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DAMPING LONGITUDINAL INJECTION ERRORS IN RHIC

E. Raka

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Introduction

The purpose of this note is to present the design consideration for the wideband cavity system whose performance specifications are given in the RHIC CDR[1]. As pointed out therein the chief purpose of the system will be to damp longitudinal injection errors arising from momentum errors or phase mis-matches. Since such errors will result in coherent dipole oscillations of each bunch damping this motion can be accomplished by phase modulating the effective rf voltage seen by an individual bunch. To do this on a bunch to bunch basis with 114 bunches (112 nsec. spacing) will require a separate cavity whose bandwidth is much greater than than of the accelerating cavities, i.e. at least $\pm 57 f_o$ around $f_{rf} = 342 f_o$.

Now effective phase modulation of the 26.7 MHz accelerating cavity voltage seen by a bunch can be accomplished by adding a small voltage in quadrature i.e. $B \cos \omega_{rf} t$ to $V_{acc} \sin \omega_{rf} t$. Since this voltage must be seen only by the bunch that is to be damped it has to be gated on and off at the maximum bunch frequency of $\approx$ 9 MHz. Also since f_{rf} is three times this frequency it is desirable to make the on gate equal to three rf periods so that the transient response of the cavity has decayed significantly before the bunch arrives at the gap. Thus the cavity exitation will consist of three cycles of ω_{rf} gated at the rotation frequency, whose amplitude will be modulated by the position error of the the injected bunch (or possibly the rf phase error of this bunch but delayed by one quarter of a sychrotron period).

Cavity Transient Analysis

We represent the cavity as a parallel L, C, R_p circuit driven by a current generator of $i_g = V_g / R_g$ so that the shunt resistance is $R_e = R_g R_p / (R_g + R_p)$. Using the LaPlace transform we can write the admittance as

$$Y = 1/R_e + 1/sL + sC = \frac{sL + R_e + s^2 LCR_e}{sR_e L} \tag{1a}$$

so that

$$V_o = \frac{i(s)\, sLR_e}{s^2 LCR_e + sL + R_e} = \frac{i(s)}{C} \frac{s}{s^2 + 2 \propto s + \omega_0^2} \tag{1b}$$

where $\propto = 1/2CR_e$ and $\omega_o^2 = 1/LC$. One can also write

$$V_o = \frac{i(s)\, s}{C(s-a)(s-b)} \qquad \begin{aligned} a &= - \propto + \sqrt{\propto^2 - \omega_0^2} \\ b &= - \propto - \sqrt{\propto^2 - \omega_0^2} \end{aligned} \tag{2}$$

For a step input of current $i(s) = V_i/R_g s$ the response is given by

$$V_0(t) = \frac{V_i}{CR_g}\frac{(e^{at} - e^{bt})}{(a-b)} \tag{2a}$$

This is non-oscillating as long as one has $\propto > \omega_o$ *or* $\sqrt{L/C} > 2R_e$ i.e. greater than critical damping.

What we are interested in is the repsonse to an input of the form $\sin\omega t$ or $\cos\omega t$ since we have not specified the timing of the on gate. We consider first $V_{in} = V\sin\omega t$ *at* $t = o$ so that

$$i(s) = \frac{V_i}{R_g}\frac{\omega}{(s^2+\omega^2)} \quad t \geq 0 \tag{3}$$

which gives

$$V_o(t) = V_i\left[\sin\omega t + \frac{\omega(e^{bt} - e^{at})}{2\sqrt{\propto^2 - \omega_o^2}}\right] \tag{3a}$$

