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The Phase Advance between AtR Flags

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Abstract

The emittance of the beam passing through the AGS-to-RHIC transfer line (AtR) is calculated from measurements of the beam width on three profile monitor flags. The tolerance of these calculations to measurement errors depends on the two phase advances between the flags. This paper demonstrates the tolerances "as is" (during 1995 commissioning) and investigates the desirability of modifying the phase advances.

Introduction

If the transfer matrix from point i to j is M, so that

$$\begin{pmatrix} x \\ x' \end{pmatrix}_j = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}_i \tag{1}$$

then the beta function at location j is related to the Twiss functions at i through

$$\beta_j = M_{11}^2 \beta_i - 2 M_{11} M_{12} \alpha_i + M_{12}^2 \gamma_i \tag{2}$$

Consequently, the beta functions of 3 neighboring flags, labeled 1, 2, and 3, are related by the equation

$$\begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix} = B_0 \begin{pmatrix} \beta_0 \\ \alpha_0 \\ \gamma_0 \end{pmatrix} \tag{3}$$

where the subscript "0" labels an arbitrary reference point, and the rows of the 3 x 3 matrix B_0 are given by the coefficients on the right hand side of equation 2. Assuming that the flags are located in dispersion free region of the line, the mean square beam size at one of them is

$$\sigma^2 = \epsilon \beta \tag{4}$$

where ϵ is the unnormalized rms emittance. If the beam size at all three flags is represented by the vector

$$s \equiv \begin{pmatrix} \sigma_1^2 \\ \sigma_2^2 \\ \sigma_3^2 \end{pmatrix} \tag{5}$$

then multiplying the left and right hand sides of equation 3 (and dropping subscripts) gives the simple and fundamental result that

$$s = B\, b \tag{6}$$

where the vector b is conveniently defined as

$$b \equiv \epsilon \begin{pmatrix} \beta_0 \\ \alpha_0 \\ \gamma_0 \end{pmatrix} \tag{7}$$

Equation 6 is fundamental in the sense that it relates two known quantities, B and s, to one unknown quantity, b.

The matrix B is known in terms of the transfer matrices between the flags, which are in turn known through the lengths and strengths of intervening drifts and quadrupoles. If the flags are close together, then the uncertainties in B are minimized. The vector s is found, within measurement error, by observing the beam profile widths on a triplet of flags, in the absence of dispersion. The AtR was designed with two such closely spaced (relatively) non-dispersive triplets - one in the U-line (UF3, UF4, and UF5), and one in the W-line (WF1, WF2, and WF3).

In the AtR, the vector b depends on the quality of the beam delivered from the AGS. It is a property of the beam at the reference point. Initially unknown, the vector is derived by inverting Equation 6 to give

$$b = B^{-1}\, s \tag{8}$$

The value of the rms emittance is derived from b using the Twiss identity

$$\beta\gamma - \alpha^2 = 1 \tag{9}$$

to show that, in terms of the components of b,

$$\epsilon^2 = b_1 b_3 - b_2^2 \tag{10}$$

In principle, both U-line and W-line flag triplets should generate the same calculated emittance, since the emittance is a constant of the motion. In practice, the two calculated emittances may vary, for example if beam is scraped off, if the emittance is blown up by passing through multiple flags, or if linear coupling or nonlinear effects are present in significant strength.

The matrix formalism presented above uses three constraints to derive three unknowns. This is not the only way to measure the beam emittance and beta functions. For example, it is possible to introduce more than three constraints (with additional width measurements, or by scanning a quadrupole strength), and then to do a least squares fit of the data. Nonetheless, the matrix formalism is well suited for the present purpose of examining and optimizing the phase advance between the AtR flag triplets, in the presence of errors.

Relative Measurement Errors

Early experience during AtR commissioning suggested that the calculated emittance is sometimes quite sensitive to measurement error. In extreme cases the value of ϵ^2 found by applying equations 8 and 10 is negative, leading to an unphysical imaginary emittance. It is easy to evaluate the robustness of the calculation to measurement errors, using the framework of equations 8 and 10, as embodied in two independent computer codes [1, 2]. The good news is that, very late in AtR commissioning, a flag calibration coefficient snafu was found that had been introducing a systematic measurement error [3]. When the early data sets are corrected for this error, imaginary emittances become very rare.

