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Overcoming the Intrinsic Spin Resonance using Resonance Island created by RF Dipole*

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Abstract

A method of overcoming intrinsic spin resonances by transporting polarized beam into a resonance island created by rf dipoles for adiabatic spin flip is discussed. Specific parameters needed in achieving polarized beam acceleration in the AGS are estimated. Numerical simulations with the AGS parameters are performed to verify the theoretical estimation.

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1 Introduction

Spin motion in synchrotrons obeys the Thomas-BMT equation [1]. For a planar synchrotron with a vertical guide field, the polarization vector precesses $G\gamma$ turns per orbital revolution, where $G = 1.792847386$ is the anomalous g-factor for protons, γ is the Lorentz factor. Thus $G\gamma$ is called the spin tune. In high energy accelerators, beam stability requires strong focusing quadrupoles. The non-vertical magnetic fields can perturb the spin away from the vertical direction. In particular, a spin depolarization resonance occurs when the spin tune is equal to the tune of spin perturbing fields.

During acceleration toward high energy, polarized beams encounter many depolarization resonances [2, 3]. Two major types of spin depolarization resonance exist in synchrotrons, i.e., the imperfection and the intrinsic resonances. The imperfection spin resonance is created by the vertical closed orbit deviation from the center of quadrupoles, and the intrinsic spin resonance is produced by the vertical betatron oscillations. The resonance condition is given by

$$G\gamma = \begin{cases} n & \text{for imperfection spin resonance,} \\ kP \pm m\nu_z & \text{for intrinsic spin resonance,} \end{cases} \tag{1}$$

where k, m and n are integers, P is the number of superperiods, and ν_z is the vertical betatron tune. For polarized protons in the AGS, we have $P = 12$, and $\nu_z \approx 8.8$. Along with these two major spin resonances, depolarization can also occur at the linear coupling resonance line (see e.g. [4]), and at the synchrotron sidebands of primary spin resonances [5].

Using the Thomas-BMT equation [1], the spin resonance strength is given by the Fourier amplitude of the spin perturbing fields in synchrotrons [2, 3], i.e.

$$\epsilon_K = \frac{1}{2\pi} \oint \left[(1 + G\gamma) \frac{\Delta B_x}{B\rho} + i(1 + G) \frac{\Delta B_{||}}{B\rho} \right] e^{iK\theta} ds, \tag{2}$$

where θ is the orbital bending angle, s is the distance along the reference orbit, ΔB_x is the radial perturbing field, $\Delta B_{||}$ is the longitudinal perturbing field, and $B\rho$ is the magnetic rigidity of the beam.

A partial Siberian snake has been successfully employed to overcome imperfection resonances at the AGS, and the IUCF Cooler Ring [4]. On the other hand, overcoming intrinsic resonances, which are traditionally corrected by the tune jump method [6], remains to be a difficult task. The difficulty in the tune jump method is that it will also cause emittance growth resulting from non-adiabatic optical mismatch and particularly the non-adiabatic betatron oscillations induced by an off-centered closed orbit in the tune jump ferrite quadrupoles [7]. For attaining polarized proton collisions in the relativistic heavy ion collider (RHIC), the polarized beam in the AGS must be accelerated up to $G\gamma = 47.5$ passing through many intrinsic resonances at $G\gamma = \nu_z, 24 - \nu_z, 12 + \nu_z, 36 - \nu_z, 24 + \nu_z, 48 - \nu_z$

and $36+\nu_z$. Thus finding a scheme to overcome intrinsic spin resonances without causing emittance dilution is an important topic in accelerator physics.

This paper studies the feasibility of polarized proton acceleration through intrinsic spin resonances by using the betatron resonance islands generated by a radio frequency (rf) dipole field. Section II provides theoretical framework for the method. Numerical simulations and the AGS modeling are given in Section III, and the conclusion is given in Section IV.

2 The theoretical framework

When the spin tune of a polarized vector is uniformly accelerated through an isolated resonance with a resonance strength ϵ, the ratio of the final polarization P_f to the initial polarization P_i is given by the Froissart-Stora formula [8]

$$\frac{P_f}{P_i} = 2e^{-\frac{\pi|\epsilon|^2}{2\alpha}} - 1, \tag{3}$$

where ϵ is the resonance strength, and

$$\alpha = \frac{d(G\gamma - kP \mp \nu_z)}{d\theta}$$

is the acceleration rate. Based on the Froissart-Stora formula, one might think that the polarization of the beam could be maintained by the adiabatic spin-flip through a strong spin resonance.

However, the intrinsic spin resonance strength is proportional to the betatron amplitude or the square root of the vertical betatron action for an intrinsic resonance. Since a beam of polarized protons is usually composed of particles with different betatron actions, the final polarization is obtained through the ensemble average of the Froissart-Stora formula over all particles with different betatron amplitudes. The final polarization after passing through an intrinsic resonance for a beam with Gaussian distribution becomes

$$\langle\frac{P_f}{P_i}\rangle = \frac{1-W_0}{1+W_0}, \tag{4}$$

where $W_0 = \pi|\epsilon_0|^2/\alpha$, and ϵ_0 is the spin resonance strength for the particle with the rms betatron action $I_{\rm rms}$ of the beam. Here the rms action of the beam is defined by the rms beam size with

$$I_{\rm rms} = \frac{\sigma_z^2}{2\beta_z}, \tag{5}$$

where σ_z is the vertical rms beam size, and β_z is the vertical betatron amplitude. This means that the adiabatic spin-flip by a slow acceleration through an intrinsic resonance cannot efficiently be used in polarized proton acceleration. Since

particles at the center of the beam bunch with small actions do not flip their spin, partial beam depolarization is usually inevitable.

