Introduction

This technical note reviews how one may trace a beam through a beam transport line from the known magnetic element transport matrices. It shows that only in a rare case may one calculate the beam position in a downstream portion of a line from the known input position and the matrix product of the elements between the input and output. A set of corollaries are obtained that one needs only to remember to trace a beam thru any beam line. Procedures are given to treat any type of magnetic element offset, tilt, or rotation in the beam line.

The Drift Space

To illustrate the procedure necessary to trace a beam thru different magnetic elements, the drift space will be studied thoroughly. One may argue correctly that this procedure is not necessary for simple lines, but it is the only way to treat bending magnets with edge focussing, gradient magnets, or magnets with horizontal or vertical offsets.

![Figure 1 - Drift Space](image)
Figure 1 shows a drift space of length L. By definition no forces are exerted on the particles in a drift space, and as a result, the particles travel in a straight line. Initially only two dimensions will be used:

Defining:

\[ x_0 \] displacement of the beam from the center line of the drift space at the entrance to the drift space (inches).

\[ \theta_0 \] angle, measured with respect to the center line of the drift space at the input, (radians)

One may easily write;

\[ x_1 = x_0 + L \tan \theta_0 \] \hspace{1cm} (1)

\[ \theta_1 = \theta_0 \]

where \( x_1 \) and \( \theta_1 \) are the values at the output of the drift space.

This value is exact and can be used for ray tracing. However, in a transport line one is also interested in the beam size as well as the beam position. In fact, in beam line design, one is more interested in beam size since this determines the beam line apertures.

It has been shown in beam line design that if all the elements are described with linear matrices, a simple matrix equation can be used to find beam sizes in a lossless beam line. As a result, one always tries to express equations like equation 1 in matrix form. It is not necessary for ray tracing, but it is necessary for beam size determinations.

Matrix form requires a form as:

\[ x_1 = Ax_0 + B\theta_0 \] \hspace{1cm} (2)

\[ \theta_1 = Cx_0 + D\theta_0 \]

where A, B, C, and D are independent of \( x_0 \) and \( \theta_0 \).

One can see that equation 1 can be put in the matrix form if:

\[ \tan \theta_0 \approx \theta_0 \] \hspace{1cm} (3)

This occurs for small angles.
Thus the well known transport matrix form for a drift space becomes:

\[
\begin{align*}
\chi_1 &= 1x_0 + L\theta_0 \quad \text{inches} \\
\theta_1 &= 0x_0 + 1\theta_0 \quad \text{radians}
\end{align*}
\]

If milliradians are used, as is common:

\[
\begin{align*}
\chi_1 &= 1x_0 + \frac{L}{1000} \theta_0 \\
\theta_1 &= 0x_0 + \frac{\theta_0}{1000}
\end{align*}
\]

Corollary #1 can now be stated:

#1 The transport matrix equation representation of a magnetic element is not the exact description of the element. It is only a simplification.

The distance and angle for the drift space are measured with respect to the center line of the drift space. This center line will now be referred to as the element centroid as in standard transport practice.

Corollary #2 can now be stated:

#2 To use the transport matrix, one must know where the centroid is for that matrix. Knowing only the transport matrix is insufficient information to solve beam ray tracing problems.

Matrix form and matrix multiplication can be used to transport a particle or a beam from the input of a magnetic element to the output of the magnetic element.

Equation 4 can be written in matrix form as:

\[
\begin{bmatrix} \chi_1 \\ \theta_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \times \begin{bmatrix} \chi_0 \\ \theta_0 \end{bmatrix}
\]

where:

\[
\begin{bmatrix} \chi_0 \\ \theta_0 \end{bmatrix}
\]

is the vector describing the input parameters:

For Figure 1:

\[
\begin{bmatrix} \chi_0 \\ \theta_0 \end{bmatrix} = \begin{bmatrix} \chi_0 \\ \theta_0 \end{bmatrix}
\]
The input vector \([\text{VX0}]\) can also be a 5 dimensional vector:

\[
[\text{VX0}] = \begin{bmatrix}
\chi_0 \\
\theta_0 \\
Y_0 \\
\phi_0 \\
\delta_0 \\
\end{bmatrix}
\] (8)

where \(\chi_0\), \(\theta_0\), \(Y_0\), \(\phi_0\), and \(\delta_0\) have standard transport definitions.