Next we take $V_{in} = V\cos\omega t$ *at* $t = o$ so that

$$i(s) = \frac{V_i s}{R_g(s^2+\omega^2)} \quad t \geq 0 \tag{4}$$

which gives

$$V_o(t) = V_i\left[\cos\omega t + \frac{(be^{bt} - ae^{at})}{2\sqrt{\propto^2 - \omega_o^2}}\right] \tag{4a}$$

since

$$2\sqrt{\propto^2 - \omega_o^2} = (a-b)$$

Now the latter terms in equations 3a, 4a represent the transient response of the cavity (assumed here to be overdamped). Depending upon the amount of damping they will become negligible after only a few cycles of the input signal. It is easy to show that if the excitation is then removed at the end of a given cycle of $\sin\omega t$ *or* $\cos\omega t$ the cavity output will cosist of only these terms but with a reversal in sign. Hence if the transient is negligible after N cycles of excitation it will also be negligible N cycles after the excitation is removed.

Cavity Parameters:

Now we assume that the cavity will be driven remotely from a 50Ω source. Hence the impedance presented by the cavity at $\pm 57 f_o$ either side of its resonant frequency of $342 f_o$ should be large compared to 50Ω. Since the Q of the cavity will be >> 1 one can show that in this case

$$|Z_c| \cong R_o Q/\tan\theta \tag{5}$$

where tan $\theta = Q(\omega/\omega_r - \omega_r/\omega)$. For $\omega = 5\omega_r/6$ one then obtains $|Z_c| \cong 30R_o/11 \approx 477\Omega\,(for R_o = 175\Omega)$ and a somewhat larger value for $\omega = 7\omega_r/6$.

With this value of $R_o = \sqrt{L/C}$ for the cavity and assuming a 50Ω shunt accross the gap so that $R_p = 50\Omega$ we have

$$\propto = \frac{1}{2CR_e} = \frac{\omega_r R_o}{2R_e} = \frac{175\omega_r}{2.25} = 3.5\omega_r \tag{6}$$

and

$$\begin{aligned} a &= -3.5\omega_r + \omega_r\sqrt{3.5^2 - 1} = -.145\omega_r \\ b &= -6.85\omega_r \end{aligned} \tag{7a, b}$$

Using these values of a and b we can determine the magnitude of the turn on or turn off transient seen at the center of a bunch when the excitation gate is opened at zero volts or at maximum drive (see figure 1).

We have

$$V_c(\tau_b) \cong \frac{V_i a e^{at}}{(a-b)} = -\frac{.145 V_i \omega_o}{6.7\omega_o}\exp(-.145 \cdot 2.5\omega_o\tau_o) = 2.2 \cdot 10^{-3}\mathrm{V_i} \tag{8a}$$

or

$$V_s(\tau_b) \cong \frac{\omega_o}{6.7\omega_o}\exp(-.145\omega_o 2.75\tau_o) = 1.22 \cdot 10^{-2}\mathrm{V_i} \tag{8b}$$

where c and s refer to equations 4a and 3a respectively. Thus for these parameters the transient would be of the order of one percent for the gate being turned on at the zero crossing of the exitation. Which phase of the excitation is chosen will depend upone bench testing of components but one should use an integral number of rf cycles so that the DC component is always zero.

We note here that the bandwidth of this configuration i.e. 50Ω source and a 50Ω load accross at cavity with $R_o = 175\Omega$ is from $.265\omega_r$ *to* $3.76\omega_r$ *i.e.* $175\omega_r/50 = 3.5\omega_r$.

Frequency Spectrum of Cavity Excitation:

The spectrum of a gated three cycle "sine" wave of frequecny $h\omega_o$ and repetition period $2\pi/\omega_o$ can be written as

$$A(n\omega_0) = \frac{2h}{\pi}\sum_{n=o}^{\propto}\frac{\sin 3n\pi/h}{(h^2 - n^2)} \tag{9}$$

For gated three cycle "cosine" excitation the spectrum in equation 9 should be multiplied by n/h. We show a plot of these two spectra in figure 2. Actually the spectrum would consist of pairs of sidebands $(n\omega_o \pm \omega_s)$ around each rotation line since the cavity excitation in either case would be amplitude modulated by $r \sin \omega_s t$ where r is the dipole oscillation amplitude and ω_s is the coherent synchrontron frequency. We note that the "cosine" excitation falls off less rapidly at frequencies above $h\omega_o$ the cavity resonant frequency while the "sine" excitation is more symetrical around $h\omega_o$. However since the loaded cavity response falls off less rapidly above resonance the "cosine" excitation is well matched by the cavity.

