RHIC/AP/2
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Required Accuracy of the RHIC Circumference

1. Introduction

The ideal RHIC **design circumference** is:

$$C_{RHIC} = \mathbf{3833.845000}\ \text{m} \qquad 1$$

The RHIC accelerator complex consists of two rings, "Blue" and "Yellow" in which two counter-rotating beams of particles will collide head-on at up to six interaction regions. There will be 57 bunches of particles filling each of the two rings (or 114 bunches, in a possible future upgrade). The number 57 is derived from the design ratio of RHIC and AGS circumferences, C_{RHIC}/C_{AGS}=19/4, multiplied by the AGS number of 12 rotating bunches. The AGS circumference was obtained from the revolution frequency measurements at the AGS top energy. A report from the AGS experiments(1) shows that the regular AGS circumference is C_{AGS} = 807.10475 m. The extraction orbit was reported(2) to be 807.12526 m. Each AGS cycle has twelve proton bunches or three heavy ion bunches. The RHIC circumference was obtained from the AGS circumference as:

$$C_{RHIC} = 19/4 * 807.1253\ \text{m} = 3833.845\ \text{m} \qquad 2$$

The actual RHIC **central circumference** propagates through the center of the quadrupoles and through the center of the curved bending magnet beam pipes. It will be different from the design circumference due to surveying errors in positioning the magnets. The ratio of RHIC and AGS central circumferences will not be exactly equal to 19/4. There are also unavoidable errors in defining and measuring the network of the ring surveying monuments. Error analysis(3) of existing network monuments indicates that the average error in measurements of the distances between the twelve monuments is ΔS_{avg} = 0.34 mm, which implies that an error of order **1.2 mm** in the RHIC circumference may already exist.

Any kind of beam transfer from the AGS to RHIC requires correction of the main bending buss, and/or of the rf frequency, in one or both accelerators. The actual synchronous **reference orbit** of an accelerator (the "closed orbit" in 6 phase space dimensions, including synchrotron oscillations) ideally has the same circumference as the **central orbit**, since the beam is then in the center of the aperture, where the magnetic field quality is highest. Any deviation from the center of the aperture puts the beam into a region of the magnetic field where the magnetic multipoles have higher values, lowering the beam dynamical aperture. In practice the synchronous orbit must be made to deviate from the central orbit, in order for injected bunches to be free from oscillations in the RHIC rf bucket.

Studies of RHIC long term particle tracking at the injection energy, as well as at other energies, have been performed. The latest tracking results(4) were performed with alignment errors of quadrupoles and dipoles, with random and systematic multipole errors within the magnets, and with the synchrotron oscillations. Results from ten different random seeds at injection were obtained after the orbit was corrected with the correction element system. The **rms** of the closed orbit errors of the ten seeds were ranging from x_{rms} = 0.124 to 0.204 mm and from y_{rms} = 0.139 to 0.189 mm. This implies that the length of the closed orbit during tracking was not the ideal central orbit ($\Delta C_{RHIC} \approx 1$ mm)

1.1 Different modes of the beam transfer from the AGS to RHIC

1.1.1 <u>The AGS and RHIC are frequency locked</u>. This is the nominal scheme for RHIC injection(5). The RF frequency of the AGS at extraction momentum of 29.22 GeV/c (or γ=31.16 for protons) is f_{AGS}= 4.46 MHz. The nominal RHIC frequency is equal to the product of the harmonic number (for RHIC h=6*12*19/4=342) and the revolution frequency:

$$F_{RHIC} = h_{RHIC} * F_{rev} = h_{RHIC}\, c\beta / C_{RHIC} = 26.7 \text{ MHz} \qquad 2$$

where C_{RHIC} is the reference synchronous circumference of RHIC. In a perfectly tuned transfer the energies and the velocities of synchronous particles in both machines are the same ($\beta_{RHIC}=\beta_{AGS}$), so that the synchronous circumferences are in the ratio

$$C_{RHIC}/C_{AGS} = h_{RHIC}/h_{AGS} = 19/4 \qquad 3$$

1.1.2 <u>Buckets are phase matched **on the fly.**</u> This is the fall back position for RHIC injection, and is routinely performed at HERA. At the instant of transfer, the center of the

AGS bucket and the center of the RHIC bucket are "side-by-side", although 4 turns of RHIC later, they are not.

a) The AGS RF could run independently from the RHIC RF system. The AGS beam could be independently controlled by the AGS rf system while the RHIC rf system adjusts the phase of the recipient bucket to be correct at the instant of transfer.

b) The AGS RF could be controlled by the RHIC RF system. The output of the RHIC RF synthesizer could be sent to the AGS RF for a phase adjustment prior to the instant of extraction from the AGS.

2. Accuracy of the Circumference Ratio

Assume that the frequencies of the AGS and RHIC are locked and the orbits are synchronous in both machines. If the synchronous orbit and the central circumferences in RHIC differ by ΔC, then it is necessary to adjust the synchronous circumference by about $-\Delta C$, relative to the design circumference, even if there is no error in the AGS circumference. The RHIC synchronous closed orbit will then be offset radially from the magnet centers, on average, by

$$\Delta R = -\Delta C/2\pi \qquad 4$$

If the good magnetic field region is defined as ±2 mm then the **allowed difference in the circumference is less than $\Delta C < 2$ mm $*2\pi \approx$ 12.6 mm.**

The longitudinal emittance is designed to be 0.3 eV-s. A bunch length for the proton beam at injection is **hbl = 6.5 ns.** A difference between the **central orbit** and the synchronous orbit of ΔC = **12.6 mm** could be presented in time as:

$$\Delta T = \Delta C/\upsilon_0 = \mathbf{0.042\ \ ns}, \qquad 5$$

where υ_0 is the speed at the synchronous orbit.