\(\chi_0\) -- horizontal displacement of input ray, in inches, with respect to the assumed central trajectory or centroid.

\(\theta_0\) -- the angle that this input ray makes in the horizontal plane with respect to the central trajectory or centroid.

\(Y_0\) -- vertical displacement of input ray with respect to central trajectory or centroid.

\(\phi_0\) -- the angle that this input ray makes in vertical plane with respect to central trajectory or centroid.

\(\delta_0\) -- \(\Delta p/p\) = fractional momentum deviation (\%) of this input ray and the assumed central trajectory or centroid.

If a 5 dimensional vector is used, the transport matrix for the magnetic element also becomes a 5x5 square matrix, \(R\).

\[
[R] = \begin{bmatrix}
R_{11} & R_{12} & R_{13} & R_{14} & R_{15} \\
R_{21} & R_{22} & R_{23} & R_{24} & R_{25} \\
R_{31} & R_{32} & R_{33} & R_{34} & R_{35} \\
R_{41} & R_{42} & R_{43} & R_{44} & R_{45} \\
R_{51} & R_{52} & R_{53} & R_{54} & R_{55} \\
\end{bmatrix}
\] (9)

The output vector then is a 5 dimensional vector.

The standard vector equation for a drift space or any other magnetic element is the following well known equation:

\[
[\text{VX1}] = [R] \times [\text{VX0}]
\] (10)
It should be emphasized that this gives the vector \([VX1]\), at the end of the magnetic element knowing the vector \([VX0]\) at the input of the element. The centroid and the \([R]\) matrix must be known. Nothing is known about the beam outside this element.

**The Beam Pipe Element**

A new transport line element will now be introduced to keep track of element centroids, called a Beam Pipe element. All beam line instrumentation such as swics or flags are assumed to be located only in the Beam Pipe element. All commonly known transport elements are separated from each other by Beam Pipe elements. Thus, two drift spaces are not allowed to be connected. They must be connected thru a Beam Pipe element.

A Beam Pipe element has a unity matrix. If the input to the Beam Pipe is the vector \([VBPO]\), the output is the vector \([VBPl]\) and:

\[
[VBPl] = [VBPO]
\]

Equation (11)

always in the same Beam Pipe element.

The centroid of the Beam Pipe element is the center line of the beam pipe.

A third corollary may be stated:

#3 All magnetic elements in a transport line are separated from each other by the unity matrix Beam Pipe element.

To show the procedure for solving beam steering problems, several examples using a drift space will be used.

**Example #1**

![Diagram of Beam Pipe Element](image)
In the Beam Pipe A

\[ [\text{VBPO}] = \begin{bmatrix} x_0 \\ \theta_0 \end{bmatrix} \]

in the horizontal plane only

Transforming this vector to the drift space, the vector at the input of the drift space is \([\text{VXO}]\).

\[ [\text{VXO}] = [\text{VBPO}] = \begin{bmatrix} x_0 \\ \theta_0 \end{bmatrix} \]

(13)

Moving thru the drift space using the transport matrix \([R]\)

\[ [\text{VX1}] = [R] \times [\text{VXO}] \]

(14)

Transforming this vector to beam pipe B

\[ [\text{VBPl}] = [\text{VX1}] = \begin{bmatrix} x_1 \\ \theta_1 \end{bmatrix} \]

(15)

This trivial example illustrates the four important vectors \([\text{VBPO}], [\text{VXO}], [\text{VX1}] \) and \([\text{VBPl}]\).

Example #2

Consider that the drift space is offset from the beam pipe in the horizontal direction by the distance \(\Delta\).

![Diagram of Beam Pipes and Drift Space]

**FIGURE 3**
In the Beam Pipe A

\[ [\text{VBPO}]_A = \begin{bmatrix} \chi_0 \\ \theta_0 \end{bmatrix} \]  \hspace{1cm} (16)

Because of the offset:

\[ [\text{VX0}] = \begin{bmatrix} \chi_0 + \Delta \\ \theta_0 \end{bmatrix} \]  \hspace{1cm} (17)

\[ [\text{VX1}] = [R] \times [\text{VX0}] = [R] \times \begin{bmatrix} \chi_0 + \Delta \\ \theta_0 \end{bmatrix} = \begin{bmatrix} \chi_1 \\ \theta_1 \end{bmatrix} \]  \hspace{1cm} (18)

\[ [\text{VBPL}] = \begin{bmatrix} \chi_1 - \Delta \\ \theta_1 \end{bmatrix} \]  \hspace{1cm} (19)

The output vector in beam pipe B is also shifted from the [VX1] vector at the output of the drift space.

