PRINCIPLE OF INTERACTION REGION LOCAL CORRECTION*

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Abstract

For hadron storage rings like the Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC), the machine performance at collision is usually limited by the field quality of the interaction region (IR) magnets. A robust local correction for the IR region is valuable in improving the dynamic aperture with practically achievable magnet field quality. We present in this paper the action-angle kick minimization principle on which the local IR correction for both RHIC and the LHC are based.

1 INTRODUCTION

For hadron storage rings like the Relativistic Heavy Ion Collider (RHIC) [1] and the Large Hadron Collider (LHC) [2], the beam size is the largest near the interaction region (IR) triplets during low-β^* operation. Furthermore, beam-beam effects often require a finite crossing angle, resulting in significant closed orbit deviation from the magnet centers. Machine performance at collision energy, measured in terms of the dynamic aperture, thus depends on achieving the highest possible magnetic field quality and alignment accuracy in the IR magnets.

Magnetic multipole correctors located in the IR region provide active means to compensate the impact of the IR magnetic errors. For hadron machines like RHIC and the LHC, the betatron phase advance across each IR triplet is negligible, and the betatron phase advance between the two IR triplet around each Interaction Point (IP) is near 180°. With these well-defined phase relations, IR-by-IR local correction can be effective and robust.

In this paper, we discuss the principle of action-angle kick minimization for IR local correction. Based on this principle, we have designed and implemented multi-layer multipole corrector packages in the RHIC IR region [3] correcting multipole errors up to the 12th-pole order. Similar correction schemes have been proposed for the LHC IR regions [4, 5, 6]. In Section 2, we review the Hamiltonian describing the particle motion under the magnetic multipole environment. In Section 3, we discuss the figures of merit for global and local error compensation. Discussions and summaries are given in Section 4.

* Work performed under the auspices of the US Department of Energy.
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2 HAMILTONIAN

Under the assumption that the effect of the longitudinal magnetic field is insignificant [7], and that the transverse amplitude of particle motion is small compared with the average bending radius, the magnetic field in a magnet can be expressed in terms of a 2-dimensional multipole expansion

$$B_y + iB_x = B_0 \sum_{n=1}^{\infty} (b_n + ia_n)(x+iy)^{n-1} \tag{1}$$

where x and y indicate the horizontal and vertical directions, respectively, B_0 is the nominal bending field, and $n=1$ is dipole term, $n=2$ is quadrupole term, etc. The Hamiltonian of the charged particle with s as the independent variable is approximately [8]

$$H(x,p_x,y,p_y;s) = -\frac{eA_s}{cp} - \frac{x}{\rho} + \frac{1}{2}\left(p_x^2+p_y^2\right) \tag{2}$$

where ρ is the local radius of curvature, **A** is given by

$$\mathbf{B} = \nabla \times \mathbf{A}, \tag{3}$$

with

$$\begin{aligned} A_s &= (\mathbf{A}\cdot\hat{\mathbf{s}})\left(1+\frac{x}{\rho}\right) \\ &= -\left(1+\frac{x}{\rho}\right) B_0 \sum_{m,n=0;m+n>0}^{\infty} (c_{mn}+e_{mn}) x^m y^n \end{aligned} \tag{4}$$

where the coefficients c_{mn} are given by

$$c_{mn} = \frac{1}{m+n} \begin{pmatrix} m+n \\ n \end{pmatrix} \begin{cases} (-)^{n/2} b_{m+n}, & n \text{ even} \\ (-)^{(n+1)/2} a_{m+n} & n \text{ odd} \end{cases} \tag{5}$$

