

BEAM-BEAM SYNCHROBETATRON RESONANCES

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Electron storage rings see dangerous beam-beam synchrotron sidebands which are almost invisible to single beams, for example the diagonal resonances $Q_z - Q_x = mQ_s$. Colliding protons and antiprotons in the SPS do not see diagonal sidebands, despite the absence of stabilisation by radiation damping. One SPS feature, proton loss or emittance blow-up at vertical tunes of less than $Q_z = (0).674$, could be a beam-beam synchrotron effect off the 4/6 resonance.

This report uses a strong-weak numerical simulation to show that an apparent tune modulation, induced by longitudinal collision point oscillations, can cause synchrotron sidebands, to a distance of about $2Q_s$ from a beam-beam resonance. The concurrent analytic description illustrates several reasons why flat beams

$$1) \quad \beta_z^* \ll \beta_x^* \quad , \quad \sigma_z^* \ll \sigma_x^*$$

behave quite differently from round beams

$$2) \quad \beta_z^* = \beta_x^* \quad , \quad \sigma_z^* = \sigma_x^*$$

and why the SPS is so resonance free. Transverse electron trajectories can become stochastic beyond a critical longitudinal amplitude limit, making microbeta optics unstable. Round beam-beam sidebands are found to be stronger near $Q_z = 2/3$ than near the diagonal. Chromaticity is more dangerous to the SPS as a source of (real) tune modulation, than the modulation due to collision point oscillations.

APPARENT TUNE MODULATION IN MINIBETA OPTICS**The approximate beam-beam impulse**

At the instant of collision, $t=0$, the strong beam is assumed to be gaussian in three dimensions, so that the vertical size is given as a function of longitudinal position by

$$3) \quad \sigma_z(s) = \left[1 + \left(\frac{s}{\beta_z^*} \right)^2 \right]^{\frac{1}{2}} \cdot \sigma_z^* = b^{-1} \left[b^2 + \left(\frac{s}{\sigma_s} \right)^2 \right]^{\frac{1}{2}} \cdot \sigma_z^*$$

while horizontally

$$\sigma_x(s) = \sigma_x^* \quad \text{flat beams}$$

4)

$$\sigma_x(s) = \sigma_z(s) \quad \text{round beams}$$

Electron rings have values of the dimensionless beta squeezing factor, $b = \beta_z^*/\sigma_s$, as low as about 2.0, while in today's SPS b is at least 3.2. In contemporary electron storage rings the beam-beam effect often limits the number of particles, N , that can be stored, leading to peak luminosity in large b lattices going as b^{-1} , not $b^{-0.5}$. However, numerical simulations, for example those of Steve Myers¹, show that the peak current is severely decreased when b is smaller than about 1.0.

A test particle passing through a flat beam with a small vertical displacement z and a longitudinal position $s = t+d \cdot \sigma_s$, gets an angular kick

$$5) \quad \Delta z' = -N \int_{-\infty}^{\infty} dt \int_0^z dz \cdot \frac{e^{-\frac{z^2}{2\sigma_z^2(s)}}}{\sigma_z^2(s)} \cdot \frac{e^{-\frac{(s+t)^2}{2\sigma_s^2}}}{\sigma_s}$$

so that integrating over a scaled time, $q = t/\sigma_s$,

$$6) \quad \frac{\Delta z'}{z} = -Nb^{\frac{1}{2}} \int_{-\infty}^{\infty} dq \cdot \frac{e^{-\frac{(2q+d)^2}{2}}}{(b^2 + (q+d)^2)^{\frac{1}{2}}}$$

This distributed kick is equivalent to an impulse at the longitudinal position corresponding to the centre of gravity of the integrand, a position which oscillates with the longitudinal displacement of a particle. If d is much greater than 1 the impulse is at $s = -d/2$, a location assumed in the simulation despite d often being of order 1, even for large longitudinal amplitudes.

The flat beam test particle feels a tune shift

$$7) \quad \xi\left(\frac{d}{2}\right) = -\frac{\Delta z'}{z} \frac{\beta\left(\frac{d}{2}\right)}{4\pi} = -Nb^{\frac{1}{2}} \left(1 + \left(\frac{d}{2b}\right)^2\right)^{\frac{1}{2}}$$

which increases linearly with large longitudinal displacements, so that large amplitude particles are only stabilised by reducing the strong beam current, N . Round beam test particles, in contrast, have a tune shift which is independent of collision position, and even of beta, since for them

$$8) \quad \frac{\Delta z'}{z} = -Nb \int_{-\infty}^{\infty} dq \cdot \frac{e^{-\frac{(2q+d)^2}{2}}}{b^2 + (q+d)^2}$$

so that

$$9) \quad \xi\left(\frac{d}{2}\right) = Nb^0$$

This is the first reason why round beams are more stable than flat beams.