It is true that since certain elements of the drift space [R] matrix are unity or zero, that these four steps could be combined. However, in general, one must always follow these four steps for each magnetic element.

Corollary #4 can now be stated:

#4 To trace a beam from the input beam pipe to the output beam pipe, four vectors [VBPO], [VX0], [VX1], and [VBPL] must be found for each element in the beam line.

Example #3

Consider that the element is rotated in the horizontal plane by an angle \( \alpha \).

\[ \begin{array}{c}
\text{Beam Pipe A} \\
\hline
\text{Beam Pipe B} \\
\hline
\end{array} \]

\[ \begin{array}{c}
\text{[VBPO]} \\
\hline
\text{[VX0]} \quad [\text{VX1}] \\
\hline
\text{[VBPL]} \\
\end{array} \]

\[ [R] \]

\[ \text{Figure 4} \]
Actually, the important fact is not that the element is rotated, but that the centroid of the element makes an angle $\alpha$ with the centroid of the beam pipe. The centroid in some elements, as in bending magnets, will move even though the physical magnet is not moved.

Following Corollary #4.

$$[\text{VBPO}] = \begin{bmatrix} x_0 \\ \theta_0 \end{bmatrix}$$

$$[\text{VX0}] = \begin{bmatrix} x_0 \cos \alpha \\ \theta_0 + \alpha \end{bmatrix}$$

For small angles a simplification may be made, but is not essential:

$$\cos \alpha = 1$$

$$[\text{VX0}] = \begin{bmatrix} x_0 \\ \theta_0 + \alpha \end{bmatrix}$$

$$[\text{VX1}] = [R] \times \begin{bmatrix} x_0 \\ \theta_0 + \alpha \end{bmatrix} = \begin{bmatrix} x_1 \\ \theta_1 \end{bmatrix}$$

$$[\text{VBPI}] = \begin{bmatrix} x_1/\cos \alpha \\ \theta_1 - \alpha \end{bmatrix}$$

or for small $\alpha$

$$[\text{VBPI}] = \begin{bmatrix} x_1 \\ \theta_1 - \alpha \end{bmatrix}$$

If the last equation is expanded:

$$x_{\text{VBPI}} = x_1$$

$$\theta_{\text{VBPI}} = \theta_1 - \alpha$$

one can see that this can not be expressed as a matrix equation, i.e., one can not write:

$$x_{\text{VBPI}} = A x_1 + B \theta_1$$

$$\theta_{\text{VBPI}} = C x_1 + D \theta_1$$

but must write

$$\theta_{\text{VBPI}} = C x_1 + D \theta_1 - \alpha$$
One can conclude that if the centroids for all the magnetic elements are not aligned, one cannot use matrix multiplication to find the output vector from the product of several element matrices and the input vector. It is necessary to step through the beam line element by element.

However, the equations for each of the elements do not have to be derived since the transport matrices for all elements have been evaluated. It is necessary to know where the centroid is for each of the elements so that one can move from the beam pipe into the element or from the element to the beam pipe.

The Quadrupole Matrix

A quadrupole is a four pole magnet as shown in Figure 5.
For a pure quadrupole field:

\[
\begin{align*}
B_x &= \frac{B_0 y}{a} \\
B_y &= \frac{B_0 x}{a}
\end{align*}
\]

where \(B_0\) is the field at the pole and \(a\) is the pole radius.

The length of the quad is \(L\).

