Sorting of High-Gradient Quadrupoles in LHC Interaction Regions

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Abstract

Sorting of superconducting high-gradient quadrupoles in the LHC interaction regions with the vector sorting scheme is found to be quite effective in enlargement of the dynamic aperture and improvement of the linearity of the phase-space region occupied by beams. Since the sorting is based on the local compensation of multipole field errors, the effectiveness of the sorting is robust.

1 INTRODUCTION

The beam dynamic of the LHC during collisions is dominated by the magnetic field errors in superconducting high-gradient quadrupoles (MQX) in the triplets of the LHC interaction regions (IRs). Sorting of magnets, in which the magnets are installed according to measured field errors so that the errors on different magnets are partially compensated with each other, has been the easiest way in many cases to reduce the detrimental effects of the random errors without introducing complications. The difficulty to achieve such an effective self compensation of the random errors is to find an optimized magnet configuration which can significantly increase the stability domain of beams, since even for a small number of magnets, the total number of possible magnet arrangements is exceedingly large. During the last decade, several sorting strategies have been proposed and studied extensively [1-8]. Most of them are, however, effective when only one multipole component in the error field is dominant. Recently, a vector sorting scheme has been developed for a systematical control of many multipole components [7,8]. Applications of the vector sorting scheme to arc dipoles as well as insertion quadrupoles of large storage rings have been found to be quite effective in increasing the dynamic aperture and improving the linearity of the phase-space region occupied by beams even when more than one multipole components are responsible for the aperture limitation [7,8]. In the low-β insertion triplets of the LHC IRs, excursion of many beam particles from the magnetic axis is very large because of large β-functions and beam separations during collisions. This makes many high-order multipoles of the field errors in MQX important. On the other hand, large β-functions in the triplets result in a very small phase advance within each triplet and the self compensation of the field errors among the quadrupoles can be relatively easy even though a limited number of interchangeable quadrupoles are available for the sorting. In this report, the effectiveness of the sorting of MQX has been studied with the latest FNL and KEK reference harmonics (version 2.0) [9].

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2 SORTING STRATEGY

The LHC has four interaction points (IPs): IP1 and IP5 are high luminosity points ($\beta^* = 0.5$ m) and IP2 and IP8 low luminosity points. The layout of the inner triplets of the four IPs is almost identical. Each inner triplet comprises four MQX of which two outer quadrupoles, Q1 and Q3, are 6.3 m long (long MQX) and the inner two, Q2A and Q2B, are 5.5 m long (short MQX). Due to the large β_{max} ($\sim$ 4700 m) in the inner triplets of IP1 and IP5, the field quality of MQX of IP1 and IP5 is far more important than that of IP2 and IP8. Therefore, the sorting primarily focuses on the selection of MQX for IP1 and IP5. Since the phase advances are close to zero within each inner triplet of IP1 and IP5, the vector sorting with 2π-cancellation [7,8] can be used for the four MQX in each triplet. The sorting of MQX must, however, observe several constraints. First, of a total of 16 long and 16 short MQX in four IRs, 8 long and 8 short MQX will be built in Fermilab and the others will be built in KEK. Due to hardware constraints such as differences in cryostats, the FNL-made and KEK-made MQX may not be interchangeable. Moreover, after cold measurements, Q2A and Q2B will be welded together so that they are not be separable afterward. Due to a large systematic b_{10} in KEK-made MQX, two different configuration, mixed and unmixed configuration, for installation of MQX are currently under consideration. Sorting of MQX are therefore studied with both of these configurations. In the unmixed configuration, the FNL-make MQX are assumed to be installed in the triplets of IP1 and IP2, and the KEK-made MQX in the triplets of IP5 and IP8. In the mixed configuration, four MQX in each triplet are mixed with two quadrupoles from Fermilab and another two from KEK. In this case, the FNL-made MQX are installed at Q2A and Q2B and KEK-made MQX at Q1 and Q3. For the unmixed configuration, the sorting has to be done with 8 long MQX and 4 pairs of short MQX for each pair of high and low luminosity IPs. For the mixed configuration, on the other hand, there are 16 FNL-made long MQX and 8 pairs of KEK-make short MQX for sorting. It should be noted that even with this small number of magnets, the number of possible magnet configurations is still very large.