In the analysis that follows, two assumptions are made. The first is that a single measurement error has been made for flag i, so that

$$\sigma_{i,meas} \quad = \quad (1 \, + \, \delta_i) \, \sigma_{i,beam} \tag{11}$$

where δ_i represents the relative measurement error. It is convenient to study the response to relative errors, since the values of $\beta_{i,beam}$ and $\alpha_{i,beam}$ do not appear in the final results. In essence, the problem is transformed to normalized phase space, where the only two independent parameters are $\Delta\phi_{21,beam}$ and $\Delta\phi_{32,beam}$, the actual phase advance from flag 1 to flag 2, and from flag 2 to flag 3. The irrelevance of all other parameters has been numerically confirmed.

The second assumption is that b corresponds to the design optics. That is, the incoming beam is exactly as expected, so that

$$\begin{pmatrix} \beta \\ \alpha \\ \gamma \end{pmatrix}_{i,beam} \quad = \quad \begin{pmatrix} \beta \\ \alpha \\ \gamma \end{pmatrix}_{i,design} \tag{12}$$

and, in particular,

$$\Delta\phi_{ji,beam} \quad = \quad \Delta\phi_{ji,design} \tag{13}$$

Flag name	β_x [m]	η_x [m]	μ_x [2π]	$\Delta\phi_x$ [deg]	β_y [m]	η_y [m]	μ_y [2π]	$\Delta\phi_y$ [deg]
UF1	65.97	-1.885	0.009		3.78	.0000	0.101	
UF2	6.40	2.278	0.338	118.4	6.33	-.0001	0.606	181.6
UF3	14.42	-0.007	0.998		2.39	.0003	1.176	
UF4	50.55	0.054	1.178	64.9	2.28	.0001	1.303	45.6
UF5	12.66	0.033	1.268	32.1	1.45	.0001	1.413	39.5
WF1	63.47	-0.031	2.599		6.70	.0008	2.800	
WF2	5.95	-0.023	2.788	68.2	2.27	.0023	2.996	70.7
WF3	99.94	-0.045	2.954	59.6	8.71	-.0001	3.124	45.8

Table 1: Design optics for the AtR in 1995, as derived from the file /rap/Holy_Lattice/YTransfer, with the time stamp Mon Jul 24 12:14:06 1995. The columns headed $\Delta\phi_x$ and $\Delta\phi_y$ record the horizontal and vertical betatron phase advances between this flag and the previous flag.

In practice, of course, the beam parameters can differ significantly from the design parameters. Nonetheless, for the sake of simplicity, Equation 13 is assumed from here on.

The AtR line, "as is" in 1995

Table 1 lists the nominal design optics during the 1995 commissioning of the AtR line. It shows that the phase advance between flags varies from 32.1 to 64.9 degrees for the U-triplet, and from 45.8 to 70.7 degrees for the W-triplet. No regular pattern (such as FODO cell) is apparent in the flag spacing, neither for the phase advance nor for the β values.

Figure 1 shows how resilient the U-triplet is to relative measurement errors. While flag UF3 is quite robust in both horizontal and vertical planes, flags UF4 and UF5 are relatively vulnerable. In the horizontal, a single error of about +45% in UF4 or -50% in UF5 leads to an imaginary emittance. The vertical situation is even worse - a single error of about +35% in UF4 or -50% in UF5 makes the calculated emittance imaginary. In practice there will be an error at each flag. Without exploring the three dimensional space any further, it is clear that the beam size must be measured at each of the U-line flags with an