If one knows the input vector to the quad, \([VX0]\), one can solve for the output vector \([VX1]\) using the above field equations. One must define a centroid for the quadrupole and, for ray tracing, this can be located anywhere within the quad, within reason. For example, the physical bottom of the quad could be the centroid location. However, as discussed for the drift space, it is desired to be able to express the output vector as a product of an input vector and an element matrix. This is necessary for beam size determination but not for beam steering. Penner has found that if the centroid of the magnet is the optical or physical center of the ideal quad, the expression relating the output vector to the input vector is a square matrix ... the commonly known Quadrupole Transport Matrix.

\[
[VX1] = [R] \times [VX0]
\]

where

\[
R = \begin{bmatrix}
\cos (KL) & \frac{1}{K} \sin (KL) & 0 & 0 & 0 \\
-K \sin (KL) & \cos (KL) & 0 & 0 & 0 \\
0 & 0 & \cosh (KL) & \frac{1}{K} \sinh (KL) & 0 \\
0 & 0 & K \sinh (KL) & \cosh (KL) & 0 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]

\[
K = \sqrt{\frac{B_0}{a}} (P/e) = \text{rigidity} = 1313.24 \text{ P with } \frac{P}{e} \text{ in Gev/c and } \frac{B_0}{a} \text{ in KG/in.}
\]

\(L\) - length in inches.
The centroid is located in the center of the quad.

Following Corollary #3, the output beam can be found for any location of the quad. If the quad axis is offset from the beam pipe axis, \([VX0]\) is found from the offset and \([VBPO]\). The quad can also be rotated or tipped. It is only necessary to know the distance between the centroid of the beam pipe and the quad centroid.

The Dipole Matrix

The most confusing element for beam line tracing was found to be the simple dipole. The reason for this confusion was a lack of knowledge of where the centroid for a dipole actually is. To transform the Beam Pipe vector, \([VBPO]\) into the input element vector, \([VX0]\), one needs to know the location of the Beam Pipe centroid and the dipole magnet centroid.

The wedge magnet matrix was first found by Penner. His derivation will now be outlined so that the most essential results can be shown. He first assumed a wedge magnet and then added corrections for a rectangular magnet. His definition of a wedge magnet is a magnet in which the central trajectory or centroid enters and leaves the magnet perpendicular to the face of the magnet. This is shown in Figure 6. The magnet field is assumed constant and in the vertical direction. As a result when the beam enters the magnet, it follows the arc of a circle in the radial or horizontal plane with a radius of curvature of \(\rho\).

It should be emphasized that this is not a difficult problem if only beam ray tracing is considered. The center of the coordinate system or the centroid could be taken as the bottom of the magnet and the output ray location and angle could be found. The problem, however, is to be able to express the output angle and position as a matrix product of the input angle and position. Penner solved this problem by measuring distance and angle from a special ray called a central trajectory or centroid. It should be apparent that as the current changes in the magnet, the bend angle \(\alpha\) will change. Thus, if the components of \([VBPO]\) are fixed, the components of the \([VX0]\) vector will change. This was the reason for introducing the Beam Pipe element.
Figure 6 shows the components of \([\text{VXO}]\) which is the vector at the input of the wedge dipole

\[
\text{VXO} = \begin{bmatrix}
\chi_0 \\
\theta_0
\end{bmatrix}
\]

One wants the components \(\chi_1\) and \(\theta_1\) of the output vector. Penner assumes that the unknown ray can also have a slightly different momentum, \(P + \Delta P\), from the central ray or centroid. If the effective length of the magnet is \(L_{\text{eff}}\) (inches) and the field is \(B\) (kilogauss) then:

\[
2 \sin \frac{\alpha}{2} = \frac{E_{\text{eff}}}{(P/e)}
\]

where \((P/e)\) is the rigidity of the particles in (Kilogauss-inches).

\[
p = \frac{L_{\text{eff}}}{\alpha}
\]

for small bending angles.

\[
\frac{\Delta P}{P} = \frac{\Delta \rho}{\rho}
\]

Following Penner's procedure, and correcting the misprint in his equation 23, one obtains the exact results:

\[
\sin \theta_1 = \frac{(\rho + \Delta \rho) \sin (\theta_o + \alpha) - (\rho + \chi_o) \sin \alpha}{(\rho + \Delta \rho)}
\]

and

\[
\chi_1 = \chi_o \cos \alpha - \rho \cos (\alpha + \theta_o) - \Delta \rho \cos (\alpha + \theta_o) +
\]

\[
+ \Delta \rho \cos \theta_1 + \rho (\cos \alpha + \cos \theta_1 - 1)
\]

