RHIC-PG-19

REGULAR CELL AND DISPERSION KILLER FOR THE 120° PHASSE ADVANCE CASE

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(BNL, December 9, 1983)

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Lattice for a Regular Cell

(Thin lenses approximation)

$$\begin{cases} L, & \text{length of half-cell} \\ \theta, & \text{bending for half-cell} \\ \varphi, & \text{phase advance per half-cell} \end{cases}$$

$$\frac{B' l_Q}{B\rho} = \frac{2 \sin\varphi}{L}$$

$$\begin{cases} \beta_{max} = \dfrac{L}{\sin\varphi} \left(\dfrac{1 + \sin\varphi}{1 - \sin\varphi} \right)^{1/2} \\[2ex] \beta_{min} = \dfrac{L}{\sin\varphi} \left(\dfrac{1 - \sin\varphi}{1 + \sin\varphi} \right)^{1/2} \end{cases}$$

$$\begin{cases} \eta_{max} = \dfrac{L\theta}{\sin\varphi} \left(\dfrac{1}{\sin\varphi} + \dfrac{1}{2} \right) \\[2ex] \eta_{min} = \dfrac{L\theta}{\sin\varphi} \left(\dfrac{1}{\sin\varphi} - \dfrac{1}{2} \right) \end{cases}$$

$$\begin{cases} \bar{\beta} = \dfrac{\beta_{max} + \beta_{min}}{2} = \dfrac{2L}{\sin 2\varphi} \\[2ex] \bar{\eta} = \dfrac{\eta_{max} + \eta_{min}}{2} = \dfrac{L\theta}{\sin^2\varphi} \end{cases}$$

$$\gamma_T = \sqrt{R/\bar{\eta}}$$ R, average radius

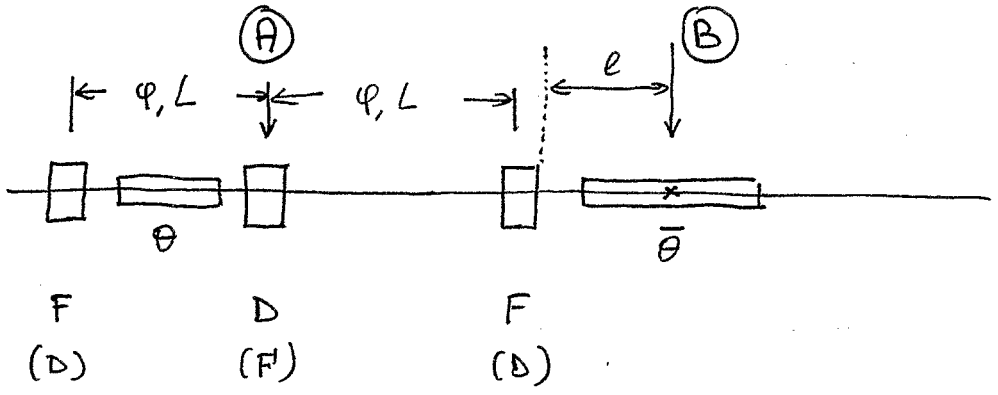
For $\varphi = 60°$

$$\frac{B'l_a}{B\rho} = \frac{1.732}{L}$$

$$\begin{cases} \beta_{max} = 4.3094\ L \\ \beta_{min} = 0.3094\ L \end{cases}$$ $\bar{\beta} = 2.3094\ L$

$$\begin{cases} \eta_{max} = 1.9107\ L\theta \\ \eta_{min} = 0.7560\ L\theta \end{cases}$$ $\bar{\eta} = 1.3333\ L\theta$

Dispersion Killer (thin lenses)



Transfer matrix from A to B

$$M = \begin{pmatrix} 1 & \ell \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -p & 1 \end{pmatrix} \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ p/2 & 1 \end{pmatrix}$$

$$= \begin{bmatrix} (1-\ell p)\left(1 + L\frac{p}{2}\right) + \frac{\ell p}{2} & L(1-\ell p) + \ell \\ p/2 - p\left(1 + \frac{Lp}{2}\right) & 1 - Lp \end{bmatrix}$$

$$p = \frac{B' l_Q}{B\rho} \quad , \quad \frac{Lp}{2} = \sin\varphi$$

At A $\quad \eta = \eta_A \quad , \quad \eta' = 0$

At B $\quad \eta = 0 \quad , \quad \eta' = 0$

For the condition $\quad \eta_B = 0$

$$(1 - \ell_p)\left(1 + L\frac{p}{2}\right) + \frac{\ell p}{2} = 0$$

from which

$$\frac{\ell}{L} = \frac{1 + \sin\varphi}{\sin\varphi \cdot (1 + 2\sin\varphi)}$$

For $\quad \varphi = 60°$

$$\ell = 0.788675 \quad L$$

$$\eta_B' = \left[\frac{p}{2} - p\left(1 + \frac{Lp}{2}\right)\right] \eta_A$$

$$= -\frac{\sin\varphi}{L}\left(1 + 2\sin\varphi\right) \eta_A$$

$$\eta_A = \frac{L\theta}{\sin\varphi}\left(\frac{1}{\sin\varphi} \pm \frac{1}{2}\right)$$

upper sign if QF' at A
lower sign if QD at A

$$\eta_B' = -\theta \frac{(1 + 2\sin\varphi)(2 \pm \sin\varphi)}{2\sin\varphi}$$

For $\varphi = 60°$

$$\eta_B' = -\theta \times \begin{cases} 4.520726 & \text{with QF' at A} \\ 1.788675 & \text{with QD at A} \end{cases}$$

Clearly the solution with QD at A is preferred.

$\bar{\theta}$, bending angle of the dispersion corrector dipole

$$\bar{\theta} = 1.788675 \times \theta$$