RHIC/AP/144
(Near Beam W/S)

MODULATIONAL EFFECTS IN ACCELERATORS *

T. Satogata,
Brookhaven National Laboratory
Building 1005S–3, PO Box 5000, Upton, NY, 11973-5000 USA

Abstract

We discuss effects of field modulations in accelerators, specifically those that can be used for operational beam diagnostics and beam halo control. In transverse beam dynamics, combined effects of nonlinear resonances and tune modulations influence diffusion rates and tail transport, and some qualitative control of loss rates with applied tune modulation has been demonstrated. In the longitudinal domain, applied RF phase and voltage modulations provide mechanisms for parasitic halo transport, useful in slow crystal extraction. Experimental experiences with transverse tune and RF modulations are also discussed.

1 INTRODUCTION

In the past several decades, the hadron beam dynamics community has expended considerable effort to understand the effects of power supply ripples and magnetic field modulations. For example, transverse tune modulation in conjunction with nonlinear resonances is well established as a significant contributor to beam lifetime. This experience has led to the suggestion of methods, both transverse and longitudinal, that use magnet modulations to control dynamic beam loss and halo population.

This paper briefly reviews modulation phenomena impacting beam halo population and transport, and long-term ($> 10^6$ turns) beam and luminosity stability, as well as recent uses of modulations in beam diagnostics.

2 EXAMPLES OF MODULATION

Noise at some level is always present in a synchrotron, and excites the beam at broad-band frequencies. The broad frequency spectrum arises from a confligration of multiple noise sources including ground motion, local industrial activity, and electrical noise. Noise is also routinely applied to transverse dampers to excite coherent beam oscillations for tune measurements.

Transverse tune modulation also arises from several sources. Dominant main bus power supply ripples range from 50–1200 Hz at 50 or 60 Hz harmonics, with tune widths ΔQ up to 10^{-4}. Chromatic tune modulation changes frequency with the synchrotron tune Q_S, typically from 50–500 Hz, and can create tune spreads up to $\Delta Q \approx 10^{-3}$ — it is worst at injection, where beams have their largest momentum spread. Controlled tune modulation can be applied up to 1 kHz (or faster, with ferrite magnets), with strengths up to $\Delta Q = 10^{-2}$.

Accelerator RF systems lack the DC regulation and spectral modulation found at power frequency harmonics in main magnets. However, RF phase and amplitude modulation are used during transfer and storage cogging, and in parametric feedback loops used to control multibunch instabilities. The frequencies most relevant to slow dynamics and halo control range from low frequencies to the synchrotron frequency, Q_S.

3 MODULATIONS AND BEAM DIAGNOSTICS

3.1 *Narrow-band excitation and response*

The SPS and LEP Q-meter systems allow application of white noise, single-frequency excitation, and frequency-swept dipole modulations (chirps) on a transverse damper. The transverse tunes are then calculated with overlapping FFTs of digitized turn-by-turn BPM signals, allowing tracking of the SPS tune through an acceleration cycle.

Calculations and experience have demonstrated that transverse emittance growth as low as 10% is achievable for 150 tune measurements through the SPS acceleration cycle, with short chirped modulations. PLL feedback and tune tracking also gives reasonable growth rates with much smaller narrow-band feedback, but requires careful tuning and balance between bandwith and precision to achieve similar results. [1]

3.2 *Beam-based instrument and optics calibration*

In several electron storage rings, quadrupole modulation has been used to dead-reckon deviations of the closed orbit from quadrupole magnet optical centers, without requiring absolute knowledge of relative magnet and BPM survey calibrations. Harmonic analysis of BPM signals at the modulation frequency can indicate magnet deviations with precisions of 100μm, using modulation strengths as low as 0.03% of the magnet strength. However, such a method requires individual shunts and power supplies for modulated quadrupoles. [2]

These methods have also been used to accurately measure relative phase advances between BPMs in LEP, and thus make lattice optics measurements parasitically during the course of operations. Such methods might be applied to measure lattice optics through acceleration ramps. [3]