A requirement for a blowup of the six dimensional emittance could be set for the injection to be less than 10% ($\Delta\varepsilon/\varepsilon \leq \mathbf{0.1}$). An error analysis[6] shows that there would be a requirement of $(\delta c/r)_{rms} \leq 0.0068$ for each individual coordinate of the six dimensional space, where the δc is one of the errors of the six dimensional space coordinates. Applying the requirement for the six dimensional blowup of $\Delta\varepsilon/\varepsilon \leq \mathbf{0.1}$ the request for the half bunch length coordinate is Δhbl = 0.0068 * 6.5 ns = **0.044 ns**

A complete analysis of rf gymnastics must take into account possible variations in the magnetic field and the orbit so that the rf frequency as well as the bending field in magnets can be adjusted properly. Possible sources of injection errors include:

1) During the extraction process in the AGS at the end of the acceleration process the rf frequency will be locked. The acceleration will continue with a change of the orbit as the guiding field is still increasing. The acceleration is stopped at the moment when the closed orbit coincides with the extraction orbit. The extracted AGS bunches might have variations in momentum due to variations of the magnetic field (with fixed rf frequency in the AGS and in RHIC) which can be presented as:

$$\Delta p/p = [\gamma^2/(\gamma^2-\gamma_t^2)]\ \Delta B_{AGS}/B_{AGS} \qquad 6$$

This leads to an AGS synchronous circumference error given by

$$\Delta C_{AGS}/C_{AGS} = 1/\gamma^2\ \Delta p/p \qquad 7$$

At injection into RHIC there will be an average instantaneous radial offset given by

$$\Delta R_{RHIC}/R_{RHIC} = 1/\gamma^2\ \Delta p/p \qquad 8$$

The injected beam would be asynchronous since $h*F_{rev}$ has changed while the rf frequency of RHIC (F_{RHIC}) has not. The bending field of RHIC needs adjustment, or longitudinal feedback must be applied, if the longitudinal emittance of the bunch is not to grow.

2) Although extraction from the AGS might be under ideal conditions and with the designed rf frequency in the AGS, there could be a variation of the bending field of the RHIC dipoles. If the variation of the RHIC field is ΔB_{RHIC} but the AGS and RHIC frequencies remain fixed, then the momentum of the RHIC injected beam remains the same as in the AGS but the *instantaneous* closed orbit in RHIC deviates from the designed orbit:

$$\Delta R_{RHIC}/R_{RHIC} = 1/\gamma^2\ \Delta B_{RHIC}/B_{RHIC} \qquad 9$$

The RHIC rf frequency needs adjustment as :

$$F_{RHIC} = h_{RHIC}\, F_{rev} = h_{RHIC}\beta c/\ 2\pi * (R_{RHIC}+\Delta R_{RHIC}) \qquad 10$$

3. Tolerable circumference errors between BLUE and YELLOW rings

There are proposals to run high intensity proton beams with different energies in the RHIC accelerators. The mode of operation is the rf **storage** mode where the proton bunches with different energies will collide at the interaction points. The rf frequency of the two beams is defined again by the revolution frequency and the harmonic number. The revolution frequency depends directly on the energy of the beam. The high energy physics experiments require for the error of the two beam energies not to be larger than the **energy spread** (dp/p=0.192% @ storage) of the beam within the bunches. This condition sets the allowed error of circumferences of the two rings. Again the rf frequencies are locked through the six cavities **common** for both rings. If there is an error in the circumference of one ring with respect to the other the synchronous orbit through one of the rings will be with an offset :

$$\eta = 1/\gamma_t^2 - 1/\gamma^2$$

$$\eta_{250GeV} = 22.8^{-2} - 268.2^{-2} = 0.019, \quad \eta_{100GeV} = 0.018 \qquad 11$$

$$\Delta C = \Delta C_1 - \Delta C_2 < 3833.845 * \eta \; dp/p = 13 \text{ mm}$$

4. Conclusions

Required accuracy of the RHIC circumference is determined at the injection while the allowed difference between the two "blue" and "yellow" ring circumferences is determined at storage. When the rf frequencies are locked a tolerable error for the circumference is $\Delta C \leq$ **13 mm**. In the storage mode according to the requirement by the experiments $\Delta C \leq$ **13 mm**. This request is believed to be easily achievable.

5. References:

1. Experimental results from the AGS Report 246, November 1988, appendix.

2. Edward Blesser, Memo for the AGS extraction 1988.

3. RHIC Internal Report,"Analysis of the Surveying Monuments Data," June 11, 1992.

4. RHIC Internal Report, "Accelerator Advisory Committee Meeting," May 25-26, 1993.

5. RHIC Internal Report, "Conceptual Design of the RHIC RF System", May 1993.

6. J. Claus and H. Foelsche, "Beam Transfer from AGS to RHIC", RHIC Technical Note No. 47, December 12, 1988, AD/RHIC-47.

7. RHIC Internal Report, "RHIC Design Manual", August 31 1992.