To circumvent this difficulty, we can give the beam a coherent betatron excitation so that all particles have large betatron actions and the resulting ensemble average of the final polarization is essentially the same as that obtained from the Froissart-Stora formula [8]. However, betatron oscillations of different particles with different betatron tunes can decohere rapidly. This will result in emittance growth and performance degradation.

A coherent betatron oscillation without emittance dilution could be maintained by creating a resonance island in the betatron phase space, generating a potential well to maintain the bunch transverse phase space profile. This idea can be achieved by a modulation rf dipole, which can create 1:1 parametric resonance islands [9, 10] in the vertical phase space. This section discusses a method in producing 1:1 parametric resonance islands.

2.1 Effect of rf dipole on betatron motion

The vertical betatron equation of motion is given by

$$z'' + K_z(s)z = -\frac{\Delta B_x}{B\rho}, \tag{6}$$

where z is the vertical betatron coordinate, the prime corresponds to the derivative with respect to the longitudinal coordinate s, $K_z(s)$ is the focusing function, and

$$\Delta B_x = \Delta B(s) \cos \nu_{\rm m}\theta$$

is the rf horizontal dipole field with $\nu_{\rm m}$ as the modulation tune given by $\nu_{\rm m} = f_{\rm rf}/f_0$. Here $f_{\rm rf}$ is the rf frequency of the rf dipole field, and f_0 is the revolution frequency. The corresponding Hamiltonian is given by

$$H = \frac{1}{2}z'^2 + \frac{1}{2}K_z z^2 + \frac{\Delta B(s)}{B\rho} z \cos \nu_{\rm m}\theta. \tag{7}$$

Using the generating function F_1,

$$F_1 = -\frac{z^2}{2\beta_z}(\tan\phi_z - \frac{\beta_z}{2}), \tag{8}$$

where $\beta_z(s)$ is the betatron amplitude function, $\phi_z(s)$ is the new conjugate phase coordinate, the canonical transformation is given by

$$z = \sqrt{2\beta_z I_z}\cos\phi_z, \quad p_z = \alpha_z z + \beta_z z' = -\sqrt{2\beta_z I_z}\sin\phi_z.$$

Here I_z and ϕ_z are conjugate action-angle phase space coordinates. The new Hamiltonian becomes

$$H(I_z, \phi_z) = \frac{I}{\beta_z} + \frac{\Delta B}{B\rho}\sqrt{2\beta_z} I_z^{\frac{1}{2}} \cos\phi_z \cos\nu_{\rm m}\theta, \tag{9}$$

where s is still serving as the time variable.

Using the generating function

$$F_2 = (\phi_z - \mu_z + \nu_z\theta)J_z, \quad \mu_z(s) = \int_0^s \frac{ds}{\beta_z},$$

where J_z and $\psi_z = \phi_z - \mu_z(s) + \nu_z\theta$ are conjugate phase space coordinates, and using the orbit angle $\theta = s/R$ as the time variable, the new Hamiltonian becomes

$$H(J_z, \psi_z) = \nu_z J_z + \frac{R\Delta B(s)}{B\rho}\sqrt{2\beta_z J_z}\cos(\psi_z + \mu_z - \nu_z\theta)\cos\nu_{\rm m}\theta, \tag{10}$$

where R is the mean radius of the accelerator.

Since $\Delta B(s)$ and $\mu_z(s) - \nu_z\theta$ are periodic function of θ, the Hamiltonian of Eq. (10) can be expanded in Fourier series:

$$H(J_z, \psi_z) = \nu_z J_z + \frac{1}{2}\sum_n \sqrt{2J_z}\left[C_{n+}e^{i(\psi_z - n\theta)} + C_{n-}e^{-i(\psi_z + n\theta)}\right]\cos\nu_{\rm m}\theta, \tag{11}$$

where we have

$$C_{n+} = \frac{1}{2\pi}\int_0^{2\pi R}\frac{\Delta B(s)}{B\rho}\sqrt{\beta_z}e^{i(\mu_z - \nu_z\theta)}e^{in\theta}ds, \tag{12}$$

$$C_{n-} = \frac{1}{2\pi}\int_0^{2\pi R}\frac{\Delta B(s)}{B\rho}\sqrt{\beta_z}e^{-i(\mu_z - \nu_z\theta)}e^{in\theta}ds. \tag{13}$$

In particular, a localized rf dipole will produce all integer harmonics in the Fourier expansion.

The resonance condition is given by $\nu_{\rm m} = n \pm \nu_z$. For example, $\nu_z \approx 8.8$, the resonance condition can be fulfilled by $\nu_{\rm m} \approx 0.2$ with $n = 9$. At a resonance condition $\nu_{\rm m} = n \pm \nu_z$, we need to retain only the resonance term

$$H(J_z, \psi_z) \approx \nu_z J_z + \frac{1}{2}\sqrt{2J_z}|C_{\rm eff}|\cos(\psi_z - n\theta + \nu_{\rm m}\theta + \chi), \tag{14}$$

where χ is the phase of the resonance strength, and

$$|C_{\rm eff}| = \frac{\Delta B\Delta L}{2\pi B\rho}\sqrt{\beta_z} \tag{15}$$

with the integrated rf dipole field strength $\Delta B\Delta L = \int \Delta B(s)ds$.