3.3 *Instability damping*

Recently, chromaticity modulation has been suggested as a means to damp the transverse head-tail instability, which limits single-bunch intensities in some machines. Modulation of the chromaticity over the RF synchroton period

* Work performed under the auspices of the US Department of Energy

would create an incoherent transverse tune spread over the beam, creating enough phase mixing to raise the threshold of instability by orders of magnitude. This is the first example of nonlinear modulations applied to beam stability, and investigations of the dynamical implications of sextupole modulations are ongoing. [4]

4 TRANSVERSE TUNE MODULATION

During storage and collisions, nonlinear beam-beam effects, as well as error fields in main magnets and nonlinear correction magnets, produce nonlinear resonances in transverse phase space. Since a complete discussion of two-dimensional resonances is inappropriate here, we shall restrict this discussion to isolated one-dimensional resonances. Motion under the influence of these resonances has been extensively studied, and is applicable to processes described in later sections. Further details and references may be found elsewhere. [5]

Motion within an isolated one-dimensional resonance is characterized by the appearance of "resonance islands" in transverse phase space (Figure 1). Moving to a coordinate system near the center of one of these islands, this behavior may be parameterized with pendular equations of motion. The frequencies of oscillation within the resonance island, around the central fixed points, range from zero near the separatrix to Q_I, the "island tune", very near the fixed point at the center of the island. Like the dynamical whisker map and simple RF synchrotron motion, motion near the separatrix has very low frequency, and thus is highly sensitive to perturbations such as tune modulation.

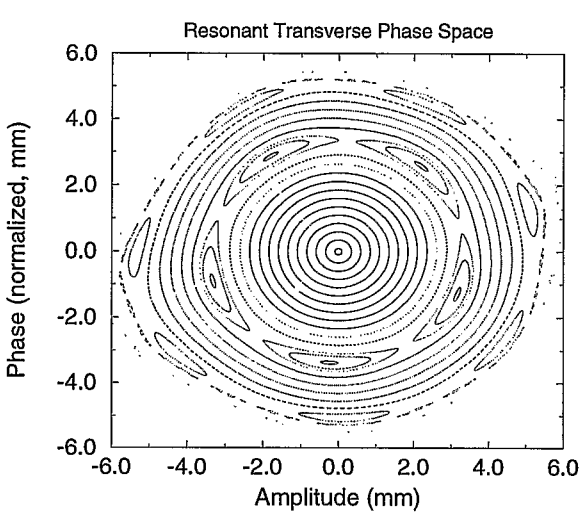

Figure 1: Two sets of resonances, $Q = 2/5$ and $Q = 3/7$, in one-dimensional normalized transverse phase space. Motion is stable up to more than 6 mm in the tails, and the core motion is regular and unperturbed by the nearby resonances.

4.1 *Parameterization and character of tune modulation*

As with driven pendulum motion, particle motion near and within resonances is highly sensitive to tune modulations with frequencies near the island tune Q_I. Weak resonances have small island tunes (on the order of $Q_I = 10^{-5}$ to $Q_I = 10^{-10}$) and small spatial extent. Very strong resonances, usually created explicitly by nonlinearities during beam dynamics studies, have larger island tunes up to $Q_I = 10^{-3}$ or more.

One-dimensional transverse tune modulation may be parameterized by

$$Q = Q_0 + q\sin(2\pi Q_M t), \quad [t] = \text{turns} \qquad (1)$$

where (q, Q_M) are the tune modulation strength (in tune units) and frequency. The behaviors of isolated one-dimensional resonances under the influence of tune modulation has been extensively studied in the past decade. This has produced complementary approaches that can be summarized by a parameterized tune modulation phase diagram, Figure 2.

This figure, when interpreted properly, is particularly powerful. Two parameters of the tune modulation, strength and frequency, and one parameter of resonance strength can be used to qualitatively predict the dynamics of a nonlinear system. When multiple resonances under the influence of tune modulation interact, e.g. at large particle amplitudes in the beam halo, controllable tune modulation can play a significant role in halo transport and slow beam loss.

