165039	PLL
3	-0.000941735	0.22683	7.37616e-05	0.225586	0.135742	PLL	0.241231	0.000125222	0.0996094	0.353516	PLL
4	-0.000565041	0.227	9.39836e-05	0.225586	0.341797	PLL	0.240469	0.000142807	0.386719	0.363281	PLL
5	-0.000187499	0.227566	9.71104e-05	0.225586	0.349609	PLL	0.240278	0.000166612	0.235352	0.333984	PLL
6	0	0.227023	0.000115365	0.225586	0.238281	PLL	0.239748	0.000112974	0.121094	0.358398	PLL
7	0.000188347	0.227412	2.94569e-05	0.225586	0.338867	PLL	0.23975	6.74208e-05	0.226562	0.12793	PLL
8	0.000565889	0.227215	4.28273e-05	0.225586	0.248047	PLL	0.240135	6.27648e-05	0.333008	0.256836	PLL
9	0.000942583	0.226587	0.000140948	0.224609	0.232422	PLL	0.240571	0.000159028	0.275391	0.0996094	PLL
10	0.00132013	0.225764	4.06e-05	0.223633	0.230469	PLL	0.241074	0.000136579	0.291992	0.196289	PLL
11	0.00169682	0.226321	0.000197488	0.222656	0.212891	PLL	0.241589	0.000239714	0.331055	0.276367	PLL

	Corr	Tune	+-Sigma	Chrom[1]	+-Sigma	Chrom[2]	+-Sigma	Chrom[3]	+-Sigma
Horizontal	0.951097	0.2273	9.15665e-05	-0.392446	0.0886934	-655.55	87.976	271765	102014
Vertical	0.971872	0.240063	7.144e-05	-0.43288	0.0691984	701.21	68.6387	84681.1	79590.8

Set Peak #1 | Set PLL | Close

Nonlinear Chromaticity

Fit Order	X				
	ν	ξ_1	ξ_2	ξ_3	Correlation
2	0.227299	0.173	-655.23	****	0.898804
3	0.227300	-0.392	-655.55	271,765	0.951097
4	0.227381	-0.392	-945.50	271,583	0.957203
5	0.227381	0.294	-944.36	-715,076	0.982091
6	0.227323	0.293	-432.87	-714,148	0.985243
	Y				
2	0.240062	-0.257	701.31	****	0.967247
3	0.240063	-0.433	701.21	84,681	0.971872
4	0.239910	-0.433	1244.89	85,023	0.991797
5	0.239910	-0.466	1244.84	132,324	0.991850
6	0.239883	-0.466	1487.28	132,764	0.992521

Harmon (Yellow design lattice, sextupoles only):

X: $\xi_1 = 0$, $\xi_2 = -466$, $\xi_3 = 118{,}139$

Y: $\xi_1 = 0$, $\xi_2 = 1401$, $\xi_3 = 112{,}566$

Nonlinear Chromaticity

- 4 families of **octupoles** for second order chromaticity correction
  - *2 families in the arcs (high dispersion)*
  - *2 families in the insertion (low dispersion), to correct amplitude dependent effects*

$$\xi_{x_2} = \sum_i \beta_{x_i} D_{x_i}^2 O_i \qquad \xi_{y_2} = 3\sum_i \beta_{y_i} D_{x_i}^2 O_i$$

$$\Delta\nu_x = \sum_i \beta_{x_i}^2 O_i \qquad \Delta\nu_y = -\sum_i \beta_{y_i}^2 O_i$$

Skew Chromaticity

Skew chromaticity arises from coupled beam dynamics.
Eigen-tunes from coupling:

$$\nu_{\pm} = \frac{1}{2}\left[\nu_x + \nu_y \pm \sqrt{(\nu_x - \nu_y)^2 + |\boldsymbol{q}|^2}\right]$$

$$\boldsymbol{q} = \frac{1}{2\pi}\sum_i \sqrt{\beta_{x_i}\beta_{y_i}}\,(K_{s_1}L)_i \begin{pmatrix}\cos(\phi_{y_i} - \phi_{x_i}) \\ \sin(\phi_{y_i} - \phi_{x_i})\end{pmatrix} \qquad \Delta Q_{min} = |\boldsymbol{q}|$$