We now transform the phase space into the resonance rotating frame by using the generating function

$$F_2 = (\psi_z - n\theta + \nu_{\rm m}\theta + \chi)J, \tag{16}$$

and the new Hamiltonian becomes

$$H(\psi, J) = \delta J + \frac{1}{2}|C_{\rm eff}|\sqrt{2J}\cos\psi. \tag{17}$$

where $\delta = \nu_z - (m - \nu_m)$ is the resonance proximity parameter, $J = J_z$ and $\psi = \psi_z - n\theta + \nu_z\theta + \chi$. Now, we add a nonlinear betatron detuning term to the Hamiltonian $H(\psi, J)$ (see Appendix A), i.e.,

$$H(\psi, J) = \delta J + \frac{1}{2}\alpha_{zz}J^2 + \frac{1}{2}|C_{\rm eff}|\sqrt{2J}\cos\psi, \tag{18}$$

where the detuning parameter α_{zz} are generated by using octupole magnets or the second order effect of sextupole magnets.

The fixed points of the resonance Hamiltonian is given by

$$\dot{J} = -\frac{\partial H}{\partial \psi} = 0, \quad \dot{\psi} = \frac{\partial H}{\partial J} = 0. \tag{19}$$

Equation (19) gives $\psi_{\rm FP} = 0$ or π and

$$\delta + \alpha_{zz}J + \frac{1}{2}C_{\rm eff}(2J)^{-\frac{1}{2}}\cos\psi_z = 0. \tag{20}$$

Defining the parameter $w = \sqrt{2J}\cos\psi_{\rm FP}$, where $\psi_{\rm FP} = 0$ or π, the equation for the fixed point becomes

$$w^3 + \frac{2\delta}{\alpha_{zz}}w + \frac{C_{\rm eff}}{\alpha_{zz}} = 0. \tag{21}$$

Letting the resonance strength parameter a and the resonance proximity b be

$$a = 2\frac{C_{\rm eff}}{\alpha_{zz}}, \quad b = \frac{2\delta}{\alpha_{zz}},$$

Eq. (20) is simplified to

$$w^3 - bw + \frac{1}{2}a = 0, \tag{22}$$

which has three solutions for $b \geq b_b = 3(\frac{a}{4})^{\frac{2}{3}}$ given by

$$w_1 = -\frac{2}{\sqrt{3}}b^{\frac{1}{2}}\cos\frac{\xi}{3}, \tag{23}$$

$$w_2 = \frac{2}{\sqrt{3}}b^{\frac{1}{2}}\sin(\frac{\pi}{6} - \frac{\xi}{3}), \tag{24}$$

$$w_3 = \frac{2}{\sqrt{3}}b^{\frac{1}{2}}\sin(\frac{\pi}{6} + \frac{\xi}{3}), \tag{25}$$

with $\xi = \arctan\sqrt{(b/b_b)^3 - 1}$. Here w_1 and w_2 are stable fixed points (SFP) and w_3 is the unstable fixed point (UFP).

Using the normalized coordinates with

$$h_i = \sqrt{\frac{3}{b_b}} w_i$$

for $i = 1, 2, 3$, the intercepts of the separatrix with the betatron coordinate are given by

$$h_a = -h_3 - \frac{2}{\sqrt{h_3}} \tag{26}$$

$$h_b = -h_3 + \frac{2}{\sqrt{h_3}}. \tag{27}$$

Figure 1 shows the fixed points of resonance islands and the intercepts of the separatrix with a phase space coordinate as a function of the resonance proximity parameter b/b_b.

2.2 Requirements for the bunched beam manipulation

Assuming a synchrotron with dynamical aperture D_0, the stability condition of particle motion is given by

$$\sqrt{\beta_z} w_a \leq D_0. \tag{28}$$

To fit the beam bunch with a 95% normalized emittance ε_N into the resonance island, the additional condition is given by

$$\sqrt{\beta_z} |w_a - w_b| \geq \sqrt{6 \frac{\varepsilon_N \beta_z}{\pi \gamma}}, \tag{29}$$

where the factor of 6 depicts the fact that the island is large enough to encompass $6\sigma_z$. In order to put the beam bunch into the center of the resonance island, the angular kick θ_k needed to transport the beam bunch from the SFP w_2 to the SFP w_1 is given by

$$\beta_{zk} \theta_k = \sqrt{\beta_{zk}} |w_1 - w_2|, \tag{30}$$

or

$$\delta = \frac{1}{2} \alpha_{zz} \beta_{zk} \theta_k^2 = \alpha_{zz} I_{\text{sfp}}, \tag{31}$$

where β_{zk} is the betatron amplitude function at the kicker location. Alternately, the beam bunch can be transported into the outer island by adiabatically ramping the betatron tune. Equation (31) provides the tune ramp needed to generate coherent betatron oscillation with a coherent betatron action I_{sfp}.

Following an angular kick θ_k, the Gaussian distribution function becomes

$$\rho(z, p_z) = \frac{1}{2\pi\sigma_z^2} \exp\{-\frac{z^2 + (p_z + \beta_{zk}\theta_k)^2}{2\sigma_z^2}\}, \tag{32}$$

where the rms beam size is $\sigma_z = \sqrt{\beta_z \varepsilon_{\rm rms}}$, and

$$z = \sqrt{2\beta_z J}\cos\psi, \quad p_z = -\sqrt{2\beta_z J}\sin\psi.$$

When the kicked beam is accelerated through an intrinsic spin resonance, the final polarization, given by the ensemble average of the Froissart-Stora formula over the beam distribution, becomes

$$\langle \frac{P_f}{P_i} \rangle = \frac{2}{1+\frac{\pi|\epsilon_0|^2}{\alpha}} \exp\{-\frac{(\frac{(\beta_{zk}\theta_k)^2}{2\sigma_z^2})\frac{\pi|\epsilon_0|^2}{\alpha}}{1+\frac{\pi|\epsilon_0|^2}{\alpha}}\} - 1 \tag{33}$$

where ϵ_0 is the resonance strength of rms particle with action $I_{\rm rms}$. In order to achieve a 99% spin flip, the kicker angle requirement is

$$\beta_{zk}\theta_k \geq 3.3(1+\frac{\alpha}{\pi|\epsilon_0|^2})^{1/2}\sigma_z. \tag{34}$$

Clearly, the method is not applicable to weak spin resonances where $\pi|\epsilon_0|^2/\alpha \ll 1$.