To have a better understanding of the sorting scheme for MQX, let's examine the section map of each inner triplet. Let $(\vec{\xi}_0, \vec{\eta}_0)$ and $(\vec{\xi}_4, \vec{\eta}_4)$ be the normalized phase-space variables just before Q1 and immediately after Q3, respectively. Since the phase advances in each triplet are almost zero, the transfer map from $(\vec{\xi}_0, \vec{\eta}_0)$ to $(\vec{\xi}_4, \vec{\eta}_4)$ is

$$\left(\vec{\xi}_4, \vec{\eta}_4\right) = \left(\vec{\xi}_0, \vec{\eta}_0 + \Delta\vec{\eta}\right) \tag{1}$$

where $\Delta\vec{\eta}$ is the nonlinear perturbation due to the multipole field errors in the four MQX. In the thin-lens approximation, the field errors are simply expressed as nonlinear kicks. Due to large variations of β-functions across the MQX, each MQX has to be sliced into a number of pieces in order to use the thin-lens approximation. For the jth piece of the ith MQX, the kick can then be written as

$$\Delta\vec{\eta}_{ij} = \sum_{n=2}^{N} \left[b_n^{(i)} \vec{F}_n^{(ij)}\left(\vec{r}_{ij}\right) + a_n^{(i)} \vec{G}_n^{(ij)}\left(\vec{r}_{ij}\right) \right] \tag{2}$$

where N is the maximal order of multipoles considered. $b_n^{(i)}$ and $a_n^{(i)}$ are coefficients of the nth-order normal and skew multipoles of the ith MQX, respectively. $\vec{F}_n^{(ij)}(\vec{r}_{ij})$ and $\vec{G}_n^{(ij)}(\vec{r}_{ij})$ are vectorial polynomials of $\vec{r}_{ij}$ in degree n, which can be obtained from the multipole expansion of the errors, and

$$\vec{r}_{ij} = (\beta_x^{\frac{1}{2}} \xi_{0x} + \delta x_{ij}, \;\; \beta_y^{\frac{1}{2}} \xi_{0y} + \delta y_{ij}) \tag{3}$$

where $(\beta_x,\, \beta_y)$ are the β-functions at the jth piece of the ith MQX and $(\delta x_{ij}, \delta y_{ij})$ is the closed-orbit offset in horizontal and vertical direction due to a crossing angle. The first-order perturbation of $\Delta\vec{\eta}$ in the transfer map (5) is then

$$\begin{aligned}(\Delta\vec{\eta})_1 &= \sum_{i=1}^{4} \sum_{j} \Delta\vec{\eta}_{ij} \\ &= \sum_{n=2}^{m} \sum_{i=1}^{4} \left[b_n^{(i)} \sum_j \left(\vec{F}_n^{(ij)}\right) + a_n^{(i)} \sum_j \left(\vec{G}_n^{(ij)}\right) \right]\end{aligned} \tag{4}$$

where the summation over j is to sum up all the kicks of a MQX. If $(\Delta\vec{\eta})_1$ can be minimized by sorting the quadrupoles, the multipole field errors in four MQX of each triplet will be partially compensated. In order to examine the magnitude of nonlinear perturbations, a $4N$-dimensional vector $\vec{S}^{(i)} = \left(S_1^{(i)}, ..., S_{4N}^{(i)}\right)$ is used to represent the nonlinear error field on each quadrupole, which is defined by

$$\begin{aligned}
S_n^{(i)}(\vec{\xi_0}) &= b_n^{(i)} \sum_j F_{nx}^{(ij)}\left(\vec{r}_{ij}\right), \\
S_{N+n}^{(i)}(\vec{\xi_0}) &= b_n^{(i)} \sum_j F_{ny}^{(ij)}\left(\vec{r}_{ij}\right), \\
S_{2N+n}^{(i)}(\vec{\xi_0}) &= a_n^{(i)} \sum_j G_{nx}^{(ij)}\left(\vec{r}_{ij}\right), \\
S_{3N+n}^{(i)}(\vec{\xi_0}) &= a_n^{(i)} \sum_j G_{ny}^{(ij)}\left(\vec{r}_{ij}\right),
\end{aligned}$$

for $n = 1, ..., N$. The magnitude of the first-order perturbation due to the field errors of the ith MQX at phase space locations of $\vec{\xi} = \vec{\xi}_0$ is defined by the normal of $\vec{S}^{(i)}$,