Frquency Spectrum of Alternate Damping Systems:

Now the spectrum of the beam current due to a single bunch preforming a coherent dipole oscillation can be approximated by that due to stationary bunch plus that due to a perturbed charge distribution that oscillates near or at ω_s the synchrotron frequency. The stationary part generates lines at $n\omega_o$ whose amplitude depends upon the bunch shape while the latter provides sidebands at $(n\omega_o \pm \omega_s)$ (for small osscillation amplitudes) whose amplitudes depend upon the assumed shape of the perturbed charge distribution.(2) The envelope of the square of the Fourier coefficients of this charge distribution is the so called form factor for dipole oscillations (m=1). It is the interaction of this component of the bunch current with the ring impedance that can produce instabilities. However, it is also possible to use this signal to damp the dipole oscillations by feeding it back to a cavity after filtering out the $n\omega_o$ lines and providing a 180° phase shift between each pair of sidebands.(3)

If, instead of using the position error of the bunch to modulate the amplitude of the gated $(cos\omega_{rf}t)$ signal, one were to feed the difference signal from a pair of pickup electrodes directly to the cavity with the correct time delay then damping could also be achieved. In this case the spectrum of the signal would be that of the beam current multiplied by $(a \propto \sin \omega_{st} + b)$ where $\propto$ is proportional to the bunch oscillation amplitude and a and b are geometircal constants of the PUE structure. Hence there would be again pairs of sidebands at $(n\omega_o \pm \omega_s)$ whose amplitude would be proportional to the stationary bunch spectrum and $\propto$. There would also be some signal at $(n\omega_o \pm\ 2\omega_s)$ but this would be of order $\propto^2$. (4)

Thus we see that no matter what method is used one must either detect, amplify or generate the sideband frequencies $(n\omega_o \pm \omega_s)$ and apply them to a cavity with the proper phase and suitable amplitude in order to produce damping of the dipole oscillations.

Damping Rates:

We shall assume Au ions are injected with a $\gamma = 12.6$ and with a bunch area of 0.3 e vsec/AMU. The bunch half length will then be $\hat{\theta} = 1.36$ rad for $V_{rf} = 185KV$. Now the spread in synchrotron frequency within a bunch can be written as:

$$\Delta\omega_s = \omega_{so}\hat{\phi}\delta\hat{\phi}/4 \tag{10}$$

where

$$\delta\hat{\phi} = \frac{\omega_{rf}\eta}{\omega_{so}}\frac{\delta p}{p} \tag{11}$$

is the phase oscillation amplitude corresponding to a momentum error of $\delta p/p$. Hence for an injection error of 10^{-4} in momentum one obtains a $\delta\hat{\phi}$ =.127 rad and a $\Delta\omega_s$ = .043 ω_{so}.

Next we consider the decoherence time[4,5] which can be written as $\Delta t = \pi/\Delta\omega_s = 1/2\Delta f_s$. For the above parameters we obtain a $\Delta t = .116 sec$ and proceed to calculate the maximum voltage required to reduce a 10^{-4} momentum error to e^{-1} in this time.

One can write for the rate of change of $(\Delta p/p)$

$$\left(\frac{\Delta p}{p}\right) \cong \frac{f_o}{2}\frac{\Delta E}{E} = \frac{V_d.79 \cdot 78.2 \cdot 10^3 \cdot 0.5}{197 \cdot 12.6 \cdot .938 \cdot 10^9} = 1.33 \cdot 10^{-6} V_d \tag{12}$$

where f_o is the rotation frequency and the factor of 0.5 is the average value of $sin^2\omega_s t$. This gives the rate for the proportional damping since the V_d will vary at $sin\omega_s t$ while the effectiveness of the successive kicks will also vary as $sin\omega_s t$.