3 Application to the AGS Polarized Beam Acceleration

The AGS lattice consists of combined function magnets with $P = 12$ superperiods, $\nu_z = 8.8$, and $\beta_{z,max} = 22$ m. The polarized proton acceleration rate is normally $\alpha = 4.5 \times 10^{-5}$. The normalized emittance of polarized proton beam is about $10 \sim 20\pi$ mm-mrad. Table 1 lists the spin resonance strength ϵ_0 for the rms particle with normalized 2.5 π mm-mrad emittance [6]. This corresponds to a beam of 15 π mm-mrad 95% emittance. The parameter $W_0 = \pi|\epsilon_0|^2/\alpha$ is listed in the third column. The 4th column is the beam size with 2.5 π mm-mrad normalized rms emittance. The required 99% spin flip kick amplitude $\beta_{zk}\theta_k$ is shown in the fifth column, and the required minimum half-aperture is listed in the sixth column, where the aperture is given by $D_{0,\rm min} = \beta_{zk}\theta_k + 3\sigma_z$. Clearly, the present method can only be used to overcome strong spin resonances at $0+\nu_z$, $12+\nu_z$, $36-\nu_z$ and $36+\nu_z$. Fortunately, weak spin resonances at $24-\nu_z$, $24+\nu_z$ and $48-\nu_z$ do not require correction for a properly tuned lattice with a small emittance beam.

The proximity parameter is chosen such that the resulting outer SFP is large enough to reach a 99% spin flip. The rf modulation field strength is chosen such that the island size is large enough to encompass the entire bunch. The 7th column of Table 1 lists the required integrated field strength of the rf dipole in Gauss-meters, which is obtained by solving Eq. (29) with a 95% normalized emittance at $\epsilon_N = 15\pi$ mm-mrad and a typical detuning parameter $\alpha_{zz} = 1000$ $(\pi\text{-m})^{-1}$. Although a larger α_{zz} parameter is preferable for the stability of the beam near the 1:1 parametric resonance, we will discuss the effect of rf dipole in the small detuning limit in next section. In the following, we discuss other issues associated with polarized proton acceleration using the rf dipole scheme.

Table 1: Intrinsic resonance strengths of the AGS

$G\gamma$	$\epsilon(2.5\pi)$	$\pi\lvert\epsilon_0\rvert^2/\alpha$	σ_z (mm)	$\beta_{zk}\theta_k$ (mm)	$D_{0,\min}$ (mm)	$(BL)_{\rm rf}$ (Gm)
$0+\nu_z$	0.0075	3.9	3.3	12.4	25	8
$24-\nu_z$	0.0003	0.006	2.5	107	–	–
$12+\nu_z$	0.0029	0.6	2.2	11.8	18	5
$36-\nu_z$	0.0062	2.7	1.9	7.3	14	5
$24+\nu_z$	0.0005	0.02	1.7	40	–	–
$48-\nu_z$	0.00077	0.04	1.6	27	–	–
$36+\nu_z$	0.0134	12.5	1.5	5.1	10	4

3.1 Numerical simulations

There are three methods to transport the beam bunch into the 1:1 parametric outer resonance island. The first method uses a single turn dipole kicker to transport the beam from the inner fixed point w_2 into the outer fixed point w_1 [9]. The second method employs slow quadrupoles to change the vertical betatron tune in order to transport the beam bunch along the w_1 SFP from the $b/b_b < 1$ region to the $b/b_b > 1$ region [10]. The third method adiabatically ramps the rf dipole field strength at $b/b_b < 1$ region in order to increase the amplitude of coherent betatron oscillations.

3.1.1 Coherent betatron oscillation with a dipole kick

Using a dipole kick, the beam bunch is transported from the inner attractor w_2 to the outer attractor w_1 by a pulsed dipole. The bottom plot of Fig. 2 shows the phase space ellipse of a beam bunch with a 95% normalized emittance of 20 π mm-mrad being transported into the outer island. The top plot shows the Poincaré surface of section at the end of 500 orbital revolutions. Note here that the mismatch of the beam ellipse can result in a substantial emittance dilution. By observing the mismatch of the equipotential contour of the inner and outer SFPs, we find that emittance dilution is inevitable for the kick method.

3.1.2 Coherent betatron oscillation with slow tune ramp

Using slow quadrupole magnets to vary the tune, the center of the bunch can be adiabatically transported along the SFP w_1 (see the bottom plot of Fig. 1 of Ref. [10]). An example of a tune ramp is shown in Fig. 3. The top left plot shows an initial distribution along with its potential contour, where the rf dipole is on without any tune ramp. The top right plot shows the beam profile after the tune is changed by 0.02 in 600 turns. The tune is held at this value to cross

the intrinsic spin resonance for another 600 turns. At the end of 1200 turns, the Poincaré surface of section is shown in the bottom left plot. After crossing the spin resonance, the tune is ramped back to its original value. The final Poincaré surface of section for the beam bunch is shown in the bottom right plot. Note here that the adiabatic process renders a minimum emittance growth for the bunch beam manipulation. Our numerical simulations show that the minimum number of turns for attaining adiabatic transport of the beam bunch is about 500 turns.

