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1. Introduction

In the SSC, it is envisioned that there will be a horizontal orbit corrector at each focussing quadrupole (QF), a vertical corrector at each defocussing quadrupole (QD) and a beam position monitor (BPM) at each QF and QD. For each plane, therefore, there are one corrector and two BPM's per cell.

Since there are about 420 cells in the SSC, a straightforward orbit correction requires inverting an 420x420 matrix in which all elements are in general nonzero. This can be avoided by applying a beam bump technique.[1] A formulation of this technique in which the matrix elements vanish except for three lines along the diagonal has been formulated in Ref.2. In section 2, we will summarize the result of the beam-bump method from Ref.2.

Section 3 gives our study results using this technique for the case of a long string of FODO cells. Section 4 applies these study results to estimate for the SSC (1) the rms corrector strength needed for orbit correction, and (2) the rms orbit distortion after correction. The orbit corrector strength suggested for the SSC is 10 times the rms value and agrees with a previous estimate.[3]

2. The Beam Bump Orbit Correction[2]

Consider either the horizontal or the vertical plane. The orbit correction arrangement is as sketched in the figure below:

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     x_{n-2}     x_{n-1}     x_n        x_{n+1}     x_{n+2}      x_{n+3}
_____*___________*___________*___________*___________*____________*_____
     θ_{n-2}                 θ_n                     θ_{n+2}
```

An asterisk means the position of a quadrupole; x_n means the orbit reading on the n-th BPM before correction; θ_n means the strength of the n-th corrector for orbit correction in units of kicking angle. The β-function at the correctors is β_{max}. The β-function at those BPM's where there is no corrector is β_{min}. Let the phase advance per half cell be φ. Let there be M cells in the long arc. We will see that the number of cells is not a crucial quantity.

Let X_0 be the 2M-dimensional vector consisting of x_n's. Let Θ be the M-dimensional vector consisting of θ_n's. The result of ref.2 applied to the case considered here is summarised below:

$$\Theta = S\,G \qquad (1)$$

where S is an MxM matrix

$$S = \begin{bmatrix} B & A & 0 & 0 & \dots & A \\ A & B & A & 0 & \dots & 0 \\ 0 & A & B & A & \dots & 0 \\ . & & & & & \\ . & & & & & 0 \\ . & & & & B & A \\ A & 0 & 0 & \dots & A & B \end{bmatrix} \qquad (2)$$

with

$$A = 1/\left(\beta_{max} \sin 2\varphi\right) \quad \text{and} \quad B = -2/\left(\beta_{max} \tan 2\varphi\right).$$

The M-dimensional vector G is given by

$$G = -A^{-1}\,T\,X_0 \qquad (3)$$

where A is an MxM tri-diagonal matrix

$$A = \begin{bmatrix} 1 & a & 0 & \dots & & a \\ a & 1 & a & \dots & & 0 \\ 0 & a & 1 & \dots & & 0 \\ . & & & & & \\ . & & & & 1 & a \\ a & 0 & & & a & 1 \end{bmatrix} \qquad (4)$$

with

$$a = (1/2)\ [\, 1 + 2\,(1+\sin\varphi)^2\,]^{-1}$$

and T is a 2MxM matrix

$$T = \begin{bmatrix} b & c & b & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & b & c & b & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & b & c & b\ \dots & 0 \\ . & & & & & & & \\ . & & & & & & & \\ b & 0 & 0 & \dots & & 0 & c & b \end{bmatrix} \qquad (5)$$

with

$$c = 1/(2d^2+1), \quad b = cd, \quad d = [\, 2(1+\sin\varphi)]^{-1}$$

The matrix A is simple enough to allow inversion analytically. The result is

$$A^{-1} = \begin{bmatrix} a_0 & a_1 & a_2 & \dots & a_1 \\ a_1 & a_0 & a_1 & \dots & a_2 \\ a_2 & a_1 & a_0 & \dots & a_3 \\ . & & & & \\ . & & & & \\ a_1 & a_2 & a_3 & \dots & a_0 \end{bmatrix} \qquad (6)$$

where

$$a_k = (-t)^k (1 - 4a^2)^{-1/2}$$

$$t = [1 - (1-4a^2)^{1/2}] / 2a$$

Given the orbit error X_0, this scheme yields the corrector strengths Θ needed to minimise the rms deviation of the orbit.