If the voltage per turn is kept constant until the remaining error is within the noise level of the system then this factor becomes $2/\pi$ = .637 which is the average value of $|sin\ \omega_s t|$. This is called bang-bang damping.

For the proportions damping case we obtain a V_d = 650 volts. If this voltage was used in the bang-bang mode then the entire error would be corrected in .091 sec i.e. less than the decoherence time. Since simulation calulations [5] indicate that in order to minimize phase space dilution resulting from rf bucket non-linearities the injection error should be damped in a time less than Δt, the latter mode should be used. In order to have some margin in the system a peak voltage capability near 1KV would be desirable. This would in turn require 10 KW of peak power. However since the duty factor would be quite small i.e. 0.1 sec/AGS period the power amplifies need have only a large pulsed power capability.

Other Options

From equation 11 we see that the momentum error and hence the position error or amplitude varies as ω_{so} for a given phase oscillation amplitued $\delta\hat{\phi}$. Hence at high energies away from injection, the onset of an insability may become more difficult to detect above the noise level, for a position sensing system, than for one that measures the individual phase error or each bunch. Thus wideband phase detection and the use of digital filters to obtain the necessary 90° phase shift should be investigated. Currently this type of system is being developed at the Fermilab booster based on the method outlines by F. Pedersen.[6]

Now during injection there can also be a mismatch of the bunch to the bucket which will give rise to bunch shape or quadrupole oscillations which will occur at $2\omega_s$. In particular for proton injection, where the bunch rotation process will be performed in order to match the bunches to higher voltage buckets (7), one can expect variations in the bunch shape at transfer. Hence a bunch to bunch quadrupole damping system may be required to minimize any dilution resulting from any mismatch. This could be accomplished by amplitude

modulating three cycle bursts of $\sin \omega_{rft}$ with the appropriate error signal and applying it to the wide band cavity in the same manner as the dipole case. The error signal can be obtained by measuring the peak amplitude of a bunch and again using digital filtering to obtain the required 90° phase shift. Thus one would generate amplitude modulation sidebands at $\pm 2\omega_s$ around the $n\omega_o$ lines which would then damp the quadrupole mode.

Finally one should also consider the frequency domain approach using periodic digital filter.[6,8] Here one used a sun pickup for the beam informaiton and a two path filter system[8], where the rf frequency and its quadruture are mixed with the bunch frequencies and sampled at 10 Mhz, the digital filter frequency. This would give a 10 MHz bandwidth and hence both the upper and lower sidebands for a 114 bunch spacing. The transmission of both sidebands are required if this mode-by-mode system is to be the equivalent of a bunch to bunch system. Here again the overdamped wide band cavity would be required since the output must be essentially flat over the range $\pm M f_o/2$ whre M is the number of bunches.