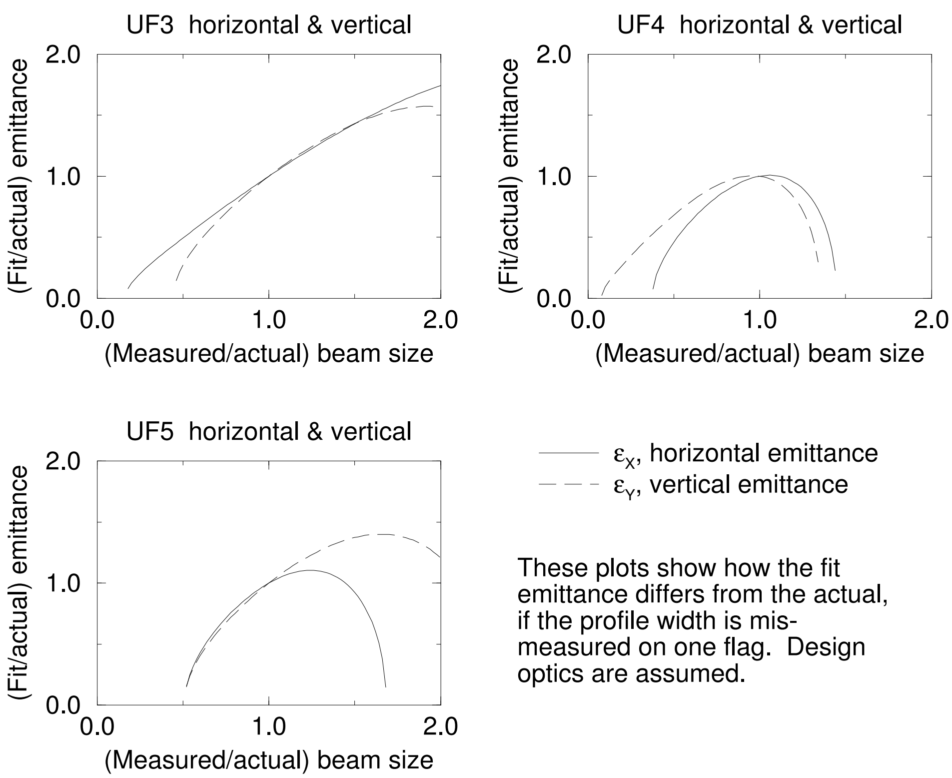

Figure 1: The tolerance of the U-line triplet, UF3, UF4 and UF5, to relative measurement errors.

accuracy of better than about 10%, in order for the calculated emittance to have much validity. This is non-trivial in practice.

The situation in the W-triplet, shown in Figure 2, is somewhat better than in the U-triplet. Nonetheless, significant vulnerabilities still exist, especially in the vertical plane. In the worst case, a single error of about +65% in the vertical beam size at flag WF3 is sufficient to make the calculated vertical emittance imaginary. Here again, the beam widths must be measured with an accuracy of around 10%, in order for the calculated emittance to not only be real and physical, but also to be quantitatively reliable.

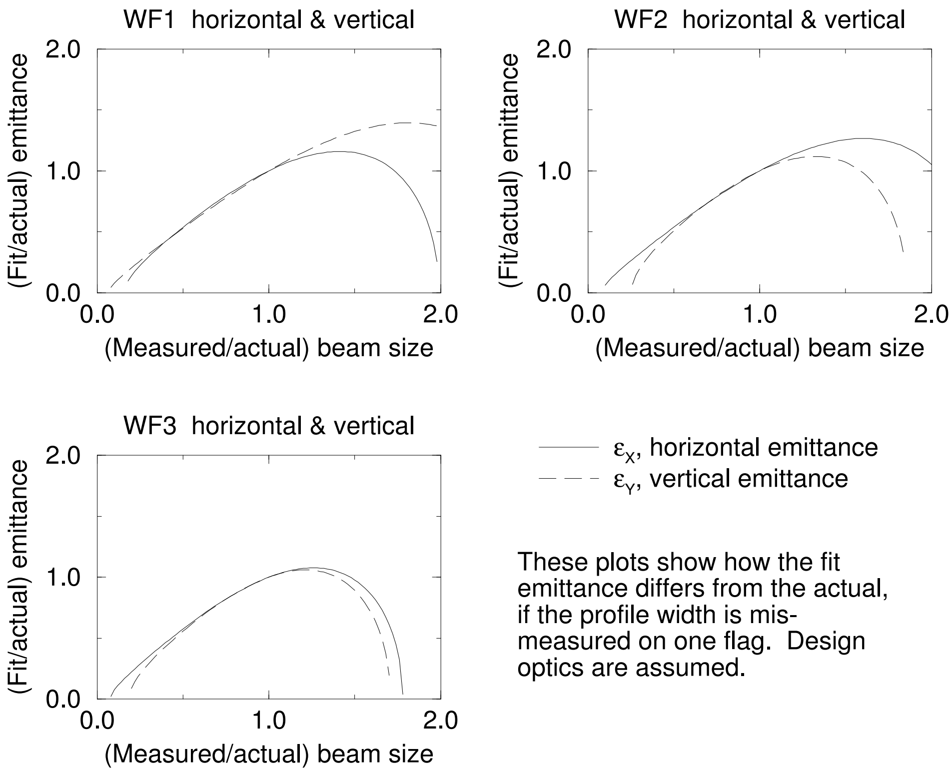

Figure 2: The tolerance of the W-line triplet, WF1, WF2, and WF3, to relative measurement errors.