In Eq. 5, the coefficients c_{mn} are deduced from the recursive equation [8]

$$\begin{aligned} &(m+2)(m+1)\rho^2 e_{m+2,n} + (n+2)(n+1)\rho^2 e_{m,n+2} \\ &+(m+1)(2m+1)\rho e_{m+1,n} + 2(n+2)(n+1)\rho e_{m-1,n+2} \\ &+(m+1)(m-1)e_{mn} + (n+2)(n+1)e_{m-2,n+2} \\ &= -(m+1)\rho c_{m+1,n} - (m-1)c_{mn} \end{aligned} \tag{6}$$

with initial conditions

$$e_{1n} = e_{0n} = 0. \tag{7}$$

We introduce a canonical transformation using the generating function

$$F_2(x, p_{x_\beta}, y, p_{y_\beta}) = (x - D_x\delta - x_c) + (y - D_y\delta - y_c), \tag{8}$$

where $\delta = \Delta p/p_0$, $p_0 c = B_0\rho_0$ is the rigidity of the beam, and ρ_0 is the nominal bending radius. The dispersion functions D_x and D_y, and the closed-orbit displacements x_c and y_c are determined by eliminating the terms in the Hamiltonian that are linear in x_β and y_β. The new Hamiltonian is expressed in terms of the betatron displacements x_β and y_β as

$$\begin{aligned} & H(x_\beta, p_{x_\beta}, y_\beta, p_{y_\beta}; s) \\ &= \frac{1}{2}\left(p_{x\beta}^2 + p_{y\beta}^2\right) + \frac{1}{2\rho_0}\left[\left(\bar{b}_2 + \frac{\bar{b}_1}{\rho}\right) x_\beta^2 - \bar{b}_2 y_\beta^2\right] \\ &+ \frac{1}{\rho_0}\left(B_{20}x_\beta^2 + B_{11}x_\beta y_\beta + B_{02}y_\beta^2 + \right. \\ &\quad \left. + B_{30}x_\beta^3 + B_{21}x_\beta^2 y_\beta + B_{12}x_\beta y_\beta^2 + B_{03}y_\beta^3 + \cdots\right). \end{aligned} \tag{9}$$

Retaining terms that are linear in the closed orbit displacements $\Delta_x = D_x\delta + x_c$ and $\Delta_y = D_y\delta + y_c$, the coefficients B_{ij} are given by [9]

$$\begin{aligned} B_{02} &= -\frac{1}{2}(\Delta b_2 - b_2\delta) - b_3\Delta_x + a_3\Delta_y \\ B_{11} &= -a_2 - 2(a_3\Delta_x + b_3\Delta_y) \\ B_{20} &= -B_{02} + \frac{1}{2\rho}(\Delta b_1 - b_1\delta) \end{aligned} \tag{10}$$

$$\begin{aligned} B_{30} &= \frac{b_3}{3} + b_4\Delta_x - a_4\Delta_y \\ B_{21} &= -3a_3 - 3a_4\Delta_x - 3b_4\Delta_y \\ B_{12} &= -3B_{30} \\ B_{03} &= -B_{21}/3 \end{aligned} \tag{11}$$

$$\begin{aligned} B_{40} &= \frac{b_4}{4} + b_5\Delta_x - a_5\Delta_y \\ B_{31} &= -a_4 - 4(a_5\Delta_x + b_5\Delta_y) \\ B_{22} &= -6B_{40} \\ B_{13} &= -B_{31} \\ B_{04} &= B_{40} \end{aligned} \tag{12}$$

$$\begin{aligned} B_{50} &= \frac{b_5}{5} + b_6\Delta_x - a_6\Delta_y \\ B_{41} &= -a_5 - 5a_6\Delta_x - 5b_6\Delta_y \\ B_{32} &= -10B_{50} \\ B_{23} &= -2B_{41} \\ B_{14} &= -5B_{50} \\ B_{05} &= B_{41}/5 \end{aligned} \tag{13}$$

$$\begin{aligned} B_{60} &= \frac{b_6}{6} + b_7\Delta_x - a_7\Delta_y \\ B_{51} &= -a_6 - 6(a_7\Delta_x + b_7\Delta_y) \\ B_{42} &= -15B_{60} \\ B_{33} &= -20B_{51} \\ B_{24} &= 15B_{60} \\ B_{15} &= B_{51} \\ B_{06} &= -B_{60} \end{aligned} \tag{14}$$

$$\begin{aligned} B_{70} &= \frac{b_7}{7} + b_8\Delta_x - a_8\Delta_y \\ B_{61} &= -a_7 - 7a_8\Delta_x - 7b_8\Delta_y \\ B_{52} &= -21B_{70} \\ B_{43} &= -5B_{61} \\ B_{34} &= 35B_{70} \\ B_{25} &= 3B_{61} \\ B_{16} &= -7B_{70} \\ B_{07} &= -B_{61}/7 \end{aligned} \tag{15}$$