$$|\Delta\vec{\eta}_i| = \left|\vec{S}^{(i)}(\vec{\xi_0})\right| = \sqrt{\sum_{n=1}^{4N} \left[S_n^{(i)}(\vec{\xi_0})\right]^2}, \tag{5}$$

and the magnitude of the first-order perturbation in the sectional map of a triplet is then

$$\begin{aligned}|(\Delta\vec{\eta})_1| &= \left|\sum_{i=1}^{4} \vec{S}^{(i)}\right| \\ &= \sqrt{\sum_{n=2}^{N} \left[\left(\sum_{i=1}^{4} b_n^{(i)} \vec{F}_n^{(i)}\right)^2 + \left(\sum_{i=1}^{4} a_n^{(i)} \vec{G}_n^{(i)}\right)^2 \right]}.\end{aligned} \tag{6}$$

The sorting of MQX is thus based on the minimization of $|(\Delta\vec{\eta})_1|$, where $\xi_{0x} = \xi_{0y} = \xi_0$ is a parameter to optimize the sorting. ξ_0 can be chosen initially in such a way that it corresponds to the dynamic aperture of the lattice without sorting. The sorting can then be optimized by tuning ξ_0. It should be noted that the minimization of the normal of the vector sum of all error fields in each triplet in Eq. (6) effectively excludes unintended cancellation of the error fields between different orders of multipoles. Any sorting scheme relying on such cancellation (e.g., cancelling sextupole field with decapole field) is harmful as the effect of sorting will then strongly depend on phase-space locations. Since the feed-down effect of high-order multipoles due to an angle crossing of beams at IPs are different for two counter-rotating beams, the sorting has to be done simultaneously with two counter-rotating beams.

3 EFFECT OF THE SORTING ON THE BEAM DYNAMICS

The LHC collision lattice V5.0 is used in this study. Only the field errors of MQX are included. The random multipole components of MQX are chosen with Gaussian distributions centered at zero and truncated at $\pm 3\sigma_{b_{n+1}}$ or $\pm 3\sigma_{a_{n+1}}$ where $\sigma_{b_{n+1}}$ and $\sigma_{a_{n+1}}$ are the rms value of the nth-order normal and skew multipole coefficient, respectively. Fermilab and KEK reference harmonics of version 2.0 is used in this study. The uncertainty of a systematic error is simply added to the systematic error in such a way that it maximizes the systematic error. The crossing angle of two counter-rotating beams is taken to be 300 μrad and the fractional parts of horizontal and vertical tunes are $\nu_x = 0.31$ and $\nu_y = 0.32$, respectively. Tracking of particle motion has been done without synchrotron oscillations and momentum deviations. The dynamic aperture (DA) has been calculated with 10^5-turn tracking. To improve the statistical significance of the simulations, we used 100 different samples of random multiple components generated with different seed numbers in a random number generator routine. All the multipoles up to 9th order in the field errors of MQX are included.

Table 1: Dynamic aperture of 5 worst cases in 100 random samples of LHC collision lattice with the mixed configuration. $\nu_x = 0.31$, $\nu_y = 0.32$, and the crossing angle is 300μrad. The unit of dynamic aperture is σ.

	Case 9	Case 39	Case 50	Case 26	Case 46	$\langle\text{DA}\rangle_{50}$	$(\text{DA})_{min}$
Original DA	6.5	6.7	6.7	6.8	7.0	8.0	6.5
2nd-order Global Correction	8.1	7.7	8.8	9.1	8.6	9.0	7.7
3rd-order Global Correction	10.1	10.0	9.9	10.7	9.8	10.2	9.2
4th-order Global Correction	10.7	10.7	10.6	10.8	10.6	11.7	9.6
5th-order Global Correction	11.3	11.0	10.6	11.0	10.9	11.3	10.1
6th-order Global Correction	11.4	11.9	10.6	11.0	10.9	11.6	10.3
Sorting (beam1)	12.0	10.0	12.8	10.8	10.0	11.0	9.0
Sorting (beam2)	10.0	11.0	9.2	9.6	10.5	10.3	9.0

