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Errors in Siberian Snakes

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ERRORS IN SIBERIAN SNAKES

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In an accelerator or storage ring for polarized protons, "Siberian snakes" may be inserted in the lattice in order to stabilize the spin dynamics, i.e. to prevent betatron oscillations and orbit distortions from depolarizing the beam.

This is accomplished by inserting an odd number N_p of pairs of snakes, spaced π/N_p apart in orbit azimuth; each snake is a spin rotator that rotates the spin by exactly 180 degrees about a precession axis in the horizontal plane. The precession axes of the two snakes of a pair should be exactly 90 degrees apart (e.g. longitudinal and transverse horizontal, or ±45 degrees from longitudinal). The preferred choice in most cases is ±45 degrees.

For RHIC, one pair of snakes ($N_p = 1$) is planned, 180 degrees apart with ±45 degree axes; in the case of the Tevatron, $N_p = 3$(six snakes) is envisaged, with the snakes spaced 60 degrees apart.

The spin rotation matrix for a snake with its axis at ±45 degrees is

$$S_{\pm} = i\frac{(\sigma_1 \pm \sigma_2)}{\sqrt{2}} = i\sigma_1 e^{\pm i\pi\sigma_3/4} \tag{1}$$

while the matrix for an arc with bending angle ϑ is

$$A = e^{i\gamma G\vartheta\sigma_3/2} \tag{2}$$

with $\vartheta = \pi/N_p$and $G = 1.7928$ the proton anomalous moment, so that the matrix for the whole ring (with no imperfections) is

$$M = (S_+AS_-A)^{N_p} = (i\sigma_3)^{N_p} \tag{3}$$

independent of energy. The spin tune ν_{sp} is determined by

$$\cos\pi\nu_{sp} = \mathrm{Tr}M/2 \tag{4}$$

which equals 0, i.e. $\nu_{sp} = \frac{1}{2}$ when M is given by (3) with odd N_p; the stable spin direction is exactly vertical.

Now what if the snakes are not perfect? We consider three types of imperfections:

(a) The rotation angle of each snake is not exactly 180 degrees, but rather $\pi + \delta$.

(b) The precession axes of the snakes are not exactly at 45 degrees from the longitudinal, but rather $\pm\pi/4 + \beta$.

(c) The bending arcs between the snakes are not exactly π/N_p, i. e. each snake is azimuthally displaced by a bending angle ϵ.

In case (a) (incorrect precession angle) the matrix for the k-th snake becomes

$$-\sin\delta_k/2 + i\frac{(\sigma_1 \pm \sigma_2)}{\sqrt{2}}\cos\delta_k/2 \tag{5}$$

To lowest order in the $\delta's$, the overall matrix is then

$$i\sigma_3 + \frac{i}{2}\delta_1\frac{(\sigma_1 - \sigma_2)}{\sqrt{2}} + \frac{i}{2}\delta_2\frac{(\sigma_1 + \sigma_2)}{\sqrt{2}}e^{i\pi\gamma G\sigma_3} \tag{6}$$

for the case of two snakes (RHIC), and

$$-i\sigma_3 - \frac{i}{2}(\delta_1 + \delta_3 + \delta_5)\frac{(\sigma_1 - \sigma_2)}{\sqrt{2}} - \frac{i}{2}(\delta_2 + \delta_4 + \delta_6)\frac{(\sigma_1 + \sigma_2)}{\sqrt{2}}e^{i\pi\gamma G\sigma_3} \tag{7}$$

for the case of six snakes (Tevatron). Thus, to lowest order in the deviations from 180 degree in the rotators, the spin tune is still exactly $\frac{1}{2}$, but the axis of the stable spin direction is tilted from the vertical by an angle of order δ. Thus if the rotators are off by a degree, the spin axis is tilted by a similar angle, which should be tolerable.

In case (b) (precession axes deviate from 45 degrees by β) the snake matrices become, instead of (1),

$$S_k = i\sigma_1 e^{i(\pm\pi/4+\beta_k)\sigma_3} \tag{8}$$

and the overall matrix is

$$(i\sigma_3)^{N_p}e^{-i(\beta_1-\beta_2+\beta_3-\beta_4+\beta_5-\beta_6)\sigma_3} \tag{9}$$

The trace of this matrix deviates from zero, and we find that the spin tune becomes

$$\nu_{sp} = \frac{1}{2} \pm (\beta_1 - \beta_2 + \beta_3 - \beta_4 + \beta_5 - \beta_6)/\pi, \tag{10}$$

i.e. the spin tune deviates from $\frac{1}{2}$ by an amount of the order of β(about 1% if the axes are off by a degree, which should be tolerable).

Finally, in case (c), where the azimuthal locations of the snakes are not evenly spaced: If the location of a snake is off by a bending angle ϵ, the spin matrix for the displaced snake is

$$e^{i\gamma G\epsilon_k\sigma_3/2}S_\pm e^{-i\gamma G\epsilon_k\sigma_3/2} = S_\pm e^{-i\gamma G\epsilon_k\sigma_3} \tag{11}$$

and the spin tune is

$$\nu_{sp} = \frac{1}{2} \pm (\epsilon_1 - \epsilon_2 + \epsilon_3 - \epsilon_4 + \epsilon_5 - \epsilon_6)\gamma G/\pi \ . \tag{12}$$

Because of the factor γG the spin tune is very sensitive to this error at high energy. If we wish the spin tune to be within .05 units of 1/2 then for RHIC ($\gamma G = 480$ at 250 GeV) we must have $|\epsilon_1 - \epsilon_2| < .02$ degrees = 0.3 milliradians; for the Tevatron ($\gamma G = 1920$) the alternating sum of the position errors must be .005 degrees or .08 milliradians of bending. These tolerances sound tight, but they will be met automatically if the orbit correction system of the ring corrects the closed orbit errors to better than a few millimeters.