3.1.3 Coherent Betatron oscillations in the zero detuning limit

When the nonlinear detuning is reduced, the amplitude of the 1:1 parametric resonance becomes much more sensitive to the proximity parameter δ. A small change of the δ parameter, can result in a large change in the parameter b/b_b in Fig. 1. Figure 5 shows the fixed points in mm at the location of $\beta_z = 22$ m for the parameter $\alpha_{zz} = 0$, 100, and 500 m^{-1} respectively. Thus the outer SFP of the 1:1 parametric resonance may lie outside the dynamical aperture (about 38 mm) in the small detuning parameter case. The procedure to induce an enhanced betatron oscillation is to operate the rf dipole with a proximity parameter of about 0.01 in the parametric region $b/b_b \leq 1$. Then the rf dipole modulation amplitude is adiabatically increased so that the beam follows the SFP of the 1:1 resonance.

The integrated rf dipole field strength in the zero detuning limit is related to the coherent betatron amplitude $z_{\rm coh}$ by

$$\int \Delta B ds\Big|_{\rm rf} = 4\pi(B\rho)\delta z_{\rm coh}/\beta, \tag{35}$$

where $z_{\rm coh}$ is the fixed point of the Hamiltonian in Eq. (17). The fifth column in Table 2 shows the integrated rf dipole strength needed to achieve the required amplitude of the coherent motion $z_{\rm coh} = \beta_z\theta_k$ of Table 1. Numerical simulations showed that the adiabaticity can be maintained easily when the modulation amplitude is varied. The resulting emittance growth factor is listed in the sixth column of Table. 2.

3.2 Physics of coherent rf spin resonance

Since the tune of the coherent vertical betatron oscillations induced by the rf dipoles at a modulation tune $\nu_{\rm m}$ in AGS is $9 - \nu_m$, coherent spin resonances are located at $G\gamma = kP \pm (9 - \nu_{\rm m})$. The coherent spin resonance is identical to the intrinsic spin resonance resulting from spin kicks due to quadrupoles in the synchrotron. At the same time, the rf dipole also generates rf spin resonance located at

$$K_{\rm rf} = n \pm \nu_{\rm m}. \tag{36}$$

Table 2: Rf dipole strength is listed together with the parameters of the strong intrinsic resonances in the AGS. The last column shows the emittance growth obtained in numerical simulations. The amplitude of the rf dipole was ramped up during 1000 turns left at the maximum value for 2000 turns and then ramped down over again 1000 turns. A betatron tune spread of ± 0.002 due to some residual non-zero chromaticity and a residual detuning parameter of 40 m^{-1} was also included.

$G\gamma$	$B\rho$ (Tm)	$\epsilon(2.5\pi)$	$z_{\rm coh}$ [mm]	$\int \Delta B ds$ [Gm]	emittance growth ratio
$0+\nu_z$	15.1	0.0075	12.4	11.	1.07
$12+\nu_z$	36.2	0.0029	11.8	24.	1.07
$36-\nu_z$	47.4	0.0062	7.3	20.	1.00
$36+\nu_z$	78.2	0.0134	5.1	23.	1.01

The induced rf resonance strength is given by

$$\epsilon_{\rm rf} = \frac{1+G\gamma}{2\pi}\frac{(BL)_{\rm rf}}{B\rho}. \tag{37}$$

Since the rf dipole field needed for polarized beam operation is very small (see Tables 1 and 2), the rf induced spin resonance has little effects on spin motion.

An alternative scheme of spin flip is to generate large rf spin resonance so that the spin motion near the intrinsic spin resonance will adiabatically follow the spin closed orbit of the rf spin resonance. If we choose the rf spin resonance location $K_{\rm rf}$ near the intrinsic spin resonance, the spin motion will be dominated by the rf resonance. To reach an rf spin resonance strength of $\epsilon_{\rm rf} = 0.025$, the rf dipole integrated field strength is about 2500 Gauss-meters. Such a high field rf dipole is hard to achieve.

The basic idea of attaining a large coherent intrinsic spin resonance is to excite coherent betatron oscillations of the beam. When the rf dipole field is adiabatically turned on, the synchronous particle with a zero betatron action is adiabatically transported to the SFP of the 1:1 parametric resonance, which has a large coherent betatron action. Particles with nonzero actions orbit around the SFP at the island tunes given by

$$\nu_{\rm island} \approx \alpha_{xz} J_x + \alpha_{zz} J_z + \cdots, \tag{38}$$

where J_x and J_z are the betatron actions of a particle relative to the center of the bunch. Thus particles with nonzero initial betatron actions will performs coherent betatron oscillations with amplitude and phase modulations around the SFP of the 1:1 parametric resonance. The amplitude modulation is averaged out

to give a coherent spin resonance strength, while the phase modulation can create many island sidebands located at

$$K = kP \pm (9 - \nu_{\rm m}) \pm \ell\nu_{\rm island},$$

where ℓ = integer. Since the island tune is small, overlapping sidebands can be summed to obtain a single coherent spin resonance.

A beam bunch is also composed of off-momentum particles. If the chromaticity is not zero, the betatron tune of the particle with an off momentum value $\Delta p/p_0$ is given by

$$\nu_{z,\rm off} = \nu_z + C_z \frac{\Delta p}{p_0}.$$

When the rf dipole is adiabatically excited at the modulation tune $\nu_{\rm m}$, all particles with an off momentum $\Delta p/p_0$ will execute coherent betatron oscillations around a SFP that differs from the SFP of the synchronous particle. The polarization of the beam after passing through the intrinsic spin resonance is the ensemble average of the beam particles.

The spin tune modulation of an off momentum particle produces synchrotron sidebands. If all synchrotron sidebands overlap with the primary spin resonance, these sidebands can be summed to give rise to the primary spin resonance with a phase shift [5].