Table 2: The same as Table 1 but with the unmixed configuration

	Case 44	Case 47	Case 12	Case 5	Case 20	$\langle\text{DA}\rangle_{50}$	$(\text{DA})_{min}$
Original DA	5.5	5.6	6.1	6.9	6.8	8.0	5.5
2nd-order Global Correction	8.1	8.8	10.0	8.8	8.6	9.0	7.7
3rd-order Global Correction	9.6	9.4	10.0	10.3	9.9	10.2	9.1
4th-order Global Correction	10.5	10.2	10.8	10.9	10.6	11.0	10.0
5th-order Global Correction	12.2	11.0	11.3	11.6	11.1	11.5	10.3
6th-order Global Correction	12.3	11.2	11.7	12.1	11.9	12.0	10.4
Sorting (beam1)	12.4	10.6	13.3	10.4	9.5		
Sorting (beam2)	11.3	10.0	13.2	11.4	9.0		

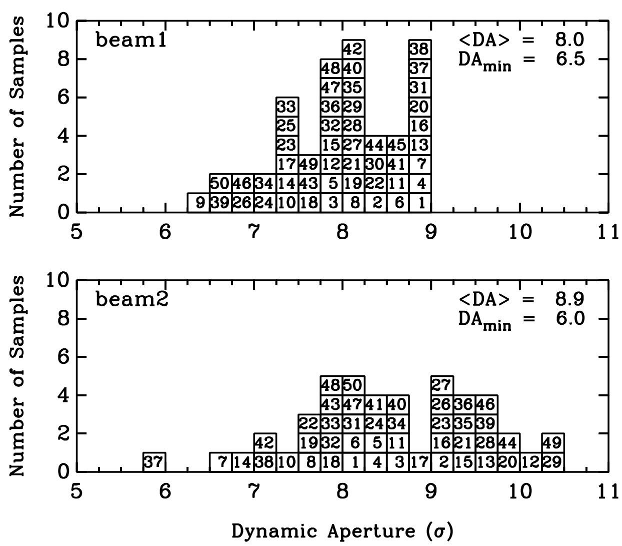

Figure 1: Dynamic aperture of two counter-rotating beams of fifty samples of the mixed configuration without the sorting and nonlinear correctors for MQX. The number in each block identifies each sample.

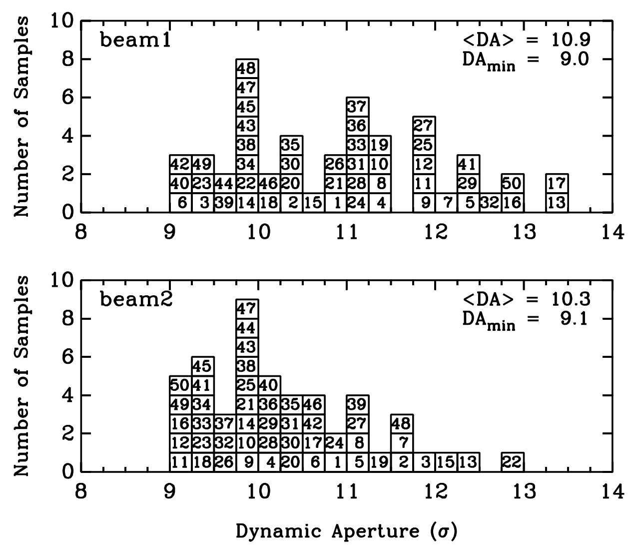

Figure 2: The same as Fig. 1 but with the sorting of MQX.

Figs. 1 and 2 plot the DA of two counter-rotating beams of fifty samples with or without the sorting of MQX for the mixed configuration. No any nonlinear corrector were used in these cases. These fifty samples were the fifty worst cases of the hundred random samples without the sorting in regarding of the DA of beam 1. Without the sorting, the smallest and the average DA of the fifty samples is 6.5σ and 8.0σ for beam 1, and 6.0σ and 8.9σ for beam 2, where σ is the transverse beam size. After the sorting, the smallest and the average DA for both beams are increased to more than 9.0σ and 10.0σ, respectively. In Figs. 3 and 4, the percentage increase of the DA after the sorting is plotted *vs.* the DA without sorting for the fifty samples of the mixed and

