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ELSHIM
Program to Simulate Elastic Processes of Heavy Ions

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ELSHIM
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Abstract

The Monte Carlo program ELSIM which simulates elastic processes of hadrons in thick targets is modified to accomodate heavy ions. These modifications are briefly indicated.

1 Introduction

The Monte Carlo code ELSIM [1] simulates elastic and quasi-elastic, i.e., of limited energy loss, processes of high energy hadrons in a thick target with particular attention to scattering off edges and the like. Its main applications concern accelerator beam loss, beam scraping, etc. Particles which only participate in elastic processes and are then reflected back into the aperture may cause problems elsewhere in the accelerator lattice—often far removed from where the beam loss occurs. Therefore ELSIM is often run in conjunction with an accelerator tracking program. It can also be used as the first stage in energy deposition studies. For example, when beam is lost in a superconducting magnet ELSIM can provide energy deposition by the incident particles along with a file specifying coordinates and momenta of the inelastic interactions. The latter can then be processed by a program

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such as CASIM [2] to complete the energy deposition simulation. A new version of this program, called ELSHIM is introduced here which extends ELSIM to include heavy ions as projectiles.

The physical processes and corresponding algorithms included in ELSIM are reasonably well documented [1] and here only the changes introduced to accomodate heavy ions are noted. For the present it is assumed throughout that the (bare) incident ion retains its initial Z and A. Processes which change Z and A by one or two units may have large probabilities attached to them and their products may cause problems of the type mentioned above for elastics. This makes ELSHIM the logical place to include them and should therefore be considered in any future upgrades. Among these processes is electron capture (and stripping) of the projectile.

Starting from scratch, an ELSHIM calculation proceeds in two parts: for a given projectile ion species (Z_P, A_P), target species (Z_T, A_T), and incident momentum, the auxiliary program HICS prepares a file which lists the necessary cross sections in a form convenient for referencing in the Monte Carlo. Such a file may serve repeatedly as input to explore various geometries and beam loss conditions. The user must also specify a number, typically between 0.9 and 1.0, which represents the fractional cut-off energy, i.e., particles which fall below this energy are no longer followed in ELSHIM. Towards the lower end of this energy range there will be important contributions from inelastic processes not treated in ELSHIM. If—in a given problem—such low cutoff particles prove to be important then the ELSHIM results may have to be supplemented by other means.

2 Updates

Two different types of basic processes are recognized in ELSIM: (a) *continuous* processes: multiple Coulomb scattering (MCS), ionization losses, and production of low energy e^+e^- pairs, versus (b) *point* processes which embrace everything else: large angle scattering (above some cutoff angle), e^+e^- pairs (above some cutoff energy), delta rays, and absorption. Bremsstrahlung is omitted here since it is expected to be relatively unimportant even at the highest RHIC energies. Diffractive target excitation is likewise not carried over into ELSHIM. It is not very significant for incident hadrons and is expected to be even less so here because it is easier to break up a nucleus than a nucleon or pion in the target dissociation process.

For heavy ions the relative importance of the basic processes differs from

that for hadrons and this necessitates some changes. Between even moderately heavy ions Coulomb scattering, ($\propto Z_P^2 Z_T^2$), will strongly dominate the angular deflection. This affects the choice of a cutoff angle between MCs and large angle scattering. Ionization losses ($\propto Z_P^2$) are also much larger and this affects the choice of a cutoff energy between continuous and point processes. As usual, a particle (ion) in the Monte Carlo is transported through the geometry from one point process to the next. At each such juncture the combined effects of the continuous processes—and the possibility of escape from the target—are taken into account. This makes the average steplength between point processes a basic parameter of the calculation.

2.1 Coulomb and Nuclear Scattering

For incident particles with unit charge, ref. [1] shows it is possible to choose a cutoff angle such that the angular region where nuclear scattering is important falls into the single scattering regime while keeping the step length between individual processes to a reasonable size (a few g/cm^2) [3]. To maintain this for heavy ions means step size would decrease with Z_P essentially as Z_P^{-2} and computation time would increase as Z_P^2. The simplest remedy is to extend the MCs regime out into the nuclear regime. The justification for this step is somewhat lenghty and is sketched in the Appendix. For simplicity, all *coherent* nuclear scattering, i.e., where both nuclei remain intact, is put into the MCs regime. *Incoherent* scattering—where either nucleus or both emits a nucleon—is included in MCs below a cutoff angle and treated event-by-event above it. The cutoff angle is placed at one tenth the rms angle determined from Coulomb plus coherent nuclear scattering for an estimated average steplength.