Note that \(\theta_1\) must be found before \(\chi_1\) can be evaluated.
These equations are sufficient for ray tracing but can be simplified for matrix calculations. If one assumes small angles and a small momentum difference:

\[ \theta_0 \ll 1 \]
\[ \chi_0 \ll \rho \]
\[ \Delta \rho \ll \rho \]
\[ \theta_1 \ll 1 \]

Then one obtains the standard transport matrix equation for a wedge magnet.

\[ \chi_1 = (\cos \alpha) \chi_0 + (\rho \sin \alpha) \theta_0 + (1 - \cos \alpha) \Delta \rho \]
\[ \theta_1 = \frac{-\sin \alpha}{\rho} \chi_0 + (\cos \alpha) \theta_0 + \frac{\sin \alpha}{\rho} \Delta \rho \]

In terms of the momentum deviation, the standard matrix becomes:

\[
\begin{bmatrix}
\cos \alpha & \rho \sin \alpha & \rho (1 - \cos \alpha) \\
-\sin \alpha & \cos \alpha & \sin \alpha \\
0 & 0 & 1
\end{bmatrix}
\]

Wedge Magnet Matrix in the Bend Plane

The wedge magnet vertical or non-bend plane matrix is the drift space matrix.

The Transport Program Manual gives the transport matrix for other elements also. There are provisions for printing these matrices with the Transport Program. The important fact for ray tracing is that these are all available and that one can easily obtain the output vector of the element, [VX1] from the input vector [VX0]. The sometimes difficult and important problem is the obtaining of the transform from the Beam Pipe input, [VBPO] to [VX0] and
the transform from the element output, [\( \text{VX}_1 \)] to the output Beam Pipe [\( \text{VBPL} \)].

One may note that the rectangular magnet has a matrix in the horizontal plane of a drift space so that these transformations can be simplified. However, these transformations must be done for any element in which the matrix is not a simple drift space matrix ... i.e., vertical focusing in a dipole or gradient fields in a dipole.

Tracing A Beam Thru A Dipole

Tracing a beam thru a dipole is similar to tracing a beam thru a drift space except that the centroid is curved rather than a straight line. Different types of dipoles exist in beam lines. The common trim dipole bends the beam a few milliradians and is used for steering corrections. This can be considered a wedge dipole for beam tracing problems. A dipole that bends a beam a few degrees is usually considered a symmetrical rectangular dipole. The matrix for this dipole includes edge effects which produce vertical focusing for a horizontal dipole. It is symmetrical because the magnet is located so that the input Beam Pipe element makes the same angle with the input magnet face as the output Beam Pipe element makes with the output magnet face.

Figure 7 shows a trim dipole lying in the beam line with the current adjusted to bend a beam a milliradian.

![Diagram of a wedge dipole](image)
The centroid for the dipole is an arc of a circle. For convenience it can be assumed to intersect at the centroid of the Beam Pipe A and Beam Pipe B. Thus:

\[
[VX_0] = \begin{bmatrix} x_0 \cos \alpha/2 \\ \theta_0 - \alpha/2 \end{bmatrix} = \begin{bmatrix} x_0 \\ \theta_0 - \alpha/2 \end{bmatrix} \quad \text{for small angles}
\]

\[
[VX_1] = [R] \times [VX_0] = \begin{bmatrix} x_1 \\ \theta_1 \end{bmatrix}
\]

Transforming out to Beam Pipe B

\[
[VBP_1] = \begin{bmatrix} x_1 \cos \alpha/2 \\ \theta_1 - \alpha/2 \end{bmatrix} = \begin{bmatrix} x_1 \\ \theta_1 - \alpha/2 \end{bmatrix}
\]

Note that if the dipole matrix is a drift space, then

\[
\theta_1 = \theta_0 - \alpha/2
\]

and

\[
VBP_1(\theta) = \theta_1 - \alpha/2 = \theta_0 - \alpha
\]

The output Beam Pipe angle differs from the input Beam Pipe angle by the bend angle of the magnet \( \alpha \). This is only true if the matrix element is equal to a drift space.

For the rectangular magnet, symmetrically located, one must include the bend angle between the two Beam Pipe elements.