$$\begin{aligned} B_{80} &= \frac{b_8}{8} + b_9\Delta_x - a_9\Delta_y \\ B_{71} &= -a_8 - 8a_9\Delta_x - 8b_9\Delta_y \\ B_{62} &= -28B_{80} \\ B_{53} &= -7B_{71} \\ B_{44} &= 70B_{80} \\ B_{35} &= 7B_{71} \\ B_{26} &= -28B_{80} \\ B_{17} &= -B_{71} \\ B_{08} &= B_{80} \end{aligned} \tag{16}$$

$$\begin{aligned} B_{90} &= \frac{b_9}{9} + b_{10}\Delta_x - a_{10}\Delta_y \\ B_{81} &= -a_9 - 9a_{10}\Delta_x - 9b_{10}\Delta_y \\ B_{72} &= -36B_{90} \\ B_{63} &= -28B_{81}/3 \\ B_{54} &= 126B_{90} \\ B_{45} &= 14B_{81} \\ B_{36} &= -84B_{90} \\ B_{27} &= -4B_{81} \\ B_{18} &= 9B_{90} \\ B_{09} &= B_{81}/9 \end{aligned} \tag{17}$$

$$
\begin{aligned}
B_{10,0} &= \frac{b_{10}}{10} + b_{11}\Delta_x - a_{11}\Delta_y \\
B_{91} &= -a_{10} - 10a_{11}\Delta_x - 10b_{11}\Delta_y \\
B_{82} &= -45B_{10,0} \\
B_{73} &= -12B_{91} \\
B_{64} &= 210B_{10,0} \\
B_{55} &= 126B_{91}/5 \\
B_{46} &= -210B_{10,0} \\
B_{37} &= -12B_{91} \\
B_{28} &= 45B_{10,0} \\
B_{19} &= B_{91} \\
B_{0,10} &= -B_{10,0}
\end{aligned} \tag{18}
$$

where Δb_1 and Δb_2 are the deviation from the design dipole $\bar{b}_1$ and quadrupole $\bar{b}_2$ fields. Regarding the multipole errors as a perturbation, the Hamiltonian given by Eq. 9 can be further rewritten in terms of the action-angle variables $(\phi_x, J_x, \phi_y, J_y)$ as

$$
H(\phi_x, J_x, \phi_y, J_y) = \sum_{i=x,y} \frac{\nu_{i0} J_i}{R_0} + \sum_{l,m=-\infty}^{\infty} A_{lm} e^{il\phi_x} e^{im\phi_y} \tag{19}
$$

using the relations

$$
z = \sqrt{2J_z\beta_z}\cos\chi, \quad p_z = -\sqrt{\frac{2J_z}{\beta_z}}(\sin\chi_z + \alpha_z\cos\chi_z) \tag{20}
$$

where $z = x, y$, and

$$
\chi_z = \phi - \frac{\nu_{z0} s}{R} + \int_0^s \frac{ds'}{\beta_z} \approx \int_0^s \frac{ds'}{\beta_z}. \tag{21}
$$

The action J_z can be written as

$$
J_z = \frac{1}{2\beta_z}\left[z^2 + (\alpha_z z + \beta_z p_z)^2\right]. \tag{22}
$$

Here, ν_{x0} and ν_{y0} are the unperturbed tunes, $2\pi R_0$ is the ring circumference, $\alpha_{x,y}$ and $\beta_{x,y}$ are the Courant-Snyder lattice functions, and A_{lm} represents the error terms which can be deduced from Eq. 9.

3 FIGURES OF MERIT

Conventionally, spread of betatron tunes has been used to guide the design of storage rings. Minimization of the tune spread is often used for global error compensation. Since skew multipoles and odd, normal multipoles do not contribute to the linear tune shift, an extension of such global method is the minimization of nonlinear components of the one-turn map.