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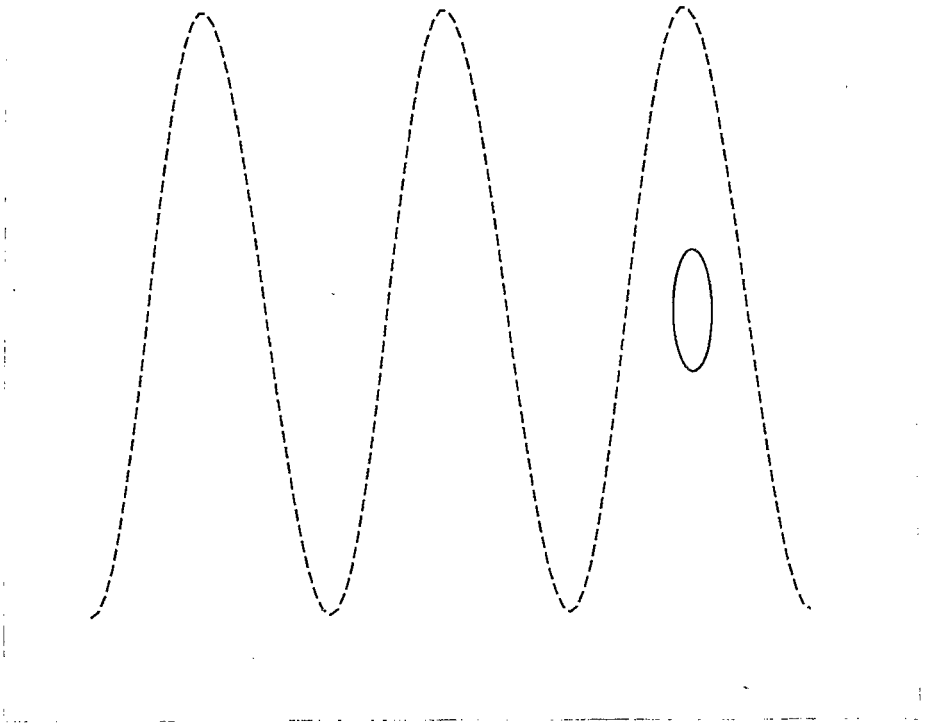
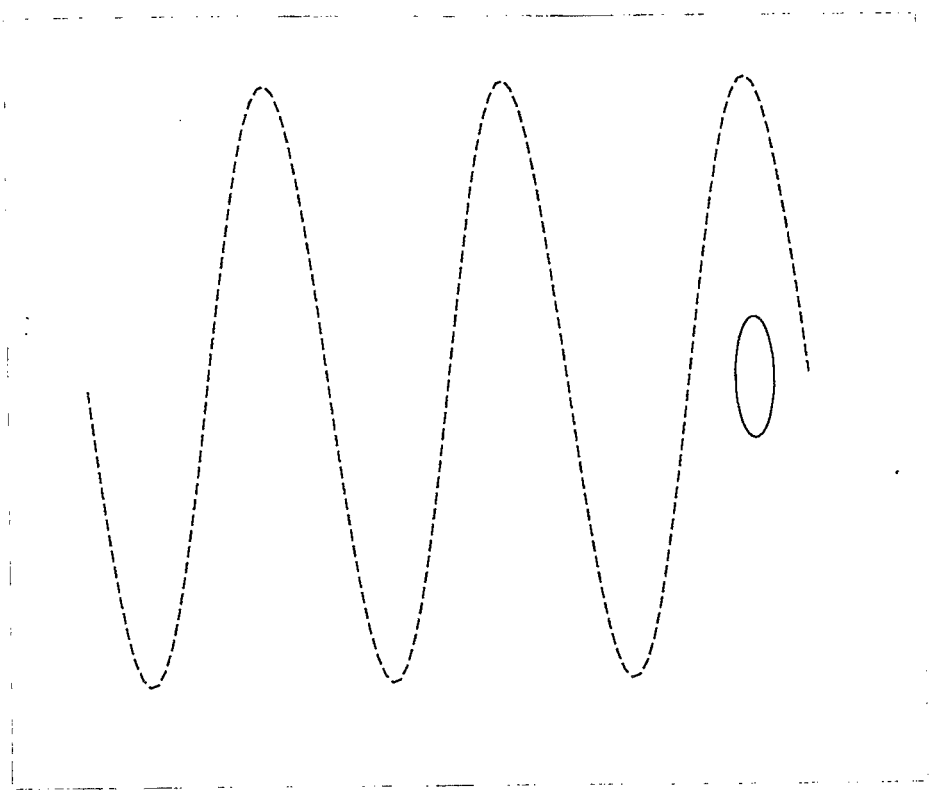


"Cosine Excitation"



"Sine Excitation"