**Figure 8**
Assume that the angle between Beam Pipe A and Beam Pipe B is $\gamma_{BP}$. This angle is fixed since it is determined by the beam line layout when the beam line is designed. The bend angle for the magnet is $\alpha$ and varies with the current in the magnet. Given the input vector in Beam Pipe A is $[VBPO]$

$$[VBPO] = \begin{bmatrix} x_0 \\ \theta_0 \end{bmatrix}$$

Transforming this to the entrance to the magnet, one must measure this vector from the centroid in the magnet. One can see that both $\gamma_{BP}$ and $\alpha$ must be included.

Assuming, for simplicity, only 2 components for $[VBPO]$; and

$$VXO(\chi) = VBPO(\chi) \cos \left( \frac{\gamma_{BP} - \alpha}{2} \right)$$

$$VXO(\theta) = VBPO(\theta) - \frac{\alpha}{2} + \frac{\gamma_{BP}}{2}$$

For small angles:

$$VXO(\chi) = VBPO(\chi) = x_0$$

$$VXO(\theta) = VBPO(\theta) - \frac{\alpha}{2} + \frac{\gamma_{BP}}{2} = \theta_0 - \frac{\alpha}{2} + \frac{\gamma_{BP}}{2}$$

or in vector notation:

$$[VX0] = \begin{bmatrix} x_0 \\ \theta_0 - \frac{\alpha}{2} + \frac{\gamma_{BP}}{2} \end{bmatrix}$$

$$[VX1] = [R] \times [VX0] = \begin{bmatrix} x_1 \\ \theta_1 \end{bmatrix}$$

And transforming to Beam Pipe B

$$[VBP1] = \begin{bmatrix} x_1 \cos \left( \frac{\gamma_{BP}}{2} - \frac{\alpha}{2} \right) \\ \theta_1 - \frac{\alpha}{2} + \frac{\gamma_{BP}}{2} \end{bmatrix} = \begin{bmatrix} x_1 \\ \theta_1 - \frac{\alpha}{2} + \frac{\gamma_{BP}}{2} \end{bmatrix}$$
One can see that the vector $[\text{VBPl}]$ cannot be expressed as a matrix product of different matrices and the input vector as long as sum and difference terms exist. Thus, only in the rare case that $\gamma_{BP}$ and $\alpha$ are equal can the output vector be expressed as:

$$[\text{VBPl}] = [R] \times [\text{VBPO}]$$

This case occurs if the bending magnet current is adjusted so that the centroid bends the same amount as the beam pipe. If trim magnets are in the line and the beam pipe does not bend, then only if the current is zero can the output be expressed as a vector product of the input.

Note that no limitations are put on the $[R]$ matrix. The only requirement is to know the location of the centroid of the $[R]$ matrix. For this reason the $[R]$ matrix could be a rectangular dipole or pitching magnet, a gradient dipole, or a tilted or rotated dipole.

The observer should note that this is not a new technique. If one were to trace a beam thru magnetic elements by integrating the magnetic fields, the same procedure would be necessary. The difference is that it is assumed that the element $[R]$ matrix gives the element output location vector from the input location vector.

**Conclusion**

This technical note outlines a procedure for tracing a beam through any beam line. This is necessary to solve beam steering problems. The most important result is that the commonly known Transport Matrices can be used to step thru each element providing the centroid is known for each element. It is not possible to multiply or combine matrices to find the output vector in terms of the input vector due to shifts in element centroids. The results can be summarized in four corollaries.

1. The transport matrix equation representation of a magnetic element is not the exact description of the element. It is only a simplification.
To use the transport matrix, one must know where the centroid is for that matrix. Knowing only the transport matrix is insufficient information to solve beam ray tracing problems.

All magnetic elements, including drift spaces, in a beam transport line are separated from each other by the unity matrix Beam Pipe element.

To trace a beam from the input Beam Pipe to the output Beam Pipe, four vectors \([VBPO]\), \([VX0]\), \([VX1]\), and \([VBPL]\) must be found for each element in the beam line.

Appendix A gives an example using this procedure for the upstream part of the U line.
References:


APPENDIX A -- A Beam Line Ray Tracing Example

As an example of this procedure, a beam will be traced through the first several magnets in the U line. The beam line is shown in Figure A1.
The characteristics of this beam line are given in Figure A2, lines 1-18.