The global compensation approaches are valuable for resonance correction as well as dynamic aperture improvement. However, in the case that dominant errors are localized in specific places like the interaction region, global multipole compensation is less robust and often practically difficult to implement during machine operation. Local IR-by-IR compensation employing multi-layer multipole correctors located in the corresponding IR quadrupole triplet region can provide effective correction.

3.1 Tune spread

The tune spread is usually defined as the spread of the tune shift of particles with various betatron amplitudes and momentum deviation. To the first order of the multipole errors, the tune shifts can be obtained by [9] averaging the time derivatives of ϕ_x and ϕ_y while keeping only the A_{00} term from the expansion,

$$
\nu_z = \langle \oint \frac{ds}{2\pi}\frac{\partial H}{\partial J_z}\rangle = \nu_{z0} + \oint \frac{ds}{2\pi}\frac{\partial A_{00}}{\partial J_z} \tag{23}
$$

where $z = x, y$, the sign $\langle\ \rangle$ denotes average over the phase variable, and the integral is performed over the circumference of the closed orbit. Retaining multipole terms up to 11th order ($n = 11$) and closed orbit terms (Δ_x, Δ_y) to the first order, the linear horizontal tune shift is

$$
\begin{aligned}
\nu_x = & \oint \frac{\beta_x ds}{2\pi\rho_0}\left\{-\frac{\Delta b_1}{2\rho} + \frac{b_1\delta}{2\rho} - C_0\right. \\
& +3C_1\beta_x J_x - 6C_1\beta_y J_y \\
& +\frac{15}{2}C_2\beta_x^2 J_x^2 - 45C_2\beta_x\beta_y J_x J_y + \frac{45}{2}C_2\beta_y^2 J_y^2 \\
& +\frac{35}{2}C_3\beta_x^3 J_x^3 - 210C_3\beta_x^2\beta_y J_x^2 J_y \\
& +315C_3\beta_x\beta_y^2 J_x J_y^2 - 70C_3\beta_y^3 J_y^3 \\
& +\frac{315}{8}C_4\beta_x^4 J_x^4 - \frac{1575}{2}C_4\beta_x^3\beta_y J_x^3 J_y \\
& +\frac{4725}{2}C_4\beta_x^2\beta_y^2 J_x^2 J_y^2 - 1575C_4\beta_x\beta_y^3 J_x J_y^3 \\
& \left.+\frac{1575}{8}C_4\beta_y^4 J_y^4\right\}.
\end{aligned} \tag{24}
$$

The linear vertical tune shift is

$$\begin{aligned}
\nu_y &= \oint \frac{\beta_x ds}{2\pi\rho_0} \{ C_0 + 3C_1\beta_y J_y - 6C_1\beta_x J_x \\
&\quad -\frac{15}{2}C_2\beta_y^2 J_y^2 + 45C_2\beta_x\beta_y J_x J_y - \frac{45}{2}C_2\beta_x^2 J_x^2 \\
&\quad +\frac{35}{2}C_3\beta_y^3 J_y^3 - 210C_3\beta_y^2\beta_x J_y^2 J_x \\
&\quad +315C_3\beta_y\beta_x^2 J_y J_x^2 - 70C_3\beta_x^3 J_x^3 \\
&\quad -\frac{315}{8}C_4\beta_y^4 J_y^4 + \frac{1575}{2}C_4\beta_y^3\beta_x J_y^3 J_x \\
&\quad -\frac{4725}{2}C_4\beta_y^2\beta_x^2 J_y^2 J_x^2 + 1575C_4\beta_y\beta_x^3 J_y J_x^3 \\
&\quad -\frac{1575}{8}C_4\beta_x^4 J_x^4 \Big\}
\end{aligned} \tag{25}$$

where the coefficients are

$$\begin{aligned}
C_0 &= \frac{\Delta b_2 - b_2\delta}{2} + b_3\Delta_x - a_3\Delta_y \\
C_1 &= \frac{b_4}{4} + b_5\Delta_x - a_5\Delta_y \\
C_2 &= \frac{b_6}{6} + b_7\Delta_x - a_7\Delta_y \\
C_3 &= \frac{b_8}{8} + b_9\Delta_x - a_9\Delta_y \\
C_4 &= \frac{b_{10}}{10} + b_{11}\Delta_x - a_{11}\Delta_y .
\end{aligned} \tag{26}$$