Generalizing to include off-momentum behavior:

$$\nu_x \to \nu_x + \xi_x \delta \qquad \nu_y \to \nu_y + \xi_y \delta \qquad \boldsymbol{q} \to \boldsymbol{q} + \boldsymbol{k}\,\delta$$

$$\nu_{\pm}(\delta) = \frac{1}{2}\left\{(\nu_x + \nu_y) + (\xi_x + \xi_y)\delta \pm \sqrt{[(\nu_x - \nu_y) + (\xi_x - \xi_y)\delta]^2 + [\boldsymbol{q} + \boldsymbol{k}\,\delta]^2}\right\}$$

S. Tepikian, M. Bai, J. Beebe-Wang, W. Fischer, S. Peggs

Skew Chromaticity

Stability Criteria (conjecture):

$$\xi_{\pm} \equiv \frac{d\,\nu_{\pm}}{d\,\delta} > 0$$

Leads to the following constraints:

$$\xi_x \xi_y > \frac{|\boldsymbol{k}|^2}{4} \qquad \xi_x + \xi_y > \frac{|\boldsymbol{k}\cdot\boldsymbol{q}|}{|\boldsymbol{q}|}$$

Skew Chromaticity

Regular and skew sextupoles contribute to skew chromaticity through feeddown:

$$\boldsymbol{k} = \frac{1}{\pi} \sum_i \sqrt{\beta_{x_i} \beta_{y_i}} \left[(K_2 L)_i D_{y_i} + (K_{s_2} L)_i D_{x_i} \right] \begin{pmatrix} \cos(\phi_{y_i} - \phi_{x_i}) \\ \sin(\phi_{y_i} - \phi_{x_i}) \end{pmatrix}$$

$|\boldsymbol{k}| \approx 3.2$ In RHIC estimated from the chromaticity sextupoles and ≈ 30cm vertical dispersion in the arcs.

$|\boldsymbol{k}| = 3.8 \pm 0.2$ Measurement in the Tevatron

Skew Chromaticity

Changing the skew chromaticity using a skew sextupole corrector in the triplets. Four correctors in the low β^* region:

Name	s [m]	β_x [m]	β_y [m]	D_x [m]	G [m^2]	$\Delta\|k\|_{max}$
bi5-sxs3	3800.605	1052.898	806.411	-0.662	194.17	7.77
bo6-sxs3	33.240	833.076	1017.840	0.573	167.95	6.72
bo7-sxs3	606.205	802.040	1030.760	0.487	140.95	5.64
bi8-sxs3	672.685	1014.004	816.837	-0.559	161.94	6.48

where

$$G = \frac{1}{\pi}\sqrt{\beta_x \beta_y}\,|D_x| \qquad |\boldsymbol{k}| = (K_{s_2} L)\, G$$

$$(K_{s_2} L)_{max} \approx 0.04 \qquad \Delta|\boldsymbol{k}|_{max} \approx 0.04\, G$$

RF Phase Modulation

- Fast RF phase modulation, $100 \sim 200Hz$
- Low synchrotron frequency, $10 \sim 30Hz$
  - Store with $200MHz$ storage cavities off
- High PLL bandwidth required $\geq 200Hz$
  - Increasing bandwidth reduces S/N ratio
  - Could lead to a less stable PLL system

O. Bruning, S. Tepikian, W. Fischer, M. Brennan, P. Cameron,
F. Pilat, J. van Zeijts

RF Phase Modulation

- RF system needs additional hardware
  - Current hardware not designed for phase modulation
  - M. Brennan requires specifications for the implementation

Summary

- Reported results on nonlinear chromaticity
  - We have not yet tried to use the octupoles to correct/change the second order chromaticity.
- Proposed measuring skew chromaticity
- RF phase modulation experiment requires additional hardware