The first element is a 95.045 inch drift followed by a wedge pitching magnet, UP09. This magnet bends the beam down by 0.03 degrees (GEL = 0.03 and positive indicates a bend down). Following UP09 is a 85.78 inch drift space. UQ1 follows which is a horizontally focussing quad with a gradient of 7.98839 KG/inch. This magnet is assumed offset in the positive horizontal direction by 0.1 inch (XSHIFT = 0.1). A 19.5 inch drift follows UQ1. This is followed by a vertically focussing quad UQ2 with a gradient of 7.69255 KG/inch. It is assumed that this magnet is offset horizontally and vertically by 0.1 inch (XSHIFT = YSHIFT = 0.1). This magnet is followed by a 18.794 inch drift. This drift is then followed by three symmetrically located rectangular dipole magnets UD1,2,3 separated by 18.1 inch drift spaces. These dipoles bend the beam 1.44220 degrees each in the horizontal direction. The beam pipe bends 1.41664 degrees in each of these magnets. It is assumed that the beam at the start can be described by an initial vector START.

\[
\text{START} = \begin{bmatrix}
0.1 \text{ inch} & \chi \\
0.1 \text{ mr} & \chi \\
0.1 \text{ inch} & y \\
0.1 \text{ mr} & y \\
0 & z
\end{bmatrix}
\]

positive is west
positive is west
positive is up
positive is up

The computer printout of Figures A2-A5 step-by-step traces the beam through this beam line.
The 5 components of the initial starting vector are printed.

The element is a drift (blank). DL should be neglected.

Transforming the starting vector into the beam pipe, one obtains [VBPO].

Since this is a drift, [VX0] = [VBPO].

The element length, DZ = 95.045 inch.

The [R] matrix for this drift space.

The vector [VX1] at the output of the drift

\[ [VX1] = [R] \times [VX0] \]

Transforming the [VX1] vector into the beam pipe to obtain [VBPI].

[VX1] repeated indicating the end of the element.

The beam pipe vector at the beginning of UP09 is the same as the beam pipe vector at the end of the previous element - [VBPO] = [VBPI].

Transforming [VBPO] into the pitching magnet to obtain [VXO]. Note that a positive iny direction is up and this magnet bends down. The beam pipe does not bend. \( \alpha = 0.03 \) deg = 0.5236 mr. Note that the mry component is changed by \( \alpha/2 \).

The vector [VX1] obtained from [VX0] and the [R] matrix for the wedge magnet.

Transforming out to the beam pipe, the mry component is changed by \( \alpha/2 \) again. (see Figure 7 on Wedge Magnet)

The [VBPI] vector for the pitching magnet becomes the [VBPO] vector for the next drift space.

For a drift space, [VX0] = [VBPO].
The output vector \([\text{VBPI}]\) for the drift space becomes the input vector \([\text{VBPO}]\) for UQ1.

The quad is offset by 0.1 inch in the horiz. direction so that the INX component changes transforming from the beam pipe to the magnet.

The horizontal focussing quad matrix.

To obtain \([\text{VBPI}]\) from \([\text{VX1}]\), the INX component is changed by the offset.

The vectors are stepped through drift spaces and a vertically focussing quad offset horiz. and vertically.

The input beam pipe vector to UD1, a rectangular dipole.

To obtain the mrx component of \([\text{VXO}]\), the MRX component of \([\text{VBPO}]\) is changed by the bend angle of the magnet and the beam pipe. The magnet bends more than the beam pipe by 0.02556 deg or 0.44611 mr. The mrx component is changed by half this angle.

The \([\text{R}]\) matrix for UD1.

The output \([\text{VX1}]\) vector. Note that the mry component of \([\text{VX1}]\) is less than \([\text{VXO}]\) illustrating vertical focussing.

Transforming \([\text{VX1}]\) out to \([\text{VBPI}]\) the MRX component is again changed by half the angle difference.

The VBPI vectors are found for each component by stepping through each element.

If one wanted to plot the x and y position for this example, the inx and iny would be plotted against the downstream z.
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**Figure A2**
\[ \text{FIGURE A5} \]
APPENDIX A — A Beam Line Ray Tracing Example

As an example of this procedure, a beam will be traced through the first several magnets in the U line. The beam line is shown in Figure A1.