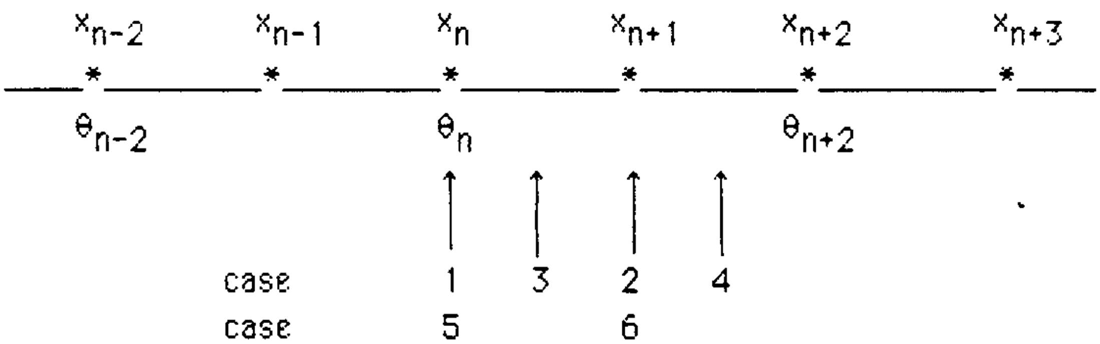
3. Studies

Consider an orbit error produced by a kick with a unit (1 mrad) kick angle. The orbit at the n-th BPM is

$$x_n = \frac{(\beta_e \beta_n)^{1/2}}{2 \sin \pi\nu} \cos (\pi\nu - | \psi_n - \psi_e |) \qquad (7)$$

where β_e and ψ_e are the beta-function and betatron phase at the error kick and $\pi\nu = M\psi$.

The orbit error (7) is fed into the correction scheme for correction. Four cases of error locations were run, indicated by the indices in the following figure:



Sources for cases 1 and 2 are quadrupole magnet misalignments. Sources for cases 3 and 4 are dipole strength error (for the horizontal orbit) and dipole roll angle error (for the vertical orbit).

We also studied two more cases (cases 5 and 6 indicated in the above figure) when the orbit is

$$x_n = 0 \text{ for all } n \text{ except } x_i = 1\text{mm for some } i. \qquad (8)$$

This represents an orbit error obtained when the i-th BPM is misaligned by 1mm.

Tables 1 to 6 give the orbit correction results for cases 1 to 6 respectively, for a phase advance per cell of $2\varphi=60°$. Tables 7 to 12 give the same for $2\varphi=90°$. Note that the orbit before correction depends on the tune of the storage ring, which in this study is determined by the number of cells M. The corrected orbit and the corrector strengths, on the other hand, are only local properties. They do not depend on M as long as M is large enough (M>10, say).

Table 1 An angular kick of 1 mrad is located at the n-th corrector. The correction just powers the n-th corrector by -1 unit, as it should, and the orbit after correction is zero. The angles are in units of mrad. The orbits are in units of mm. The last row gives the rms's to be used later. $2\psi=60°$.

i	θ_i	x_i (before correction)	x_i (after correction)
n-6	0.	0.000	0.0000
n-5		50.000	0.0000
n-4	0.	150.000	0.0000
n-3		100.000	0.0000
n-2	0.	150.000	0.0000
n-1		50.000	0.0000
n	-1.000	0.000	0.0000
n+1		50.000	0.0000
n+2	0.	150.000	0.0000
n+3		100.000	0.0000
n+4	0.	150.000	0.0000
n+5		50.000	0.0000
n+6	0.	0.000	0.0000
$[\sum (\,)^2]^{1/2}$	1.00		0.0000

Table 2 Angular kick error is located at the (n+1)-th BPM. $2\varphi=60°$.

i	θ_i	x_i (before correction)	x_i (after correction)
n-6	0.0003	-50.000	-0.0065
n-5		0.000	0.0214
n-4	-0.0028	50.000	0.0706
n-3		50.000	-0.2331
n-2	0.0308	100.000	-0.7698
n-1		50.000	2.543
n	-0.3359	50.000	8.397
n+1		0.000	-27.74
n+2	-0.3359	50.000	8.397
n+3		50.000	2.543
n+4	0.0308	100.000	-0.7698
n+5		50.000	-0.2331
n+6	-0.0028	50.000	0.0706
n+7		0.000	0.0214
n+8	0.0003	-50.000	-0.0065
$[\sum(\ )^2]^{1/2}$	0.477		30.4

Table 3 Kick error between the n-th and the (n+1)th BPM. $2\psi=60°$.