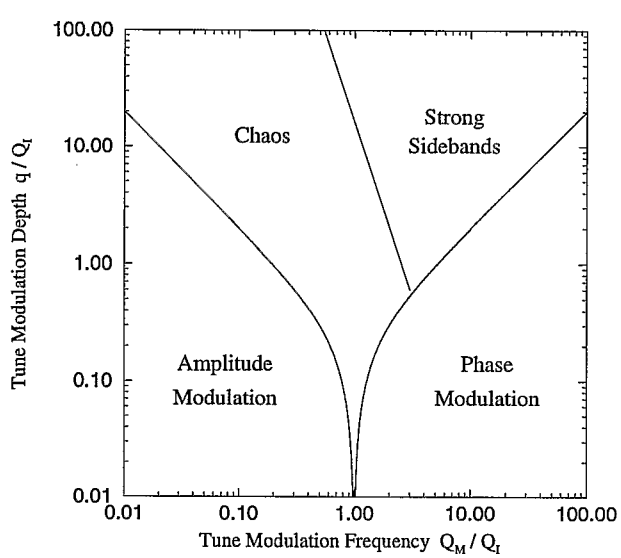

Figure 2: A parameterized tune modulation phase diagram. Resonant motion phases include stable motion (amplitude and phase modulation), the creation of isolated sideband resonances (strong sidebands), and thick bands of bounded stochastic motion (chaos). [6, 7, 8].

4.2 *Modulational diffusion*

Overlapping resonances and stochastic motion create amplitude growth and beam loss over timescales ranging from tens to millions of machine turns. Since strongly resonant and stochastic motion are avoided in the course of operations, practical interests in slow tail transport and beam loss concentrate on slow diffusive mechanisms. Though transverse diffusion in hadron colliders is partly created by noise growth, efforts have concentrated on other sources (such as tune modulation) that provide operational access to correction and control.

The most promising of these sources is modulational diffusion, originally applied by Chirikov. Here tune modulation creates small bands of stochastic motion in one-dimensional resonances, for resonances that have appropriate small strengths and island tunes. (See Figure 2.) This stochastic motion, though bounded, serves as a noise source that can be coupled into other dimensions of particle motion, thus creating slow diffusion. Simulations of modulational diffusion with realistic magnet and beam-beam nonlinearities and chromatic tune modulation have agreed within factors of two with observation. [5, 8]

4.3 *Experience: FNAL, IUCF, CERN, HERA-p*

Beam capture onto resonances has been observed in experiments at FNAL [6] and IUCF [7], and measurements of strong resonances created by controlled nonlinearities have been demonstrated to agree with simulations and theory.

In a series of experiments at CERN and DESY, resonant slow diffusion has also been observed, similar to experiments performed in FNAL's E778 experiments [8, 9]. Loss rates were demonstrated to depend strongly on the presence and character of tune modulation, and transverse diffusion coefficients were measured. However, these loss rates were also shown to depend strongly on machine conditions, creating difficulties with reproducibility.

Diffusion and loss rates in the HERA-p halo have been controlled by compensating 100 and 300 Hz quad bus ripple lines with external tune modulation [10, 8]. After tuning, this compensation reducted proton losses by up to 40%. This experiment also demonstrated the use of PLL circuits to measure tune modulation from beam-based measurements, instead of inferring tune modulation from measurements of quad bus ripple.

More recently, another compelling argument has proposed that modulational diffusion is the source of the operational HERA-p dynamic aperture [11].

Transverse tune modulation, combined with knowledge of magnet and beam-beam nonlinearities, can be used to qualitatively control beam loss and halo growth. However, the many sensitive dependencies on machine parameters (e.g. base tunes, chromaticities, beam momentum spread, magnetic nonlinearities) make quantitative control difficult at best, and do not provide a very promising or sophisticated way to control beam halo and long-term beam loss.