The rms scattering angle, $\langle\theta^2\rangle$, which is the only parameter of the Fermi distribution, is evaluated numerically in HICS since it includes all of the Coulomb, nuclear, and interference regions. In ref. [1] Glauber theory [4] is used to calculate the angular distributions. This should work for heavy ions as well [5]. One difference here is the assumed nuclear densities, which in ref. [1] are the harmonic oscillator well densities for light nuclei and Wood-Saxon densities for heavier nuclei. Here the densities are all taken to be of the former type:

$$\rho(r) = \alpha\left(1 + \beta r^2/\sigma^2\right)\exp\left(-r^2/2\sigma^2\right) \tag{1}$$

but with β and σ treated as adjustable parameters while α provides correct normalization. The choice of $\beta = 0.30A^{1/3} - 0.47$ (but $\beta = 0$ for $A \leq 4$) and

$\sigma = 0.35A^{1/3} - 0.44$ ensures that eq. 1 predicts rms and half-density radii close to experiment everywhere. The values of β are such that only a slight depression of $\rho(r)$ results in the central core. Eq. 1 has the advantage that the 'profile functions' for two colliding nuclei can be evaluated analytically, contrary to those of, e.g., Wood-Saxon densities.

Incoherent nuclear scattering between two ions can be divided into three components, viz., (1) scattering between the (unaltered) projectile and a target *nucleon* which is ejected from the nucleus, (2) the obverse: target nucleus on projectile nucleon, and (3) elastic scattering between a projectile nucleon and a target nucleon where both are liberated. Within the assumptions already made only the first case is to be included here (though (2) and (3) being of the low ΔZ, low ΔA variety are of potential interest). This part of the incoherent cross section is calculated using Glauber theory for nucleon-nucleus scattering multiplied by an 'effective number' of target nucleons $N_A = 1.6A_T^{1/3}$ [6].

2.2 Ionization Losses

The cutoff energy between continuous ionization losses and the δ-ray point process is set here much higher than 10 MeV as in [1] for reasons similar to those cited above with respect to MCs. The cross sections for this process also increases with Z like Z^2. Maintaining such a low cutoff would thus also become quickly prohibitive in computer time and, because of the increase in overall energy loss, there is not much reason for it, i.e., the energy loss in a typical individual event is dwarfed by the continuous energy loss. For this reason the cutoff is placed at one tenth the estimated energy loss in an average steplength between point processes. This results in energetic δ-ray production cross sections which are readily accomodated event-by-event. (For heavy enough nuclei all ionization loss becomes continuous.) In evaluating cross sections and average energy loss, especially towards the high energy end of the ejected electron energy (which is proportional to the square of the 4-momentum transfer $t = -2m_eT_e$) a nuclear form factor must be included. It is convenient to use the Fourier transform of eq. 1 for this since this is easily evaluated analytically.

2.3 Pair Production

The total energy loss per unit length due to pair production is assumed to equal that for protons multiplied by Z_P^2. The energy loss distribution in a

pair production event is assumed to follow the parametrization of ref. [7]. The particulars of the latter make it convenient to lump energy loss below $10^{-4}E_0$ with the continuous energy losses while treating larger losses as individual events. To account for energy loss fluctuations the amount lost in a Monte Carlo step is chosen from a Landau distribution with average equal to the combined average ionization and pair production loss within the step. Angular deflections associated with pair production are neglected.

2.4 Absorption

The *nuclear* absorption cross section is represented by the Bradt-Peters formula [8]

$$\sigma = \pi r_0^2 \left(A_P^{1/3} + A_T^{1/3} \right)^2 \tag{2}$$

with $r_0 = 1.2$ fm. To this is added the cross section for electromagnetic fragmentation [9] which makes a significant contribution. In the latter process the final state is usually close in Z and A to the initial one and, as mentioned in the Introduction, this may therefore be promoted to a separate process in a future ELSHIM upgrade. Absorption can be treated either by removing the projectile from further consideration or by exponentially downweighting along the trajectory. The former has the advantage that all weights are equal to unity (and hence can be suppressed) while the latter method generally benefits statistics.

3 Remarks

As pointed out in ref. [1] the present approach is capable of solving elastic scattering problems in thick targets to the same degree of approximation as that of the Fermi distribution in a homogeneous targets plus the effects of the point processes which apply—in principle—to arbitary accuracy. This holds also for heavy ions. Compared to its hadron counterpart, considerable more uncertainty is attached to the heavy ion version. The nuclear models used above to quantify density distributions, etc., weaken the overall reliability of the program somewhat. While having the advantage of simplicity it is not always satisfying to represent the entire collection of known nuclides by a few parameters. One might therefore consider introducing *ad hoc* parameters for frequently studied projectile and target species—if one has more confidence in them. For proton projectiles ELSIM is recommended over the new version mostly because the nuclear scattering model has better

experimental backing. [10] As mentioned in several places above, inclusion of low ΔZ, low ΔA processes would improve the usefulness of the program.