Fourier analysis of the apparent tune

If a particle on turn n gets its beam-beam impulse at a displacement s_n , where

$$10) \quad s_n = s_0 \cos(2\pi Q_s n)$$

then the kick is received at a betatron phase

$$11) \quad \phi_n = \int_0^{s_n} \frac{ds}{\beta(s)}$$

and the particle perceives an instantaneous tune

$$12) \quad Q = Q_0 + \frac{1}{2\pi} \frac{d\phi_n}{dn} = Q_0 - Q_s \frac{s_0}{\beta(s_n)} \sin(2\pi Q_s n)$$

Fourier decomposition of the apparent tune

$$13) \quad Q = Q_0 + Q_s \sum_{m=0}^{\infty} F_{2m+1}(a_s) \cdot \sin((2m+1)2\pi Q_s n)$$

reveals harmonics at odd multiples of the synchrotron frequency, Q_s . The amplitudes of these harmonics, plotted in Figure 1, are functions of the dimensionless longitudinal collision amplitude, $a_s = s_0/\beta^*$, and are given by

$$14) F_{2m+1}(a_s) = (-1)^{m+1} a_s \frac{4}{\pi} \int_0^{\pi/2} d\alpha \frac{\cos((2m+1)\alpha)\cos\alpha}{1+a_s^2 \sin^2\alpha}$$

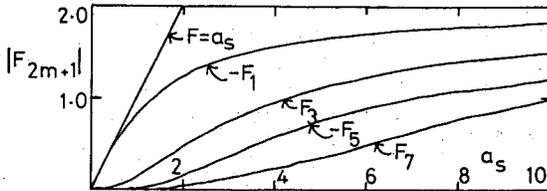


Figure 1 Harmonic amplitudes

At small a_s the dominant effect is tune modulation of depth $a_s Q_s$ at a frequency of Q_s , but when a_s is larger than, say, 4.0, many harmonics are important. It has already been shown, in theory^{2,3} and by simulation⁴, that real tune modulation near a nonlinear magnetic resonance of order n causes synchrotron sidebands separated by Q_s/n . These sidebands get weaker beyond the distance at which the resonance is no longer repeatedly crossed. Since the asymptotic harmonic amplitude is $F=2$, beam-beam sidebands should never be strong further than $2Q_s$ from a resonance.

When round beams collide, both tunes are modulated together, so the tune point motion is parallel to the diagonal. This is a second important contrast to flat beams, where only the vertical tune is modulated and the coupling resonance is crossed at 45 degrees, strengthening the diagonal sidebands.

Useful lifetimes are only obtained with electrons if trajectories with synchrotron amplitudes upto, say, 8 standard deviations are stable. In a lattice with $b=1$, electrons with $a_s=4$ should be stable. Meanwhile, although protons are not helped by radiation damping, neither are they hurt by radiation diffusion, and do not need large amplitude stability. SPS particles which are only just in the r.f. bucket, wavelength 1.5 metres, seeing a beta of 0.75 metres, still have only a modest value of $a_s=0.5$. This is a third reason why hadron collisions are comparatively stable.

FLAT BEAM SIMULATION RESULTS

Particles start off with 'typical' vertical and horizontal amplitudes of $\sqrt{2}\sigma_z$ and $\sqrt{2}\sigma_x$, with a horizontal tune $Q_x=0.402$, and with a single kick per turn of strength $\xi=0.02$. Radiative effects are not included. Figure 2 shows the maximum vertical amplitude seen by a particle during 500 synchrotron periods of 27 turns, about one damping time in CESR, as a function of the unshifted vertical tune, Q_{z0} .

When $a_s=0$ only two resonances appear, $Q_z=1/3$ and $Q_z=Q_x$. Several more arise when $a_s=1$, most notably the family $2Q_x-2Q_z=mQ_s$, separated by $Q_s/2$ because the vertical kick is modulated by the horizontal displacement⁵ at even multiples of Q_x . Figure 3 shows violent instability below $Q_z=Q_x$ when $a_s=3$, due to the mixing of many harmonics.