3.3 Detuning Parameter in the AGS

The bunched beam manipulation requires detuning parameter α_{zz} to ensure that SFPs of the 1:1 parametric resonance remain inside the dynamical aperture. The detuning parameter arises mainly from sextupole and octupole fields. The bottom plot of Fig. 4 shows the measured chromaticities as a function of the beam momentum in (GeV/c) [11]. Solid lines are chromaticities obtained from a fit with the integrated sextupole field strength

$$S_b = -0.0003875 + 0.03228/p - (0.00008p + 0.000001p^2 + 0.0000009p^3)\ (\mathrm{m}^{-1}),$$
$$S_e = -0.01\ (\mathrm{m}^{-1}),$$

for short AGS bending magnets (2.0066 m). For long magnets (2.3876 m) in the AGS, the integrated sextupole strength of the S_b term is assumed to be proportional to their length. Here p is the beam momentum in (GeV/c), S_b is the integrated sextupole field in each dipole distributed in the whole dipole, and S_e is the integrated sextupole field distributed only at the end of each dipole. The first term in S_b may be considered as the systematic error term. The eddy current contribution, which is inversely proportional to the beam momentum, depends on the $\dot{B}$. In this specific experiment [11], we have $\dot{B}$ = 2 T/s. The saturation term is nonlinear with respect to the momentum p. The end sextupole in the S_e term is assumed to be momentum independent.

It is worth pointing out that the above sextupole model may not be fully justified by fitting only the data from the chromaticity measurements. Detailed beam measurements are needed to obtain a better model.

There are two family of chromatic sextupoles in the AGS. The vertical chromatic sextupoles are located at all 7th straight sections. The horizontal chromatic sextupoles are located at the 13th straight sections except the C, F, I, and L superperiods. Using these two families of sextupoles, the chromaticities can be adjusted to zero easily. The resulting detuning parameters, α_{xx}, α_{xz}, and α_{zz} are shown on the top plot of Fig. 4. Since α_{zz} is less than zero for all momentum, the detuning parameters seen by the beam can be reinforced by choosing octupoles with negative detuning octupole field gradients. To minimize vertical tune spread of the beam, it is preferable to have a machine with zero chromaticity and zero α_{xz}.

4 Conclusion

The method of overcoming intrinsic spin resonances by using the 1:1 parametric resonance island created by rf dipoles is studied. Analytic formula and numerical simulations are used to evaluate the feasibility and difficulties of this method. We find that the beam should be adiabatically transferred to the outer parametric resonance island. The chromaticity should be corrected to minimize the betatron tune spread vs the particle momentum. The required betatron amplitude detuning parameter can be generated by octupoles. Using the realistic AGS lattice, the method can indeed be used to provide polarized beam acceleration through intrinsic resonances. Experimental tests should be carried out in order to study the feasibility of the method present in this study.

Appendix

A Detuning coefficients

The particle Hamiltonian in the presence of octupole magnets is given by

$$H = \frac{1}{2}(x'^2 + z'^2) + \frac{1}{2}(K_x(s)x^2 + K_z(s)z^2) + \frac{1}{24}\frac{B'''(s)}{B\rho}(x^4 - 6x^2z^2 + z^4), \quad (39)$$

where K_x and K_z are focusing functions, $B'''(s) = B'''\Delta L|_i \delta(s - s_i)$ is the strength function of the i-th octupole magnet. Here $B'''\Delta L|_i$ is the corresponding integrated octupole strength. Using the Floquet transformation with

$$x = \sqrt{2\beta_x J_x}\cos(\psi_x), \quad z = \sqrt{2\beta_z J_z}\cos(\psi_z), \quad (40)$$

and using the angle $\theta = s/R$ as the time variable, the nonlinear detuning to the Hamiltonian is given by

$$H = \nu_x J_x + \nu_z J_z + \frac{1}{2}\alpha_{xx}J_x^2 + \alpha_{xz}J_xJ_z + \frac{1}{2}\alpha_{zz}J_z^2 + \cdots, \quad (41)$$

where

$$\alpha_{zz} = \frac{\beta_z^2}{16\pi B\rho}\sum_i (B'''\Delta L)_i, \quad (42)$$

$$\alpha_{xz} = -\frac{\beta_z\beta_x}{8\pi B\rho}\sum_i (B'''\Delta L)_i, \quad \alpha_{xx} = \frac{\beta_x^2}{16\pi B\rho}\sum_i (B'''\Delta L)_i. \quad (43)$$

Besides the nonlinear detuning, octupoles generate also half-integer and 4th order nonlinear resonances. The AGS lattice is made of 12 superperiods with 5 nearly identical FODO cells per superperiod. It has a weak 60 FODO periods. Since the betatron tunes are about 8.8, the "best" octupole arrangement is a configuration with 4, 6 or 12 octupoles located symmetrically with respect to the AGS machine. Let N be the number of octupoles, i.e. $N = 4, 6$ or 12. The driving terms for the $2\nu_x - 2\nu_z = 0$ and $4\nu_z = 36$ resonances are respectively given by

$$H_{2,-2;0} = -\{\frac{1}{16\pi B\rho}N(B'''\Delta L)_i\beta_x\beta_x\}J_xJ_z\cos(2\phi_x - 2\phi_z), \quad (44)$$

$$H_{0,4;36} = -\{\frac{1}{96\pi B\rho}N(B'''\Delta L)_i\beta_x^2\}J_z^2\cos(4\phi_z - 36\theta). \quad (45)$$

The effect of these resonances can be minimized by moving the betatron tunes away from these resonance lines.