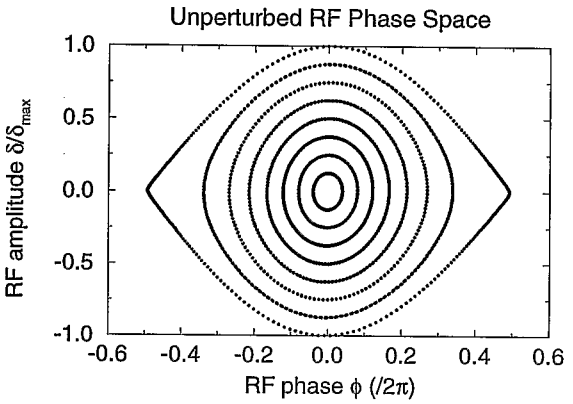

Figure 3: Typical RF phase space at beam store. This motion is parameterizable as a pendulum, with synchrotron frequencies ranging from $Q_{S0} = 0.008$ at the center fixed point to zero at the separatrix.

5 LONGITUDINAL VOLTAGE MODULATION

During beam storage and collisions, longitudinal phase space is significantly simpler than the transverse. Typically a single storage RF voltage is applied, creating RF buckets that are dynamically equivalent to free pendula. The synchrotron frequency $Q_S(\delta)$ depends smoothly on momentum offset $\delta \equiv \Delta p/p$, ranging from the base synchrotron frequency Q_{S0} at small amplitudes to zero at the separatrix, $\delta = \delta_{\max}$. (See Figure 3.)

Due to the character of RF feedback and control, the two simplest quantities to modulate are RF voltage and synchronous RF phase. RF phase modulation, however, moves RF bucket fixed points, and has the unfortunate side effect of modulating the location of experimental interaction points. RF voltage modulation instead modulates the beam momentum width by varying the bucket size. Useful voltage modulation strengths incur changes much smaller than the total beam momentum width, and should not have any observable effects on colliding beam experiments.

When RF voltage modulation of the form

$$\frac{\Delta V}{V} = q\cos(2\pi Q_M t) \quad [t] = \text{turns} \tag{2}$$

is applied, the pendular RF phase space of Figure 3 becomes that of a parametrically driven pendulum [12]. Since pendulum motion is nonlinear, a primary nonlinear resonances is driven at amplitude δ_{res} where $Q_M = 2Q_S(\delta_{\text{res}})$, and the region near the RF separatrix becomes stochastic. Multiple voltage modulations may be applied, creating several resonances,

Modulation strengths significantly less than 1% of the RF voltage can create large resonance islands, as shown in Figure 4. Furthermore, beam within these resonance islands can be moved radially in the RF bucket with adiabatic changes to the voltage modulation frequency.

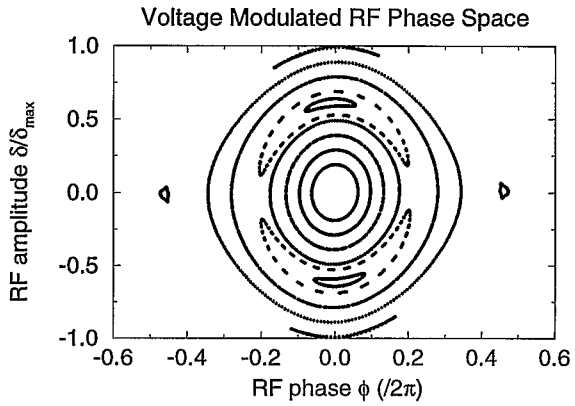

Figure 4: RF phase space as in Figure 3, with RF voltage modulation of depth $q = 0.002$ and frequency $Q_M = 1.8Q_{S0}$. Note the two large primary resonance islands, with small higher-order islands crated near the separatrix.

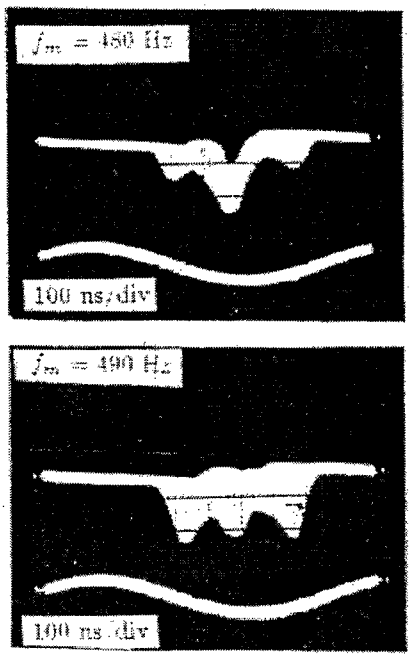

Figure 5: The longitudinal beam distribution for many turns, acquired from a high-dispersion BPM sum signal over many synchrotron periods as observed in IUCF for two different voltage modulation frequencies [15]. Beam capture in resonance islands is clearly visible.