i	θ_i	x_i (before correction)	x_i (after correction)
n-6	0.0002	-36.60	-0.0047
n-5		21.13	0.0156
n-4	-0.0021	100.00	0.0517
n-3		78.87	-0.1706
n-2	0.0225	136.6	-0.5635
n-1		57.74	1.861
n	-0.6685	36.60	6.147
n+1		21.13	-20.30
n+2	-0.2459	100.00	6.147
n+3		78.87	1.861
n+4	0.0225	136.6	-0.5635
n+5		57.74	-0.1706
n+6	-0.0021	36.60	0.0517
n+7		-21.13	0.0156
n+8	0.0002	-100.00	-0.0047
$[\Sigma(\,)^2]^{1/2}$	0.713		22.3

Table 4 Kick error between the (n+1)-th and the (n+2)th BPM. $2\varphi=60°$. Results are closely related to those of table 3 by symmetry.

i	θ_i	x_i (before correction)	x_i (after correction)
n-6	0.0002	-100.00	-0.0047
n-5		-21.13	0.0156
n-4	-0.0021	36.60	0.0517
n-3		57.73	-0.1706
n-2	0.0225	136.6	-0.5635
n-1		78.87	1.861
n	-0.2459	100.00	6.147
n+1		21.13	-20.30
n+2	-0.6685	36.60	6.147
n+3		57.74	1.861
n+4	0.0225	136.6	-0.5635
n+5		78.87	-0.1706
n+6	-0.0021	100.00	0.0517
n+7		21.13	0.0156
n+8	0.0002	-36.60	-0.0047
$[\sum()^2]^{1/2}$	0.713		22.3

Table 5 The n-th BPM is displaced by 1mm. $2\psi=60°$.

i	θ_i	x_i (before correction)	x_i (after correction)
n-6	-0.00003	0.000	0.0006
n-5		0.000	-0.0021
n-4	0.0003	0.000	-0.0070
n-3		0.000	0.0231
n-2	-0.0031	0.000	0.0763
n-1		0.000	-0.2519
n	0.0033	1.000	0.1679
n+1		0.000	-0.2519
n+2	-0.0031	0.000	0.0763
n+3		0.000	0.0231
n+4	0.0003	0.000	-0.0070
n+5		0.000	-0.0021
n+6	-0.00003	0.000	0.0006
$[\sum ()^2]^{1/2}$	0.00544		0.410

Table 6 The (n+1)th BPM is displaced by 1mm. $2\psi=60°$. Note that the orbit "correction" on this type of errors is very ineffective.

i	θ_i	x_i (before correction)	x_i (after correction)
n-6	-0.000008	0.000	0.0002
n-5		0.000	-0.0006
n-4	0.000084	0.000	-0.0021
n-3		0.000	0.0070
n-2	-0.000924	0.000	0.0231
n-1		0.000	-0.0763
n	0.000077	0.000	-0.2519
n+1		1.000	0.8321
n+2	0.000077	0.000	-0.2519
n+3		0.000	-0.0763
n+4	-0.000924	0.000	0.0231
n+5		0.000	0.0070
n+6	0.000085	0.000	- 0.0021
n+7		0.000	-0.0006
n+8	-0.000008	0.000	0.0002
$[\Sigma(\,)^2]^{1/2}$	0.00132		0.912

Table 7 Kick error is located at the n-th corrector. $2\psi=90°$.

i	θ_i	x_i (before correction)	x_i (after correction)
n-6	0.	-170.7	0.0000
n-5		-100.0	0.0000
n-4	0.	-170.7	0.0000
n-3		0.000	0.0000
n-2	0.	170.7	0.0000
n-1		100.0	0.0000
n	-1.000	170.7	0.0000
n+1		100.0	0.0000
n+2	0.	170.7	0.0000
n+3		0.000	0.0000
n+4	0.	-170.7	0.0000
n+5		-100.0	0.0000
n+6	0.	-170.7	0.0000
$[\sum(\,)^2]^{1/2}$	1.00		0.0000

Table 8 Error is located at the (n+1)-th BPM. $2\varphi=90°$.

i	θ_i	x_i (before correction)	x_i (after correction)
n-6	0.0001	0.000	-0.0027
n-5		-29.29	0.0101
n-4	-0.0015	-100.0	0.0372
n-3		-29.29	-0.1370
n-2	0.0212	0.000	-0.5048
n-1		29.29	1.861
n	-0.2743	100.0	6.857
n+1		29.29	-25.27
n+2	-0.2743	100.0	6.857
n+3		29.29	1.861
n+4	0.0212	0.000	-0.5048
n+5		-29.29	-0.1370
n+6	-0.0015	-100.0	0.0372
n+7		-29.29	0.0101
n+8	0.0001	0.000	-0.0027
$[\sum(\,)^2]^{1/2}$	0.389		27.2

Table 9 Error between the n-th and the (n+1)th BPM. $2\psi=90°$.