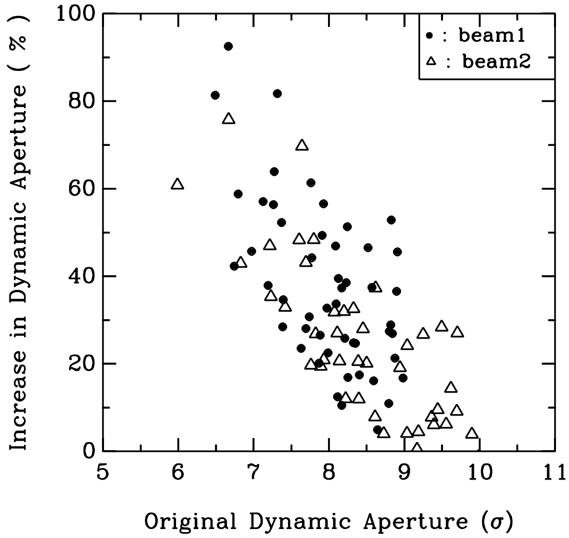

Figure 3: The increase of the DA after the sorting vs. the DA without the sorting for two counter-rotating beams of the fifty samples of the mixed configuration.

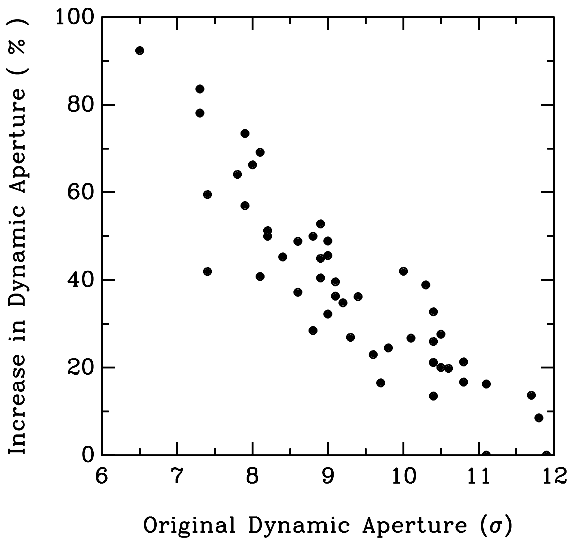

Figure 4: The increase of the DA after the sorting vs. the DA without the sorting for beam 1 of the fifty samples of the unmixed configuration.

unmixed configuration, respectively. It shows that, in general, the smaller the unsorted DA, the larger the increase of the DA after the sorting. For example, before the sorting, two worst cases of the mixed configuration, case 9 for beam 1 and case 37 for beam 2, have a DA of about 6σ. After the sorting, the DA becomes larger than 9.5σ for both cases, which is more than 60% gain in the DA. As the DA without the sorting increases, the gain of the DA after the sorting diminishes. It is understandable that if the original system is already quite linear, the sorting will not result in a substantial improvement. In Table 1 and 2, we list the DA with or without sorting for five samples of the mixed and unmixed configuration. These are the five worst cases in the 100 random samples of the LHC collision lattice with the mixed or unmixed configuration. The DA after the global correction is also listed for a comparison [10]. It shows that the DA of the LHC collision lattice can be increased to 9σ with the sorting of MQX.

4 SUMMARY

The sorting scheme for the insertion quadrupoles of the LHC IRs based on the self compensation of random field errors in each triplet has been shown to be a very effective means to increase the dynamic aperture of the LHC during collisions even though only a limited number of quadrupoles are available for the sorting. Since the sorting scheme is based entirely on the local compensation of multipole field errors in each triplet, it is very robust, i.e. the sorted lattice should be superior to unsorted one even when other factors are included. The effectiveness of the sorting has also been demonstrated with different working points of the LHC [8]. It should be noted that the sorting of magnets requires a reliable cold measurement of multipole components of all the magnets. It is assumed that the cold measurements will be conducted for all MQX. In this study, we assumed that all 32 MQX of the LHC are available for the sorting, i.e. the cold measurement of all MQX can be completed before installing any of them. Practically, however, there will be constraints from the construction and installation schedules which could prevent the pool of the quadrupoles available for sorting from being large. If that was the case, sorting would be less effective. The merit of sorting, however, lies in the fact that it can coexist with any other correcting measures without introducing any harmful side effects. It therefore provides an additional measure for controlling the effects of magnetic field errors.

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