I. Introduction

To obtain polarized protons at low energy, process of the fast tune jump through many intrinsic depolarizing resonances is clearly unavoidable. Unfortunately, the emittance of the beam will increase due to the excitation of these fast quadrupoles. In this short note we shall discuss the emittance problem related to the tune jump.

II. Effect of Fast Quadrupoles

The excitation of fast quadrupoles change suddenly the betatron amplitude around the closed orbit. The amount of this nonadiabatic change can be expressed as

\[ \Delta R = - \frac{Q}{4\pi} \int J_p e^{i\phi} ds \]

where

\[ J_p = \int_0^{2\pi R} \beta(s)k(s)e^{-i\phi(s)} ds \]

\[ \Delta Q = -\frac{1}{4\pi} \int_0^{2\pi R} \beta(s)k(s) ds \]

The nonadiabatic sudden change in the betatron amplitude function results in the mismatch of the beam size. These nonadiabatic increments of beam size will result in a corresponding increase in the emittance. To calculate the increase in the beam size, one can either use the Synch
Program\textsuperscript{2} or use Section 4 of ref. 1. A program in BASIC, written by E.D. Courant, is included in the appendix.

Because the AGS machine is operating at $Q_x \equiv 8.6$, $Q_y \equiv 8.8$ with superperiod $P=12$, the important terms in the summation of eq. (1) are $p=12$, 17, and 18. For a smaller $\Delta Q$, the strength $k(s)$ of tuning quadrupoles needed become smaller. The corresponding integral $J_p$ will also be smaller. With even numbers of fast tuning quads located in each consecutive cells, the contribution of $J_{18}$ is zero (because the superperiod of AGS is $P=12$). However, the contribution of $J_{17}$ and $J_{12}$ are important.

At present, there are 10 fast pulsed tuning quadrupoles in AGS. It is better to put these 10 tuning quadrupoles consecutively in each cell and leave two cells without tuning quadrupoles close together (such as EF in the present arrangement). Table I lists the emittance (beam size) increase due to the sudden excitation of fast quadrupoles. We observe that the experimental value\textsuperscript{3} of the increase in the beam size is approximately a factor of $2 \sim 3$ larger than that predicted from the betatron mismatch calculation.

On the other hand, when a set (12) of slow quads are used before the fast quads are turned on, we observe a reasonably good agreement between the experimental result and the theoretical prediction.

III. Possible Coupling Effect

The disagreement between the experiments and the theory on the increase of the beam size in the process of fast quad tune jump may be attributed to the coupling resonance.

Figure 1 shows that the betatron tunes $Q_x$ and $Q_y$ as a function of the excitation strength of the fast quadrupole. We note first that the $Q_x = Q_y$ resonance has to be crossed to reach $\Delta Q = 0.2 \sim 0.25$. Because of the coupling resonance, the vertical beam size may increase due to a much larger horizontal emittance. It is, however, puzzling to observe that the amount of increase in the horizontal beam size is about the same as that of vertical beam size.

On the other hand, when the slow quads were turned on before the fast quads are pulsed, the experiment agrees well with the theoretical prediction (see Table I).

Figure 2 shows similar plot as that of Fig. 1, except that a set of 12 slow quads are turned on to shift the tune $Q_x - Q_y$ apart before the fast quads are turned on. We observe that the increase in the betatron beam size is about the same as that of Fig. 1. However, the coupling resonance, $Q_x - Q_y = 0$, may be less important.
<table>
<thead>
<tr>
<th>Table I: Mismatch and emittance blow-up with $\Delta Q=0.25$</th>
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<tr>
<td><strong>Tune jump with Fast Quad only</strong></td>
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<tr>
<td><strong>Calculation</strong></td>
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<tr>
<td>$(\Delta \beta/\beta)_V$</td>
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<td>.44</td>
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<td>$(\Delta \beta/\beta)_H$</td>
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<td><strong>Experiments</strong></td>
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<tr>
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<tr>
<td>$(\sigma/\sigma_o)_H$</td>
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<td>1.37±.07</td>
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<td><strong>Tune jump with slow and fast quads</strong></td>
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<td><strong>Calculation</strong></td>
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<td><strong>Experiments</strong></td>
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<td>1.11±.04</td>
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IV. Tune Jump for the Resonance at 36-ν

At resonance $γC = 36-ν$, the betatron tune of the machine has to be jumped upward to avoid the resonance. Figure 3 shows the betatron functions in the process of tune jump with and without slow quads.

Without slow quadrupoles, $Q_x$ and $Q_y$ are approaching 8.5 and 9.0 respectively. The emittance can blow-up by a factor of $2 \sim 3$ easily (see ref. 4). With slow quadrupoles, the betatron tune of the machine can be controlled to be a distance away from the stopbands. The resulting emittance mismatch can then be controlled. With the slow quads, the emittance will blow-up about 10%.

We note in Fig. 3 that the condition, $Q_x < Q_y$, is still satisfied when the slow quads are excited. The requirement of this condition depends solely on the importance of the coupling resonances. If the coupling resonance, $Q_x - Q_y = 0$, is not important, then the degree of freedom for the slow quads can be increased to obtain even better performance.

IV. Conclusion

The emittance blow-up due to the excitation of the fast quadrupoles are estimated based on the mismatch in the betatron amplitude. We found that the dominant harmonic components are $p = 12, 17$ and 18. The harmonic 18 happens to cancel each other due to the even number of fast quadrupoles and 12 superperiodicity of AGS, provided that these fast quadrupoles are arranged in pairs. The contribution of the harmonic 17 is least when these fast quadrupoles are arranged in each consecutive cell (the current arrangement).