Figure 1

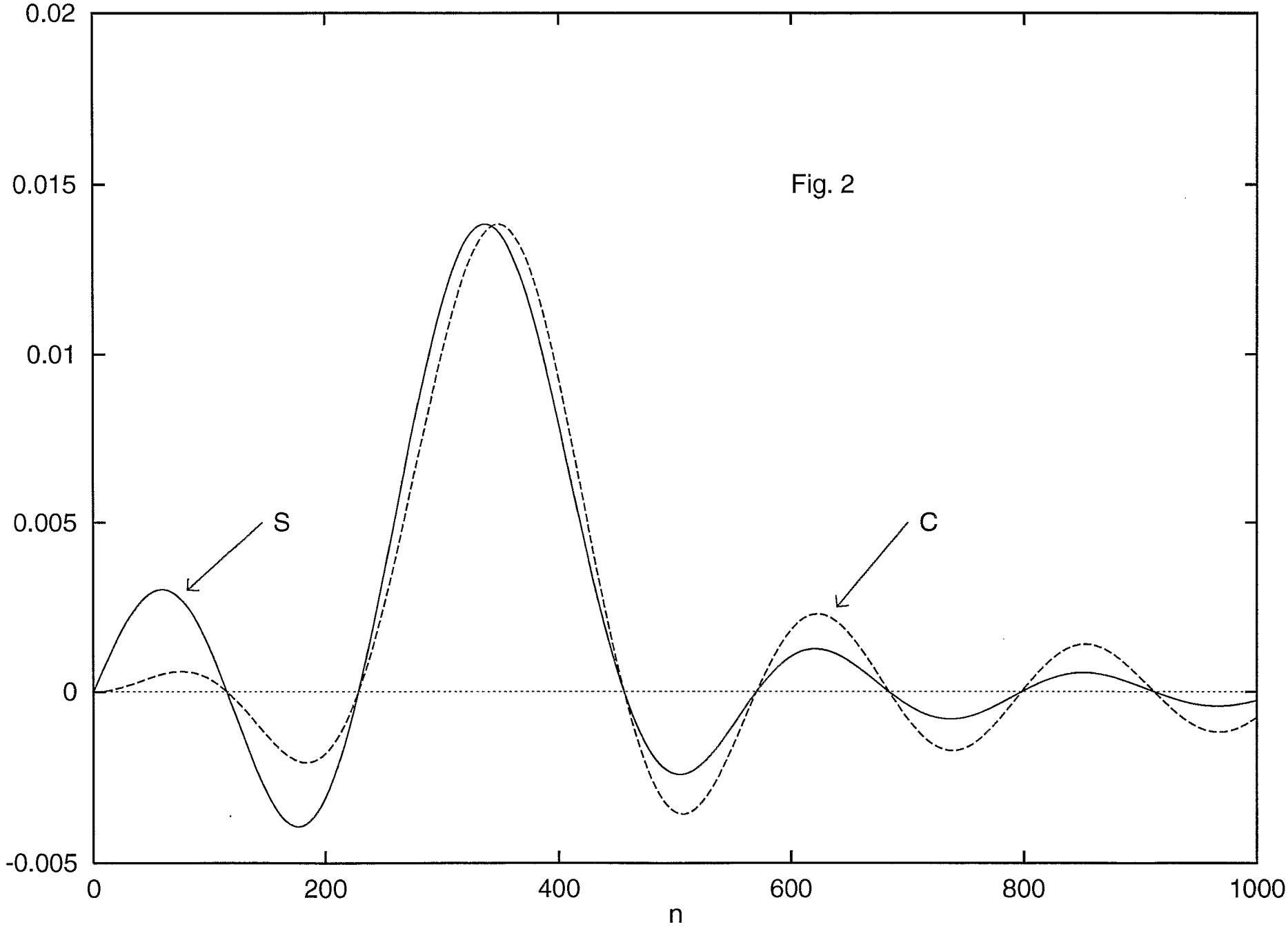


Fig. 2