The caveat of ref. [1] applies here also: a typical surface of, e.g., a collimator, encountered in practice may differ considerably from the ideal flat one assumed to be present in edge scattering. It applies even more strongly when a complicated surface is represented by a combination of flat ones. Questions of alignment, etc., may also have important bearings on the problem. Such uncertainties do not in any way invalidate ELSHIM yet impair its predictive power. It is difficult to remedy this situation since usually one does not know precisely what those imperfections, etc., are. And even if one had such information it would more likely be used to try and rectify the actual device instead of improving the simulation. In many instances one also lacks sufficient detail about beam loss to achieve a high degree of realism in that part of the calculation. ELSHIM can nonetheless be a valuable tool in exploring thick target scattering problems. Idealized simulations may give order-of-magnitude answers, e.g., to reveal whether or not a problem exists or to help point to a solution. They have broader applicability and are usually easier to interpret than ultra-realistic ones. The latter kind may often be bracketed between two or more idealized cases or it may suffice to address them on a 'worst case' basis.

Appendix

It remains here to justify the approximation of treating all of the (multiple) nuclear and Coulomb scattering jointly by the Fermi distribution. First it is argued that this makes sense for a sufficiently long step length and then it is shown that a typical ELSHIM step thus qualifies.

The Fermi distribution rests on the same approximations as those resulting in a Gaussian distribution for either (projected) angle or displacement in MCs (cfr. Appendix of ref. [1]). This involves the Central Limit Theorem which states that the distribution of the sum of a sufficiently large number of random variables, chosen from a common distribution, is Gaussian.[11] Some restrictions to the underlying distribution apply but a sufficient condition is that it has finite variance. This is true here, though it requires invoking a large angle cutoff which enters here naturally from considerations of nuclear size. The coherent part (nuclear-plus-Coulomb) is to be regarded as a single distribution on physical grounds. Under certain broad conditions the Central Limit Theorem holds also for random variables derived from dif-

ferent distributions with the same mean [11] and the small incoherent part included in MCs easily meets these conditions.

The question now is whether the number of scatterings in one step (in ELSHIM, between point processes) is sufficiently large for the Theorem to apply. To test for this the spread of the Gaussian in such a step will be compared to θ_{max}, the maximum single scattering angle. For Coulomb scattering it can be shown [12] that—regardless of step length—for a *projected* angle, ϑ, beyond about $2.5\langle\theta^2\rangle^{1/2}$, the single scattering tail contributes more than the MCs Gaussian.[13] However, for a sufficiently long step, $2.5\langle\theta^2\rangle^{1/2}$ will exceed ϑ_{max}, the maximum projected single scattering angle, and the approximation may thus be regarded to pass the test if $\vartheta_{max} <\sim 2.5\langle\theta^2\rangle^{1/2}$. To make simple estimates of $\langle\theta^2\rangle^{1/2}$ and ϑ_{max} the nuclear part of the scattering is ignored. Especially for heavy ions the Coulomb part determines almost all of $\langle\theta^2\rangle^{1/2}$. The usual expression for $\langle\theta^2\rangle^{1/2}$ is: [12]

$$\langle\theta^2\rangle = 6.2\cdot 10^{-7}\frac{Z_P^2 Z_T^2}{A_T p^2}\ln\frac{174}{Z_P^{1/3}+Z_T^{1/3}}\cdot\ell \tag{3}$$

where p is the total momentum of the ion in GeV/c, and ℓ is the steplength in g/cm^2. For all Z of interest $3 < \ln 174/(Z_P^{1/3}+Z_T^{1/3}) < 4.5$ and simply replacing this factor by 4 yields

$$\langle\theta^2\rangle = 2.5\cdot 10^{-6}\frac{Z_P^2 Z_T^2}{A_T p^2}\cdot\ell. \tag{4}$$

The maximum angle is expected to occur at an impact parameter where the two nuclei just touch. The corresponding projected angle is

$$\vartheta_{max} = \frac{0.1}{(A_P^{1/3}+A_T^{1/3})p}. \tag{5}$$

This is a standard expression for Coulomb scattering but it should still hold approximately when the nuclear part is added because absorption quickly becomes dominant at lesser impact parameters. Choosing 10 g/cm^2 as a typical value for ℓ and comparing ϑ_{max}^2 with $2.5^2\langle\theta^2\rangle$ results in

$$\frac{Z_P^2 Z_T^2 (A_P^{1/3}+A_T^{1/3})^2}{A_T} > 64. \tag{6}$$

For practical purposes, beryllium is the lightest target material to be considered. For $Z_T = 4, A_T = 9$, eq. 6 is already satisfied for proton projectiles. Even though ELSIM would be preferred for that case, it proves the point about the validity of using the Gaussian and Fermi distributions.

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[13] This shows that the large angle cutoff (finite variance) is also a *necessary* condition for the Central Limit Theorem to be applicable here.