Comparing our calculation to that of experimental results, we find the coupling resonance may be important in the process of the emittance blow-up when the slow quads are not excited. Some experiments would be very useful to pinpoint the effect of coupling resonance.

1. Using skew-quads in the fast quadrupole tune jump, one may find that the emittance blow-up depends on the polarity and excitation of skew quads.

2. At a smaller tune shift, e.g. $ΔQ = 0.15$, the fast quadrupole tune jump will pass the coupling resonance, $Q_x - Q_y = 0$. On the other hand, excitation of the fast quadrupoles incorporating with the slow quadrupoles will not go through the coupling resonance.

3. If it is indeed that the coupling resonances are important to the emittance blow-up, a possible solution may be a fast excitation of these fast quadrupoles and a fast de-excitation at a later time. Technically, this method may be difficult.
Finally, the slow quads are especially important for the resonance at 36-\nu. Without slow quads, the emittance can blow-up by a factor of 2 ~ 3. With slow quads, the tune of the machine can be controlled. Therefore, the emittance blow-up can also be controlled.

References


3. L. G. Ratner et al., AGS-Fast quad experiments and private communications.


Figure Captions

Fig. 1. The betatron functions and the betatron tune of the AGS is plotted as a function of the excitation strength of the fast quadrupoles $\int B'dl/B\rho$ (m$^{-1}$). The initial tune was chosen to be $Q_x = 8.6$, $Q_y = 8.8$.

Fig. 2. The same as that of Fig. 1, except that 12 slow quads at 5 feet straight section (S5) are excited with $\int B'dl/B\rho = -0.00355$ m$^{-1}$ and are excited to shift $Q_y$ away from $Q_x$. The betatron functions are similar to that of Fig. 1. The betatron tunes of the machine $Q_x$, $Q_y$ differ from that of Fig. 1.

Fig. 3 The same as that of Fig. 1, except that the fast quads shift the $Q_y$ upwards for the 36-\nu resonance. The dashed lines correspond to the excitation of the fast quads without the slow quadrupoles. The solid line corresponds to the excitation of slow quads at S15-5' straight section with strength $\int B'dl/B\rho = 0.00610$ m$^{-1}$. Note that the betatron functions are drastically different in these two cases.
REM PROGRAM DB CALCULATES BETA MODULATION BY QUADS AT GIVEN AZIMUTHS

DEFINT I-M
30 CLS: KEY 5,"cont"+CHR$(13): KEY 6,"GOTO 113"+CHR$(13)
40 DEFDBL N,T,D
50 DIM TH(I2), DI(I2), ITH(I2), Y(I2), DD(I2)
60 INPUT"nu";NU
70 INPUT"m";M
80 TWPOI=8*ATN(1#)
90 FOR I = 1 TO M
100 PRINT USING"theta(##)":I;
110 INPUT TH(I): NEXT
130 FOR I=1 TO M: TH(I)=TH(I)-360*INT(TH(I)/360): NEXT
140 FOR I=1 TO M: FOR J=1 TO I
150 IF TH(I)<TH(J) THEN SWAP TH(I),TH(J)
160 NEXT J: NEXT I
180 MU#=TWPOI*NU: DEN=M*SIN(MU#)/TWPOI
190 TH1=TH(I)
200 FOR J=1 TO M: TH(J)=TH(J)-TH1
210 ITH(J)=TH(J)*2/9:IF ITH(J) < 1 THEN ITH(J)=1
220 NEXT J: FOR J=1 TO M
230 D=0
240 FOR I=1 TO M
250 THS=MU#*(1-ABS(TH(I)-TH(J))/180)
260 IF J = I THEN THS=THS
270 D=D+COS(THS)
280 DS=DS*SIN(THS)
290 NEXT I
300 D = SQRT(D^2+DS^2)/DEN : Y(J)=112*ABS(D) : DD(J)=D
310 NEXT J
320 CLS: KEY OFF: LINE(1,1)-(1,342): LINE (1,342)-(719,342)
330 FOR K = 1 TO 5: L=25-4*K
340 LOCATE L,1: PRINT USING".##":5*K
350 NEXT L: LOCATE 25,1:PRINT"0":LOCATE 25,77:PRINT "360":
355 LOCATE 2,60: PRINT USING "Nu=##.##":NU
360 FOR I=1 TO M: IF I=1 THEN GOTO 420
370 IF ITH(I)<1 THEN IL=ITH(I)-1 ELSE IL=1
380 LOCATE 25,IL
390 PRINT USING"##.##":TH(I)
400 X1=TH(I)*2+1: IF I<M THEN X2=TH(I+1)*2 ELSE X2=719
410 Y1=342-Y(I): LINE (X1,Y1)-(X2,Y1): IF Y1 < 1 THEN Y1=1
420 IF I=M THEN Y2=342-Y(I+1) ELSE Y2=342-Y(I)
430 IF Y1 < 1 THEN Y1=1
440 LINE (X2,Y1)-(X2,Y2)
450 NEXT I
460 FOR J=1 TO M: LOCATE J,20: PRINT USING"THETA =##.##":TH(J):PRINT USING ",D
B/DNU = ##.###":DD(J):NEXT
480 END
490 PRINT "New Parameters?": END :CLS: GOTO 130
Figure 2.

\[ \hat{\beta}(m) \]

\[ \beta_Y \]

\[ \beta_X \]

\[ \nu \]

\[ \nu_Y \]

\[ \nu_X \]

\[ \int \frac{B'}{B_P} (m^{-1}) \]
Figure 3.