Figure 2. First and second order matrices for the helical snake at $\gamma = 27$ (injection)

Figure 3. First and second order matrices for the helical snake at $\gamma = 268$ (storage)

The first row in Table 1 gives the linear coupling for 2 snakes in the ring, represented by analytical matrices as described in section 3.2. In the second and third row the matrices for the 2 snakes are obtained numerically (see section 3.1). The results in the second row refer to the configuration when the full numerical linear matrix (see Figure 1) is used, while the last result are for a numerical matrix where the focusing terms Σ_{21} and Σ_{43} have been set to zero and the diagonal terms to Σ_{ii} to 1, to verify that in absence of focusing terms the trace results coincide with the difference between the cosines of the nominal tunes.
The results concerning the analytical model have been independently verified [6] by inserting the snake matrices in the RHIC lattice and using the code SYNCH to calculate ΔQ_{min}. The results in this case is ΔQ_{min}=0.00935, in very good agreement with the results obtained with the 1-turn matrix.

The linear coupling obtained by the 2 models ($\Delta Q_{min} \sim 10^{-2}$) at injection is well within the range of capability of the RHIC decoupling system. At storage, the coupling introduced by the snakes is negligible ($\Delta Q_{min} \sim 10^{-4}$).
There is about a factor 2 between the coupling effect predicted by the numerical and analytical matrices: the 2 models use different approximations for the fields and equations of motions, so a perfect agreement was not expected.
Work is in progress to evaluate the higher order effects of the snake on the beam dynamics, and the non-linear behavior will have to be compared to the linear effects.

References

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[6] E. Courant, Private Communication, October 1994

The rotation matrix between the location 1 and 2 is (2 to 1 is obtained by swapping indexes):

$$\begin{bmatrix} R_{12}^{x} & 0 \\ 0 & R_{12}^{y} \end{bmatrix}$$

where

$$R_{12} = \begin{bmatrix} \sqrt{\frac{\beta_2}{\beta_1}}(\cos\pi Q + \alpha_1 \sin\pi Q) & \sqrt{\beta_1\beta_1}\sin\pi Q \\ -\frac{1+\alpha_1\alpha_2}{\sqrt{\beta_1\beta_1}}\sin\pi Q + \frac{\alpha_1 - \alpha_2}{\sqrt{\beta_1\beta_1}}\cos\pi Q & \sqrt{\frac{\beta_1}{\beta_2}}(\cos\pi Q - \alpha_2 \sin\pi Q) \end{bmatrix}$$

As described in more detail in [1], it is possible to derive from the 1-turn matrix T the linear coupling effect, quantified by the distance of minimum approach of tunes (ΔQ_{min}) in the following way. By writing the 4x4 matrix T as $\begin{bmatrix} M & m \\ n & N \end{bmatrix}$, one can demonstrate that:

$$\left[\frac{1}{2}Tr(A-B)\right]^2 = \left[\frac{1}{2}Tr(M-N)\right]^2 + detH$$

$$[\cos(2\pi Q_A) - \cos(2\pi Q_B)]^2 = [\cos(2\pi Q_x) - \cos(2\pi Q_y)]^2 + \Delta Q_{min}{}^2$$

where H = m + n^+ and A and B are the eigenmatrices and Q_A and Q_B the eigentunes of the coupled motion. From detH one can derive ΔQ_{min}.

5. Results and discussion

The results for the linear model at injection obtained with the numerical matrices and the analytical matrices for the snake are summarized in Table 1.