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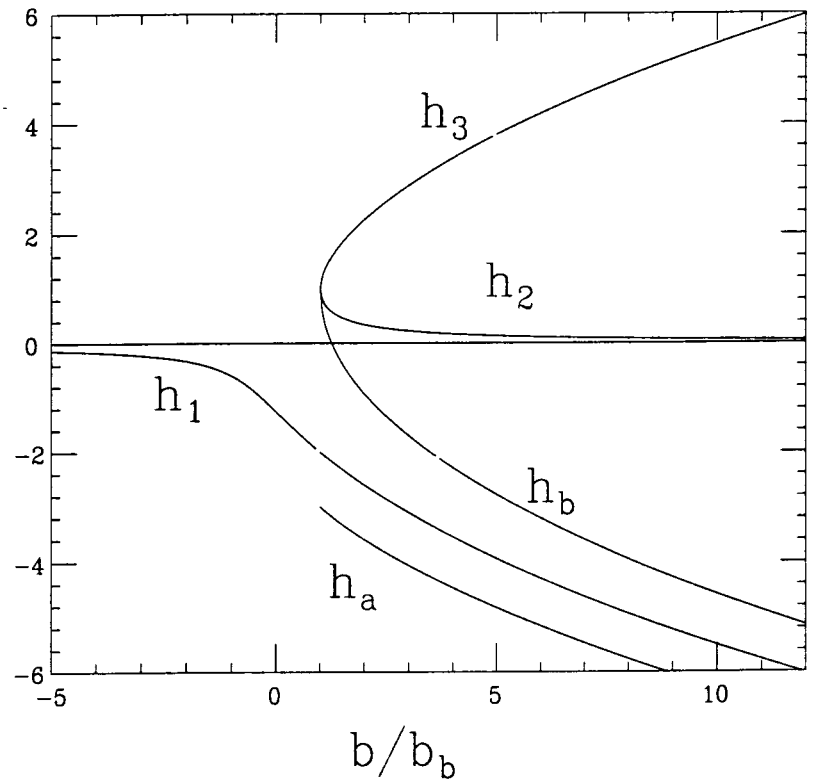

Figure 1: The fixed points, in the normalized coordinate, of the 1:1 parametric resonance Hamiltonian is plotted as a function of the modulation frequency parameter $x = b/b_b$.

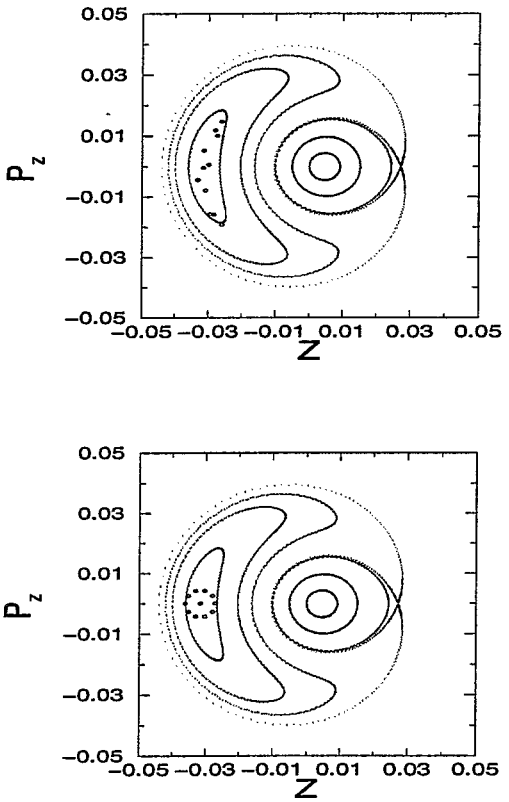

Figure 2: The bottom plot shows the beam bunch with a 95% normalized emittance of 20 π mm-mrad being kicked into the outer island of the 1:1 parametric resonance. The top plot shows the evolution of the ellipse after 500 orbital revolutions. The mismatch of the Hamiltonian contour can cause a substantial emittance dilution.

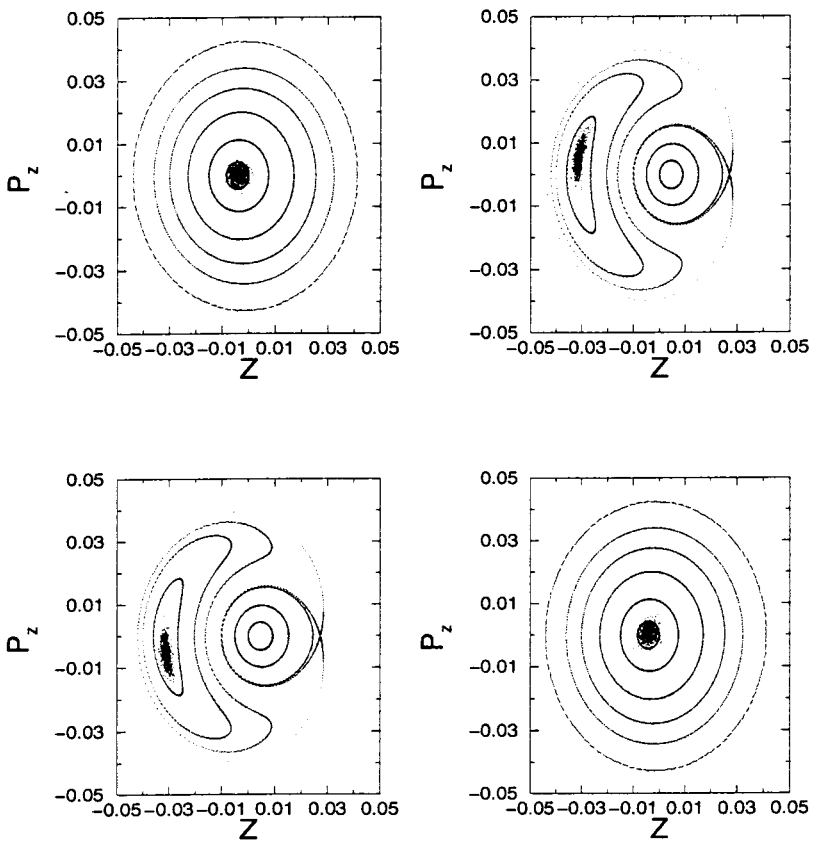

Figure 3: The top-left plot shows the beam distribution in the Hamiltonian contour. The top-right plot shows the beam profile and the Hamiltonian contour after the betatron tune is adiabatically ramped by 0.02 in 600 revolutions. The bottom-left plot shows the beam profile in the Poincaré surface of section when the betatron tune is stationed for the spin resonance crossing. The bottom-right plot shows the beam profile after the betatron tune is adiabatically restored to its original value. Note that the emittance is preserved in these beam manipulations.

