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RHIC PROJECT

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β–Functions in the Presence of Linear Coupling

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1. Introduction

One effect of random skew quadrupole field errors is to perturb the β–functions. This effect may be large in proton accelerators using superconducting magnets, because of the relatively large random skew quadrupole field errors that are expected in these magnets. The effect is also increased by the required insertions in proton colliders which generate large β–functions in the insertion region.

This effect has been studied in the RHIC accelerator (the Relativistic Heavy Ion Collider proposed at Brookhaven National Laboratory). For RHIC, large changes in the β–functions were found, as large as 100% increase in the β–function in one worse case. The β–functions were computed for ten different distributions of the random field errors, and for two values of β^*, the β–function at the collision points.

The random skew quadrupole field errors also produce large shifts in the tune or the betatron oscillation frequencies.[1,2] For RHIC a tune splitting as large as 0.2 was found in one worse case. A correction system has been developed[1] for correcting this large tune splitting which uses two families of skew quadrupoles as correctors. The correction of the tune–splitting will in most cases also improve the error in the β–functions. However, the error in the β–functions is still appreciable, and in some cases the error in the β–functions actually is larger after the tune–splitting correction is made. A skew quadrupole correction system has been developed that appears able to correct a large part of the changes in the β–functions. The application of this correction system to correct the β–function changes caused by the random skew quadrupole errors indicates that the important harmonics that need to be controlled are the harmonics that would drive the nearby sum resonances, $\nu_x + \nu_y =$ integer.

2. Computed Results for the β–Functions

The random quadrupole field errors expected in RHIC are used in this study. The effect of the random a_1, the skew quadrupole error, is to couple the x and y motions. The x motion and the y motion can each be written as the sum of 2 normal modes which have the ν–values ν_1 and ν_2. Each of the normal modes also have β–functions denoted by β_1 and β_2. For a given distribution of field errors, β_1 and β_2 can be computed using the results of Edwards and Teng.[3]

Table 1 lists the largest β–functions found for ten different distributions of random quadrupole errors. Two lattices were studied having $\beta^* = 2$ and $\beta^* = 6$ at the six beam crossing points. The table lists the maximum β_1 found at the focusing quadrupoles, QF, in the arcs, the maximum β_2 found at the defocusing quadrupoles, QD, in the arcs, and the maximum β_1, β_2 found around the accelerator, which occurs at the high–β quadrupoles in the insertions.

Table 1: Maximum β–Functions for 10 distributions of random quadrupole errors for $\beta^* = 6$ and $\beta^* = 2$ lattices

$\beta^* = 6$

Error Field Distribution	$\beta_{1,max}$ at QF	$\beta_{2,max}$ at QD	$\beta_{1,max}$	$\beta_{2,max}$
1	57	55	247	236
2	59	61	267	265
3	56	57	237	244
4	57	61	254	249
5	58	58	255	260
6	54	53	230	223
7	58	52	250	233
8	66	70	305	284
9	55	54	237	243
10	58	56	255	248

$\beta^* = 2$

Error Field Distribution	$\beta_{1,max}$ at QF	$\beta_{2,max}$ at QD	$\beta_{1,max}$	$\beta_{2,max}$
1	60	58	696	684
2	53	58	650	643
3	55	66	683	740
4	65	62	725	799
5	82	64	868	752
6	58	61	625	655
7	100	80	950	964
8	68	61	816	763
9	57	61	688	656
10	67	74	772	668

The largest β–function found in the arcs is $\beta_1 = 100$ for $\beta^* = 2$ and $\beta_2 = 70$ for $\beta^* = 6$. The unperturbed largest β functions in the arcs is $\beta_x = 50$ and $\beta_y = 50$. In the insertions, at the high–β quadrupoles the largest β–functions are $\beta_1 = 305$ for $\beta^* = 6$ and $\beta_2 = 964$ for $\beta^* = 2$. The unperturbed largest β–functions at this place is $\beta = 220$ for $\beta^* = 6$ and $\beta = 625$ for $\beta^* = 2$.

The results listed in Table 1 include the effect of the random b_1 quadrupole error. This random b_1 can generate changes in the β–functions of about 20% in a worse case when no a_1 is present.

Loosely speaking, one may say that the changes in β– functions caused by the random a_1 are associated with the two nearby sum resonances which are $\nu_x + \nu_y = 58$ and $\nu_x + \nu_y = 57$ for RHIC. The β–function changes are caused primarily by the 57 and 58 harmonics in a_1, which are the harmonics in a_1 that would drive the nearby sum resonances.

3. Correction of the β–Function Changes

A correction system has been studied which consists of 12 skew quadrupole correctors, one near each of the high–β quadrupoles in the insertions, and all separately excited. Some of these quadrupoles are also used for the tune splitting correction system. By connecting these quadrupoles in just four families, to correct the tune splitting and to generate the harmonics near $n = \nu_x + \nu_y$, it was found that the change in the β–function could be reduced to the 20% level in the worse case. It seems likely that by exciting the b_1 correction system to reduce the contribution of the random b_1, and by using the full capability of the 12 skew quadrupole correctors, one can further reduce the remaining error in the β–functions.

References

1. G. Parzen, BNL Report AD/RHIC–AP–72 (1988).
2. G. Parzen, BNL Report AD/RHIC–AP–81 (1989).
3. D. Edwards and L. Teng, IEEE 1973 PAC, p. 885.