i	θ_i	x_i (before correction)	x_i (after correction)
n-6	0.0001	-70.71	-0.0027
n-5		-70.71	0.0101
n-4	-0.0015	-170.7	0.0372
n-3		-29.29	-0.1370
n-2	0.0202	70.71	-0.5048
n-1		70.71	1.861
n	-0.6885	170.7	6.857
n+1		70.71	-25.27
n+2	-0.2743	170.7	6.857
n+3		29.29	1.861
n+4	0.0202	-70.71	-0.5048
n+5		-70.71	-0.1370
n+6	-0.0015	-170.7	0.0372
n+7		-29.29	0.0101
n+8	0.0001	70.71	-0.0027
$[\sum(\ )^2]^{1/2}$	0.742		27.2

Table 10 Error between the (n+1)-th and the (n+2)th BPM. $2\psi=90°$.
Results are closely related to those of table 9 by symmetry.

i	θ_i	x_i (before correction)	x_i (after correction)
n-6	0.0001	70.71	-0.0027
n-5		-29.29	0.0101
n-4	-0.0015	-170.7	0.0372
n-3		-70.71	-0.1370
n-2	0.0202	-70.71	-0.5048
n-1		29.29	1.861
n	-0.2743	170.7	6.857
n+1		70.71	-25.27
n+2	-0.6885	170.7	6.857
n+3		70.71	1.861
n+4	0.0202	70.71	-0.5048
n+5		-29.29	-0.1370
n+6	-0.0015	-170.7	0.0372
n+7		-70.71	0.0101
n+8	0.0001	-70.71	-0.0027
$[\sum()^2]^{1/2}$	0.742		27.2

Table 11 The n-th BPM is displaced by 1mm. $2\psi=90°$.

i	θ_i	x_i (before correction)	x_i (after correction)
n-6	-0.00001	0.000	0.0003
n-5		0.000	-0.0013
n-4	0.0002	0.000	-0.0047
n-3		0.000	0.0172
n-2	-0.0025	0.000	0.0635
n-1		0.000	-0.2341
n	0.00037	1.000	0.1371
n+1		0.000	-0.2341
n+2	-0.0025	0.000	0.0635
n+3		0.000	0.0172
n+4	0.0002	0.000	-0.0047
n+5		0.000	-0.0013
n+6	-0.00001	0.000	0.0003
$[\sum ( )^2]^{1/2}$	0.00362		0.370

Table 12 The (n+1)th BPM is displaced by 1mm. $2\varphi=90°$.

i	θ_i	x_i (before correction)	x_i (after correction)
n-6	-0.000004	0.000	0.0001
n-5		0.000	-0.0003
n-4	0.000051	0.000	-0.0013
n-3		0.000	0.0047
n-2	-0.000689	0.000	0.0172
n-1		0.000	-0.0635
n	-0.000635	0.000	-0.2341
n+1		1.000	0.8629
n+2	-0.000635	0.000	-0.2341
n+3		0.000	-0.0635
n+4	-0.000689	0.000	0.0172
n+5		0.000	0.0047
n+6	0.000051	0.000	- 0.0013
n+7		0.000	-0.0003
n+8	-0.000004	0.000	0.0001
$[\sum ( )^2]^{1/2}$	0.00133		0.929

4. Corrector Strength and Corrected Orbit

Assuming all errors are uncorrelated, the rms corrector strength needed to make orbit corrections is

$$<\theta^2>^{1/2} = \{ (1.00)^2 < (\delta x_{qf}/f)^2> + (0.477)^2 <\delta x_{qd}/f>^2$$
$$+ (0.713)^2 <\delta\theta^2> + (0.713)^2 <\delta\theta^2>$$
$$+ (0.00544)^2 <\delta x_{bpm}{}^2> + (0.00132)^2 <\delta x_{bpm}{}^2> \}^{1/2} \qquad (9a)$$

$$<\theta^2>^{1/2} = \{ (1.00)^2 < (\delta x_{qf}/f)^2> + (0.389)^2 <\delta x_{qd}/f>^2$$
$$+ (0.742)^2 <\delta\theta^2> + (0.742)^2 <\delta\theta^2>$$
$$+ (0.00362)^2 <\delta x_{bpm}{}^2> + (0.00133)^2 <\delta x_{bpm}{}^2> \}^{1/2} \qquad (9b)$$

In Eq.(9a), the numerical coefficients are obtained from the bottom rows of Tables 1 to 6 for phase advance per cell of $2\varphi=60°$. Eq.(9b) is obtained similarly from Tables 7 to 12 for $2\varphi=90°$. In these expressions, δx's with subscripts refer to the misalignments of the corresponding elements; f is the focal length of the FODO quadrupoles; $\delta\theta$ is the kicking angle error that occurs in the dipole magnets.