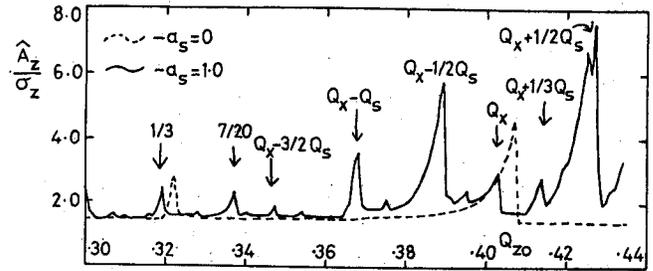


Figure 2 Maximum flat beam amplitudes

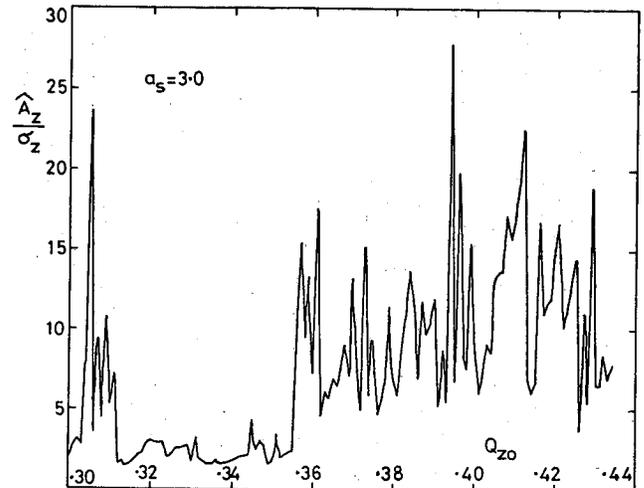


Figure 3 Flat beam sensitivity when $a_s=3.0$.

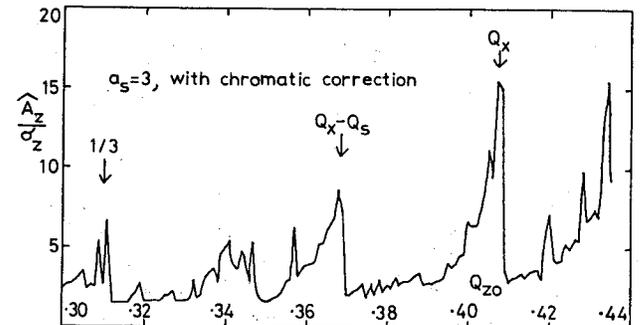


Figure 4 Stabilisation of $a_s=3$ by chromaticity

Stabilisation by chromaticity

A particle with a collision point amplitude of a_s also has a relative energy amplitude of a_e , which couples with chromaticity to generalise the instantaneous apparent tune from 13) to become

$$15) Q = Q_0 + Q_s \sum_{m=0}^{\infty} F_{2m+1}(a_s) \sin((2m+1)2\pi Q_s n) + \left(\frac{dQ}{dE} \frac{E_0}{Q_0} \right) Q_0 a_e \sin(2\pi Q_s n)$$

Since F_1 is negative, the amplitude of the first tune modulation harmonic can be reduced to zero at a given a_s by a positive vertical chromaticity,

$$16) \frac{dQ}{dE} \frac{E_0}{Q_0} = \frac{-F_1(a_s)}{a_e} \cdot \frac{Q_s}{Q_0}$$

Small amplitude particles, which least need it, are all stabilised together by a chromaticity larger than that needed to control the head-tail effect, since then F_1 , a_s and a_e are proportional. Figure 4 shows that the tune plane is still far from featureless when the first harmonic is removed with

$a_s=3$ (cf. Figure 3), due to the unstabilised higher harmonics. The resonance $Q_z=Q_x-Q_s/2$ disappears, perhaps significantly.

Chaotic phase space

According to whether two trajectories, started infinitesimally close together in phase space, diverge linearly or exponentially as a function of time, a region of phase space is called non-chaotic or chaotic. Figure 5 tracks the divergence of two trajectory pairs, with a tune of $Q_z=0.362$ on the resonance $Q_z=Q_x-Q_s$, over many orders of magnitude. The initial relative amplitude displacement is 10^{-13} , the double precision noise limit of a NORD-500 computer, while the first pair has $a_s=0.76$, the second $a_s=0.78$, and every other parameter is 'typical'. Chaos sets in at a small amplitude on this synchrotron resonance.

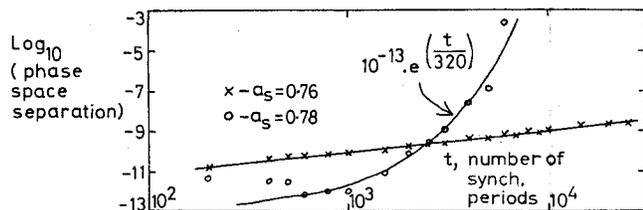


Figure 5 Chaotic and non-chaotic divergence.