3.2 Action-angle kick

The figures of merit for local minimization are the action-angle kicks produced by the IR magnets at each specified multipole order. The action kicks can be expressed as

$$\begin{aligned}
\Delta J_x &= -\int ds \frac{\partial H}{\partial \phi_x} = -\sum_{l,m=-\infty}^{\infty} il\Delta J_{lm} \\
\Delta J_y &= -\int ds \frac{\partial H}{\partial \phi_y} = -\sum_{l,m=-\infty}^{\infty} im\Delta J_{lm}
\end{aligned} \tag{27}$$

where

$$\Delta J_{lm} \approx \int ds\, A_{lm} \exp\left(il\int_0^s \frac{ds'}{\beta_x}\right) \exp\left(im\int_0^s \frac{ds'}{\beta_y}\right). \tag{28}$$

The correction scheme is simplified by the fact that the action is approximately a constant of motion at the time scale of the revolution period, and that the relative betatron phase is well defined within the high-β IR region. Minimization is performed on every significant multipole error b_n (or a_n). Since the available physical space is usually limited in the high-β region, corrector packages containing multi-layer corrector elements of various multipole content are used. For each multipole order c_n (either a_n or b_n), (a minimum of) two correction elements are implemented for every IR, each located at symmetric locations around the IP. Due to the anti-symmetry of the IR optics, one of the two elements is near the maximum β_x location, and the other is near the maximum β_y location, resulting in an effective compensation. The strengths of these correction elements are determined by minimizing the two quantities

$$\int_L ds\; C_z\; c_n + (-)^n \int_R ds\; C_z\; c_n, \quad z = x, y \tag{29}$$

taking advantage of the negligible betatron phase advance within each triplet, and approximate 180° phase advance between the triplets. The integral is over the entire left-hand-side (L) or right-hand-side (R) triplet. In general, the weights C_z in Eq. 29 are chosen according to the multipoles as:

$$C_x = \begin{cases} \beta_x^{n/2} & \text{for } b_n \\ \beta_x^{(n-1)/2}\beta_y^{1/2} & \text{for } a_n \end{cases} \tag{30}$$

and

$$C_y = \begin{cases} \beta_y^{n/2} & \text{for even } b_n \text{ or odd } a_n \\ \beta_x^{1/2}\beta_y^{(n-1)/2} & \text{for odd } b_n \text{ or even } a_n \end{cases} \tag{31}$$

4 DISCUSSIONS AND SUMMARY

Compared with the tune shift, the action (and angle) kick has similar dependence on the lattice optics β_z for each multipole. Consequently, minimization of action-angle kicks results in a reduction of tune spread and an improvement of the dynamic aperture. The compensation scheme is usually not sensitive to the change of β^*, as long as β^* is low at the IP (usually the only relevant case) so that β at a distance s from the IP satisfies the relation $\beta\beta^* \cong s^2$. In the case of two beams sharing the same IR magnets, the compensation is equally effective for both intersecting beams, since the optics of the interaction region is anti-symmetric. Although closed-orbit deviation (e.g. due to finite crossing angle) is not taken into account, the correction is usually effective since the effect of the magnet feed-down is partially compensated by the feed-down from the correctors.

The most straightforward approach for local correction on multipoles of $n = 3$ and higher order is the dead-reckoning method, setting the corrector strength according to Eq. 29 using bench-measured magnetic multipole errors. Up to 10% of measurement errors and quench/thermal cycle dependent multipole variations can usually be tolerated [3, 5, 6]. The method is also immune to moderate closed-orbit errors and corrector misalignments [6].

Multipole errors of order $n = 1, 2$ produce closed orbit deviation, tune perturbation, and coupling. The effects are usually compensated using beam-based tuning. In the case that skew quadrupole components and quadrupole misalignment of the IR triplets is significant, local decoupling utilizing the a_2 corrector in the IR can be effective [10]. The corrector strength obtained from the local decoupling scheme is similar to those given by Eq. 29. Beam-based corrections for higher order multipoles have also been pursued by several authors recently [11, 12].

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