It is quite possible that Equation 13 is far from the truth, and that using $\Delta\phi_{ji,beam}$ in the simulations instead of $\Delta\phi_{ji,design}$ would make the response to measurement errors even worse. Unfortunately, the $\Delta\phi_{ji,beam}$ values are not very well known. If they were well known, there would be no need to do an error sensitivity analysis!

The optimum phase advance

Figure 3 demonstrates the tolerance to a single relative measurement error when the two flag phase advances are both equal to $\Delta\phi_{beam}$, for values of $\Delta\phi_{beam}$ between 40 and 70 degrees. The first and third flags behave identically, due to the symmetry of the situation. The performance on these two flags unambiguously deteriorates for values above 60 degrees. To the contrary, the second flag deteriorates for values below 60 degrees. (Note that the horizontal scale extends further than for the previous two figures.)

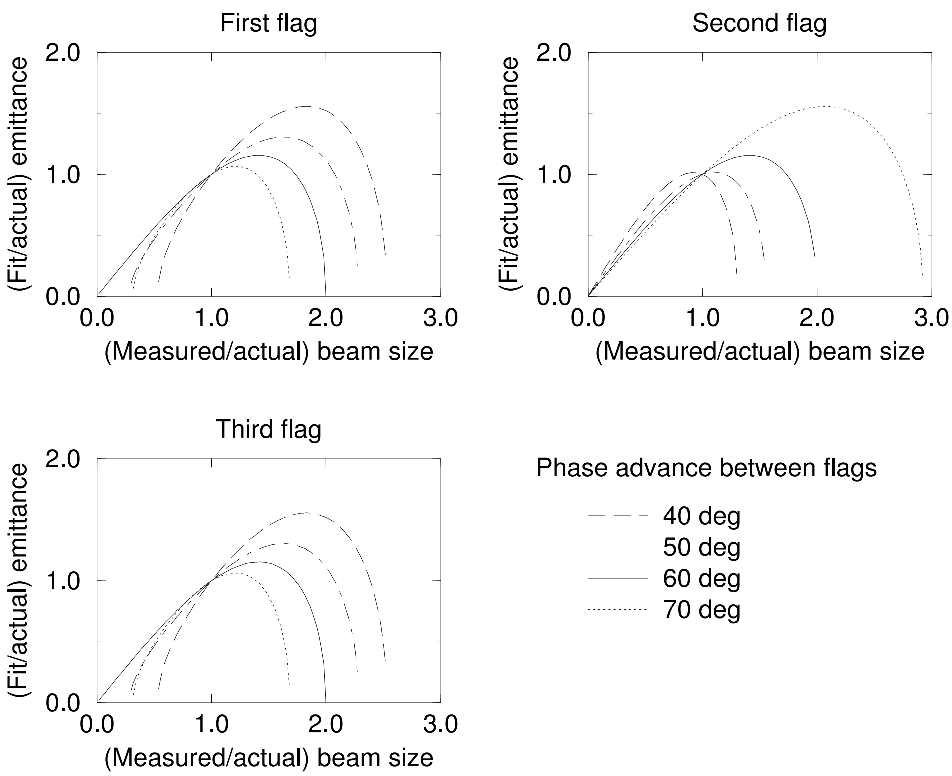

Figure 3: The tolerance of a flag triplet to relative measurement errors, parameterized by the common flag phase advance.

When $\Delta\phi_{beam}$ = 60 degrees, all three flags behave identically. All three are superficially tolerant to relative errors as big as $\delta_i = \pm 100\%$. This seems like the natural optimal phase advance.

Conclusions

In the absence of competing optical constraints, it is desirable to modify the AtR optics so that the phase advance between neighboring flags in each flag triplet is 60 degrees, in both the horizontal and the vertical.

References

[1] S. Peggs, the computer code FLAGON, not publicly released, 1995.

[2] N. Tsoupas, the computer code EMIT, not publicly released, 1995.

[3] L. Hoff, e-mail, December 19, 1995.