5.1 RF bucket halo transport

Following Gabella, et al. [13], one can use the resonances created by several RF voltage modulations to construct an integrated slow extraction system for crystal extraction. Particles are adiabatically moved outwards from the beam core to a "drive" resonance at a large momentum amplitude in the RF bucket. They then diffuse into the drive resonance through a web of weak stochastic resonances. Once within the drive bucket, particles are smoothly moved outwards to amplitudes that achieve penetration depths consistent with efficient crystal extraction.

Such a system requires several simultaneous voltage modulations of varying frequencies and amplitudes. The drive bucket frequency is low and constant, to place the drive bucket at a large momentum offset in RF phase space. The "feed" bucket, which captures particles near the core and moves them outwards, must ramp in frequency from $Q_M \approx 1.95Q_{S0}$ down to near the drive bucket frequency. Theoretical hamiltonian considerations give a maximum frequency ramping rate of

$$\frac{dQ_M}{dt} < 2\pi Q_S^2(\delta_{\text{res}}) \tag{3}$$

to maintain adiabatic capture. Furthermore, the feed bucket must be powered suddenly at the beginning of every ramp, to capture beam nonadiabatically in the core. Three to five additional modulations are added to create weak stochasticity around the drive resonance. Gabella, et al. state that this improves transfer efficiency.

Though such an extraction system is complicated by the many modulation parameters, it allows elegant control of extraction parameters such as spill rate. Because this system extracts particles by moving them to large momentum, it also requires that extraction be performed at a point of high horizontal dispersion.

5.2 Experience: IUCF, FNAL, CERN SPS

5.2.1 IUCF experiments and theory

Following the development of methods to track the synchrotron motion of an electron-cooled proton beam, RF voltage and frequency modulation have been extensively studied in a series of nonlinear dynamics experiments performed at IUCF[14, 15]. Beam capture in RF resonance islands has been observed (Figure 5), and locations of resonance islands have been shown to agree very well with theory. Many of their results on parametric oscillators and driven resonances are applicable to both the transverse and longitudinal domains.

Other relevant work on RF dynamics investigated by the IUCF group includes double RF systems and barrier bucket dynamics. When combined with resonant behavior and RF voltage modulations, the results may be applied to RF manipulations ranging from transition crossing to efficient rebucketing. [16, 17, 18]

5.2.2 FNAL and CERN crystal extraction

Experiments with crystal extraction, such as those performed at the CERN SPS and Fermilab Tevatron, have produced extraction efficiencies ranging up to approximately 30%. These experiments have suffered from the necessary limitation of crystals placed at low-dispersion areas, disallowing the opportunity to investigate RF-based extraction schemes such as those described above. Instead, tails were

populated in the full phase space by application of noise on transverse dampers, or the beam was kicked into large betatron oscillations to physically overlap circulating beam tails with the extraction crystal.[19]

6 SUMMARY

Single-frequency and narrow-band modulations are becoming more common with the advent of high-sensitivity systems in beam instrumentation. Minute responses to small excitations are locked and tracked while minimizing beam disturbances, and create the opportunity for understanding and experience with beam response during operations. Previous approaches using broad-band noise are unfriendly to luminosity requirements of colliding-beam experiments.

There have been qualitative and quantitative successes in the realms of nonlinear dynamics and magnet modulations, both transversely and longitudinally. In particular, tune modulation and transverse nonlinear dynamics have been combined to qualitatively affect slow tail transport and loss, though there are complex dependencies on even small changes in machine parameters. Transverse tune modulation may be controlled via feedback, but it is not a promising mechanism in the search for delicate slow extraction mechanisms during collider operations. The changing nature of beam sizes and beam-beam forces, and the resonances they create, is enough to limit the functionality of this approach.