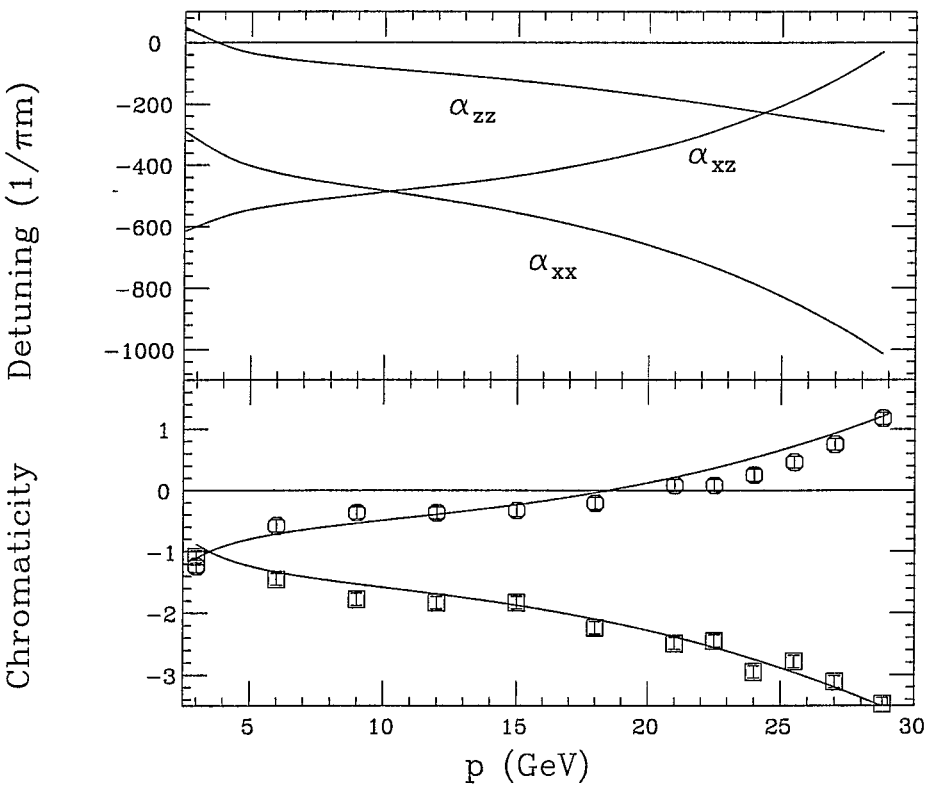

Figure 4: The bottom plot shows the measured chromaticities of the AGS [11] as a function of the beam momentum. This set of data is used to deduce the intrinsic sextupole field in A the AGS Ring. The top plot shows the nonlinear detuning parameters when the chromaticity of the AGS is corrected to zero with two families of chromatic sextupoles.

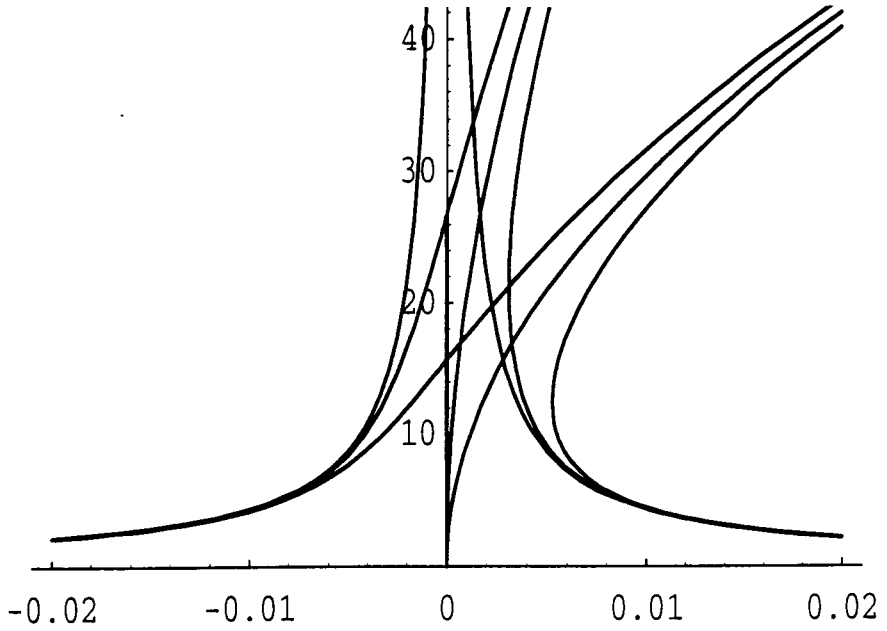

Figure 5: The distance (in mm) of the fix points from the reference orbit is shown as a function of the proximity parameter δ. The three set of curves show the fix point locations and nonlinear detuning curve with detuning parameter of 0, 100, and 500 m^{-1} from left to right, respectively.