Table 1: Coupling for the helical snake (linear models)

configuration	|1/2Tr(M-N)|	|cos(2πQ$_x$)-cos(2πQ$_y$)|	ΔQ_{min}
2 snakes / analytical	$5.893\ 10^{-2}$	$5.765\ 10^{-2}$	0.00926
2 snakes / numerical	$8.876\ 10^{-2}$	5.765 10-2	0.01809
2 snakes / numerical no focusing terms diagonal terms = 1	$5.765\ 10^{-2}$	$5.765\ 10^{-2}$	0.02371

where $\delta = 1/\left(2k\rho^2\right)$, $\lambda_o = \sqrt{\varepsilon^2 + \delta^2}$ where $\varepsilon^2 = 1/\left(2\rho^2\right)$ and $\rho = (B\rho)/B_o$.
For the snake design at the injection γ of 27 the matrices for the 1.458 T and the 4 T modules are respectively :

$$\underset{\text{module } B_o = 1.458\text{ T}}{\overset{\text{M1}}{\begin{bmatrix} 0.9996 & 2.3996 & -0.00014 & -0.00033 \\ -0.00035 & 0.9996 & 0.00000 & -0.00014 \\ 0.00014 & 0.00033 & 0.9996 & 2.3996 \\ -0.00000 & 0.00014 & -0.00035 & 0.9996 \end{bmatrix}}} \qquad \underset{\text{module } B_o = 4\text{ T}}{\overset{\text{M4}}{\begin{bmatrix} 0.9967 & 2.397 & -0.00102 & -0.00247 \\ -0.00269 & 0.9967 & 0.000002 & -0.00102 \\ 0.00102 & 0.00247 & 0.9967 & 2.397 \\ -0.000002 & 0.00102 & -0.00269 & 0.9967 \end{bmatrix}}}$$

The matrix for the full snake can be obtained by matrix multiplication of the modules M_1 and M_4, separated by a drift matrix D of 0.32m, the design distance between modules, i.e. :

$$\Sigma = M_1 \otimes D \otimes M_4 \otimes D \otimes M_4 \otimes D \otimes M_1 \otimes D$$

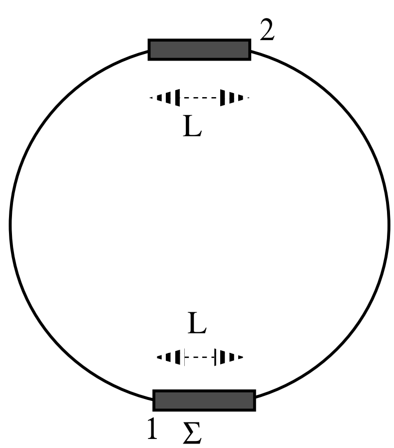
The resulting snake matrix Σ at $\gamma = 27$ is:

$$\begin{bmatrix} 0.9659 & 10.7248 & -0.00225 & -0.02500 \\ -0.00606 & 0.9679 & 0.000014 & -0.00225 \\ 0.00225 & 0.02500 & 0.9659 & 10.7248 \\ -0.000014 & 0.00225 & -0.00606 & 0.9679 \end{bmatrix}$$

4. The one turn map

The model to obtain a linear representation of the ring (1-turn map) is simply to place the 2 snakes in their lattice position, project the snakes and connect the 2 ring locations by a phase space rotation: the 1 turn matrix T is obtained by multiplication of the matrices representing these operations (See Figure 4).

Figure 4. Model for the 1-turn map.