The focal length f is given by

$$1/f = 4 \sin\varphi / L_{cell} \qquad (10)$$

with L_{cell} the cell length. The error $\delta\theta$ depends on whether it is the horizontal or the vertical plane being considered:

$$<\delta\theta^2>^{1/2} = (\delta B/B)\, \theta_B/m^{1/2} \quad \text{for horizontal plane}$$
$$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (11)$$
$$(\text{roll})\, \theta_B/m^{1/2} \quad \text{for vertical plane}$$

where $\delta B/B$ is the error in dipole strength, (roll) is the roll angle misalignment of the dipole magnets, θ_B is the bending angle per half cell and m is the number of independent dipole magnets per half cell.

For the SSC, we take[3]

$$
\begin{aligned}
\delta x_{qf} = \delta x_{qd} = \delta x_{bpm} &= 0.2\text{mm} \\
L_{cell} &= 200 \text{ m} \\
\theta_B &= 8.0 \text{ mrad} \\
\delta B/B &= 10^{-3} \\
m &= 5 \\
\text{roll} &= 0.5 \text{ mrad}
\end{aligned}
\qquad (12)
$$

The cell length value gives f = 100m for $2\psi=60°$ and f= 71m for $2\psi=90°$. When the values (12) are inserted in (9a) and (9b), we obtain the needed rms corrector strength of

	horizontal	vertical	
$2\psi=60°$	4.38	3.07	(unit: μrad)
$2\psi=90°$	4.88	3.64	

These values agree quite well with a previous estimate[3] of 4.2 μrad in the horizontal plane and 3.3 μrad for the vertical plane.

Again following Ref.3, we take 10 times the rms values(*) for the needed corrector strength for the SSC. At 20 TeV beam energy, this means an integrated corrector field shown below

	horizontal	vertical	
$2\psi=60°$	3.0	2.0	(unit: tesla-meter)
$2\psi=90°$	3.4	2.4	

(*) Taking 10 times rms may be generous but also note that the errors assumed in eq.(12) are rather tight.

The rms orbit distortion after correction, as read by the BPM's, is given by

$$<x^2>^{1/2} = \{ (0.00)^2 < (\delta x_{qf}/f)^2> + (30.4)^2 <\delta x_{qd}/f>^2$$
$$+ (22.3)^2 <\delta\theta^2> + (22.3)^2 <\delta\theta^2>$$
$$+ (0.410)^2 <\delta x_{bpm}{}^2> + (0.912)^2 <\delta x_{bpm}{}^2> \}^{1/2} \tag{13a}$$

$$<x^2>^{1/2} = \{ (0.00)^2 < (\delta x_{qf}/f)^2> + (27.2)^2 <\delta x_{qd}/f>^2$$
$$+ (27.2)^2 <\delta\theta^2> + (27.2)^2 <\delta\theta^2>$$
$$- (0.370)^2 <\delta x_{bpm}{}^2> + (0.929)^2 <\delta x_{bpm}{}^2> \}^{1/2} \tag{13b}$$

Substituting the numerical values, we obtain

	horizontal	vertical	
$2\psi=60°$	0.24	0.22	(unit: mm)
$2\psi=90°$	0.25	0.23	

These values of orbit distortions seem quite reasonable.

The beam sees a different orbit as read by the BPM's. When the BPM misalignments are subtracted from the readings, the orbit distortion is that seen by the beam. The rms values, however, remain the same as the orbit distortion seen by the BPM's because all the misalignment subtraction does is to switch Tables 5 and 6 for $2\psi=60°$ and switch tables 11 and 12 for $2\psi=90°$.

References

1. A beam bump technique has been adopted before by K.Steffen in the program PETROS. It was applied by a numerical iteration procedure rather than the closed form formulation used in Ref.2.
2. S.Peggs, thesis, Cornell Univ., 1981.
3. C.Moore, T.Murphy, J.Norem and M.Zisman, Ann Arbor Workshop on Accelerator Physics Issues for a SSC, December, 1984, UM HE 84-1, page 78.