RF space particle transport is more promising than the transverse, owing to the simpler nature of RF dynamics, the lack of strongly driven resonances, and the slower characteristic frequencies of motion. RF voltage modulation is accessible at frequencies and strengths required for low-intensity tail repopulation, and a promising parasitic mechanism has been proposed for crystal extraction [13]. However, RF extraction methods are only applicable for crystals placed in high-dispersion areas. As of the present there are no known plans to experimentally investigate high-dispersion crystal extraction.

7 REFERENCES

[1] Boccard, C. et al., "Tune Measurements in the SPS as Multicycling Machine", in *Proceedings of the 1996 European Particle Accelerator Conference* (IEEE, Sitges, 1996), Vol. 2, p. 1600.

[2] Deregel, J. et al., "Proposal of a K-Modulation System for the LHC Quadrupoles", LHC Project Report 4, 1996.

[3] Castro-Garcia, P. , "Luminosity and Beta Function Measurement at the Electron-Positron Collider Ring LEP", Ph.D. thesis, CERN-SL-96-070-BI, 1996.

[4] Cheng, Wen-Hao, A.M. Sessler, and J.S. Wurtele, "Damping of the Transverse Head-Tail Instability by Periodic Modulation of the Chromaticity", Phys. Rev. Lett. **78**, 4565 (1997) .

[5] Satogata, T. "Nonlinear Resonance Islands and Modulational Effects in a Proton Synchrotron", Ph.D. thesis, Northwestern University, February 1993.

[6] Satogata, T. et al., "Driven Response of a Trapped Particle Beam", Phys. Rev. Lett. **68**, 1838 (1992) .

[7] Wang, Y. et al., "The Effects Of Tune Modulation On Particles Trapped In One-Dimensional Resonance Islands", Phys. Rev. E **49**, 5697 (1994) .

[8] Fischer, W. "An Experimental Study on the Long-Term Stability of Particle Motion in Hadron Storage Rings", Ph.D. thesis, Universitat Hamburg, DESY 95-235, December 1995.

[9] Chen, T. et al., "Measurements of a Hamiltonian System and their Description by a Diffusive Model", Phys. Rev. Lett. **68**, 33 (1992) .

[10] Brüning, O.S. and F. Willeke, "Reduction of Particle Losses in HERA by Generating an Additional Harmonic Tune Modulation", in *Proceedings of the 1995 IEEE Particle Accelerator Conference* (IEEE, Dallas, 1995), Vol. 1, p. 420.

[11] Zimmermann, F. "Transverse Proton Diffusion", Particle Accelerators 49(N2) 67-104, 1995.

[12] Peggs, S., "Proton Mining — Dual Frequency Amplitude Modulation", May 9, 1991, FNAL AP note 91-001.

[13] Gabella, Rosenzweig, Kick, Peggs, "RF Voltage Modulation at Discrete Frequencies, with Applications to Crystal Channeling Extraction", Particle Accelerators, 1993, Vol 42(3-4), pp. 235-257.

[14] Ellison, M. et al., "Driven Response of the Synchrotron Motion of a Beam", Phys. Rev. Lett. **70**, 591 (1993) .

[15] Li, D. et al., "Experimental measure of resonance islands induced by RF voltage modulation", Phys. Rev. E **48**, R1638 (1993) .

[16] Lee, S.Y. et al., "Parametric Resonances in Synchrotrons with Two RF Systems", Phys. Rev. E **49**, 5717 (1994) .

[17] Lee, S.Y. and K.Y. Ng, "Particle Dynamics in Storage Rings with Barrier RF Systems", Phys. Rev. E **55**, 5992 (1997) .

[18] Liu, J.Y. et al., "Bifurcation of Resonance Islands and Landau Damping in the Double-RF System", Phys. Rev. E **50**, R3349 (1994) .

[19] Murphy, C.T et al., "First Results From Bent Crystal Extraction at the Fermilab Tevatron", in *Proceedings of the Workshop on Channeling and Other Coherent Crystal Effects at Relativstic Energies* (Aarhus, Denmark, 1996).