A Standard FODO Lattice with Adjustable Momentum Compaction

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Abstract
An existing lattice made of identical FODO cells can be modified to have adjustable momentum compaction. The modified lattice consists of repeating superperiods of three FODO cells where the cells have different horizontal phase advances. This allows tuning of the momentum compaction or (transition) to any desired value. A value of the \( \gamma_t \) could be an imaginary number. A drawback of this modification is relatively large values of the dispersion function (two or three times larger than in the regular FODO cell design). This scheme also requires an additional quad bus for the modified cells.

1 INTRODUCTION
Particles travel along the reference orbit in an accelerator ring with momentum \( p_0 \) and period of revolution \( T_0 \). If they have a momentum deviation \( \Delta p \), the time of the arrival at the point of observation will be different. An offset in the revolution period \( \Delta T \) is given by:

\[
\frac{\Delta T}{T_0} = \left( \alpha - \frac{1}{\gamma^2} \right) \frac{\Delta p}{p_0},
\]

where \( \alpha \), the momentum compaction is a property of the lattice, and \( \eta = \alpha - \gamma^{-2} \) is called the phase-slip factor; \( \gamma \) is the Lorentz relativistic factor for the on-momentum particle. The momentum compaction factor is a measure of the path length difference between the off-momentum particle and the on-momentum particle. The transition energy \( \gamma_t \) is the energy at which \( \eta \) vanishes, i.e. it equals \( 1/\sqrt{\alpha} \). In many accelerators \( \gamma_t \) lies in the acceleration range. We shall show that an existing FODO lattice can be modified so as to make \( \gamma_t \) either very large or even imaginary (negative \( \alpha \)). This could be used to for example to avoid having to cross transition, or to make zero momentum compaction isochronous storage rings.

The momentum compaction of a lattice, to the first order, is an integral of the dispersion function \( D \) through the dipoles:

\[
\alpha = \frac{1}{C_0} \int \frac{D(s)}{p(s)} ds,
\]

where \( \rho \) is the radius of curvature and \( s \) is the longitudinal path length measured along the reference orbit with a circumference \( C_0 \). There are many ways to devise an accelerator lattice with either fixed or adjustable value of the momentum compaction \([1],[3],[4],[5]\). Vladimirski and Tarasov [1] propose use of reverse bend dipoles to make the momentum compaction negative. Teng [6] shows that a straight section with a phase advance of \( \pi \) can make the dispersion closed orbit negative at dipoles. Iliev [3] and Guignard [4] use a harmonic approach, where the betatron function is modulated to produce negative values of the momentum compaction by way of resonance conditions. We have reported earlier [5] and [8] the use of flexible-momentum compaction lattices to minimize dispersion values.

2 NORMALIZED DISPERSION FUNCTION
The dispersion function \( D \) needs to be adjusted through the FODO cell to obtain a different integral of its values through dipoles. Because the dispersion function satisfies a second order inhomogeneous differential equation of motion \([7]\) it is useful to use the normalized dispersion function with components \( \xi \) and \( \chi \) as previously defined [5]:

\[
\xi = \sqrt{\beta_x} D - \frac{\beta'_x}{2\sqrt{\beta_x}} D, \quad \chi = \frac{1}{\sqrt{\beta_x}} D, \quad (3)
\]

where \( \beta_x \) and \( \beta'_x \) are respectively the horizontal betatron amplitude function and its derivative \([7]\), \( \xi \) and \( \chi \) are projections of the normalized dispersion vector.

3 FODO CELLS WITH ADJUSTABLE MOMENTUM COMPACTION
As an example we modify the lattice of the Relativistic Heavy Ion Collider (RHIC) at Brookhaven National Laboratory, where the dipole length is \( L_d = 9.45m \) and the
cell length is $L_c \approx 29.6 m$. In our example we use super-periods of three cells as previously proposed by Guignard [4]. Figure 1 shows the Courant-Snyder functions for three cells under the standard operating conditions.

We chose a point of reflection symmetry of the orbit functions at the middle of the three cells. We modify the quadrupole strengths so as to obtain a negative momentum compaction. The quadrupole strengths which make the momentum compaction negative, with $\gamma_t = 1358$, are presented in Table 1.

The normalized dispersion plot for this case is shown in figure 2.

Figure 3 shows the orbit functions $\beta_x$, $\beta_y$ and D in the modified cells (the centers of symmetry are at QDA and QFC).

We see that the penalty paid for making the dispersion negative is almost a doubling of the maximum $\beta_x$ and $\beta_y$, and of the dispersion function. In addition the tunes $\nu_x$ and $\nu_y$ are changed substantially.

4 CONCLUSION

The momentum compaction of the standard FODO cell lattice could be adjusted by the modulating the betatron functions to any desired values with the drawback of larger values of the dispersion and betatron functions. A range of dispersion function offsets, obtained by the quad adjustments, falls within twice of the optimum FODO cell dispersion values. The maximum values of $\beta_x$ and $\beta_y$ are less than two times the values at optimum betatron tunes ($\nu = \pi/2$). The beam size was less than $\sqrt{2}$ larger. This report shows that a resonance condition ([4], [3]) was not necessary to achieve different values of the momentum compaction within standard FODO cells. We used an existing FODO lattice to accommodate the momentum compaction value, but we do not recommend it the lattice of a new accelerator.

5 REFERENCES