$$T = R(2\rightarrow1) * L^{-1} * \Sigma * R(1\rightarrow2) * L^{-1} * \Sigma$$

3. The snake matrix

3.1 Numerical approach

The SNIG program [2], which is being used for snake design and optimization, allows the computation of particle trajectories in the snake by integrating the equations of motion in the magnetic field of the snake. The helical field is expressed analytically as a continuous superposition of wigglers, an expression which has the right symmetry and satisfies Maxwell equations. A third order expansion of this field proved accurate enough for trajectory calculations. For a detailed discussion of the field and equations of motion in the helical snake see [5]. The SNIG program has been extended to allow the derivation of first and second order transfer matrices from the integration of particle trajectories [4]. A distribution of particles is randomly generated in an ellipse, whose parameters are defined by user-specified twiss functions and emittances at the entrance of the snake. The initial conditions, typically 50 to 100, are tracked through the snake with SNIG and a polynomial fit of the dependence of final from initial conditions is performed. That allows one to derive first and second order transverse matrices, as well as the statistical errors associated. A typical result for the present snake design at injection (γ=27) is listed in Figure 2.
The dependence of the numerical matrix on input parameters, as *number of particles* tracked, *shape* and *size* of the initial *ellipse*, random *seed*, *offset of the ellipse center* (closed orbit at the snake entrance), and *energy* has been systematically checked. The fit results proved to be insensitive (variation of matrix terms < 1%) to most of the parameters varied, with the exception of the ellipse offset and energy. However, in order to have appreciable effects on the matrix (>>1%) the ellipse center offset has to be ~3cm, an unrealistic value for the closed orbit at the entrance of the snake in a corrected machine. The matrix obviously changes with energy. Results for the design snake at storage energy (γ=268) are listed in Figure 3. At higher γ the diagonal terms are ~1, the length ~12m and focusing and coupling terms of the linear matrix as expected decrease with energy.

3.2 Analytical approach

It is possible to derive a first order transfer matrix for 1 module of the helical snake by expanding around the reference helical orbit, approximating the helical dipole field and simplifying the equations of motion by averaging sin-like and cosine-like terms. For a detailed discussion of the derivation see [3]. I will only repeat here the final form of the matrix, for a helical field where L = λ = $2\pi/k$, with λ and k respectively wave length and wave number of the helix.

$$
\begin{bmatrix}
\cos(\delta L) & 0 & -\sin(\delta L) & 0 \\
0 & \cos(\delta L) & 0 & -\sin(\delta L) \\
\sin(\delta L) & 0 & \cos(\delta L) & 0 \\
0 & \sin(\delta L) & 0 & \cos(\delta L)
\end{bmatrix}
\quad
\begin{bmatrix}
\cos\left(\lambda_o L\right) & \dfrac{\sin\left(\lambda_o L\right)}{\lambda_o} & 0 & 0 \\
-\lambda_o \sin\left(\lambda_o L\right) & \cos\left(\lambda_o L\right) & 0 & 0 \\
0 & 0 & \cos\left(\lambda_o L\right) & \dfrac{\sin\left(\lambda_o L\right)}{\lambda_o} \\
0 & 0 & -\lambda_o \sin\left(\lambda_o L\right) & \cos\left(\lambda_o L\right)
\end{bmatrix}
$$

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Linear Coupling effect of the Helical Snake in RHIC

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1. Introduction

The following note describes a simple way to calculate the linear coupling effect of helical snakes in RHIC by calculating the minimum tune separation (ΔQ_{min}) from the one-turn linear map[1]. The latter is derived by using a strictly linear model: the snakes are represented by matrices and the RHIC lattice by the transfer matrices between the location of the snakes. The snake matrix obtained by numerical integration of an ensemble of trajectories [2] is compared to the matrix obtained analytically by simplification of the equations of motions in the snake [3] and the coupling in the 2 cases is calculated. The linear coupling generated in RHIC by the Siberian snakes seems well within the capability of the decoupling correction system at injection, and negligible at storage energy. A more detailed analysis of the effect of snakes on the machine where higher order effects are taken into consideration is in progress.

2. The present snake design

The present nominal design for the RHIC helical snake [4] consists of 4 modules of 2.4 m length, where the helix wavelength equals the module length (see scheme in Figure 1). The B_o field for the outer modules is 1.458 T and for the inner ones 4 T, a configuration that minimizes the closed orbit excursions in the snake ($\Delta y < 27$ mm at the injection γ of 27).
Two snakes will be installed in each RHIC ring at locations separated in betatron phase by πQ_x and πQ_y (The nominal tunes for RHIC are $Q_x = 28.19$ and $Q_y = 29.18$). A drift is reserved for snake installation in the lattice database RHIC92.0.4 next to the Q7 quadrupoles in the 10 o'clock and 4 o'clock interaction regions.

Figure 1. Schematic view of the RHIC helical snake.