ROUND BEAM SIMULATION RESULTS

With collision point oscillations

Particles stored in the SPS for one second are simulated by tracking with $Q_x=0.685$ for 200 synchrotron periods of 199 turns, with only one collision per turn, not six. Figure 6 plots the maximum amplitudes against unshifted vertical tune, in a possible future SPS with $\beta=0.4$ metre and $a_s=1.0$, and with a tune shift of 0.02, for direct comparison with the flat beam results of Figure 2. Typical ($\sqrt{2}\sigma$) and large ($3\sqrt{2}\sigma$) initial vertical amplitude particles are tracked, with initial horizontal amplitudes of $\sqrt{2}\sigma$.

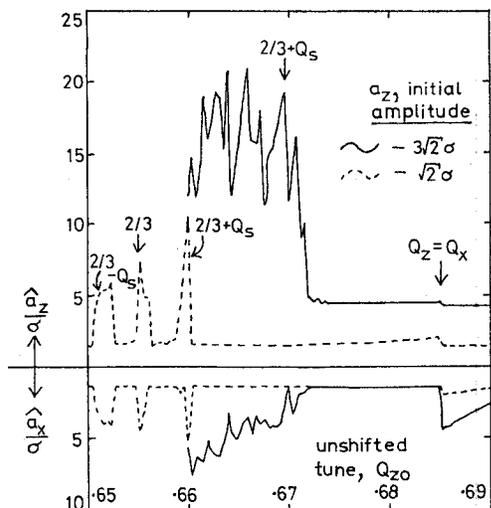


Figure 6 Round beam response, $a_s=1$

Typical particles respond strongly to $Q_z=2/3$ ($4/6$), and to sidebands displaced by Q_s , but the diagonal resonance $Q_z=Q_x$ is slight and broad, with no evidence of sidebands, in marked contrast to the flat beam case. Large amplitude particles are very sensitive to the $2/3$ resonance, upto an

unshifted tune cutoff at $Q_{z0}=0.672$, or a shifted tune of 0.675. Higher tune particles, with a modulation depth of about $a_s \cdot Q_s=5 \times 10^{-3}$, no longer cross and recross the $2/3$ resonance.

A fourth difference between round and flat beam collisions is that the footprint, (Q_x, Q_z) versus (a_x, a_z) , is more compact for a round beam than for a flat beam, as shown in Figure 7. The round beam footprint is in quantitative agreement with analytic prediction⁶.

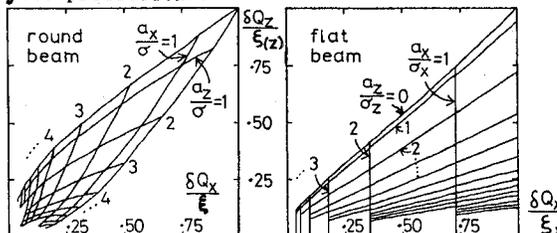


Figure 7 Round and flat beam footprints

With vertical chromaticity

Chromaticity is a more dangerous source of tune modulation than collision point oscillations, in today's SPS, because few particles have a_s larger than about 0.3. Figure 8 shows the large amplitude response to a vertical tune modulation of amplitude 5×10^{-3} at a pure frequency of Q_s , that is, to a relative energy amplitude of 7×10^{-4} and a perfectly constant vertical chromaticity of 0.27.

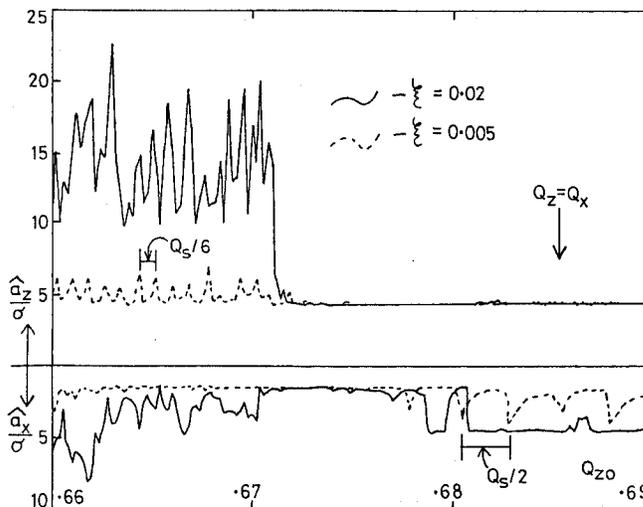


Figure 8 Round beam response to chromatic modulation

The behaviour with a tune shift of 0.02 is reminiscent of apparent tune modulation to the same depth, Figure 6. A more realistic tune shift of 0.005 shows significant sidebands of $Q_z=2/3$, with a spacing $Q_s/6$, upto $Q_{z0}=0.672$, and shows sidebands of $Q_x=Q_z$ with a spacing of $Q_s/2$.

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