DAMPING COHERENT QUADRUPOLE
OSCILLATIONS IN THE AGS

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September 20, 1994
INTRODUCTION

Quadrupole oscillations appear during the beam acceleration, as well as before or after transition. These oscillations can cause emittance growth. The larger emittance leads to beam loss when gamma transition jump operates. After pointing out the origin of these oscillations in the AGS, this paper describes them theoretically and how they can be damped. A practical solution that gives good results has been implemented.

1. ORIGIN OF THE OSCILLATIONS

The AGS quadrupole oscillations seem to be due to the B ripple. The Fourier transform of the B signal shows a peak which appears at the same frequency as a peak in the Fourier transform of the beam signal. Those two signals are strongly correlated.

Fig. 1 shows B, the beam signal (from the peak detector), and their respective Fourier transforms.
2. BEAM TRANSFER FUNCTION

As far as quadrupole oscillations are concerned, the beam can be considered as a pure oscillator at twice the synchrotron frequency $\omega_s$ [1]. If $B(s)$ is the transfer function relating the relative variations of the RF voltage $V_{rf}$ to the relative variations of the bunch length $l$, we have:

$$\frac{\Delta l}{l} = B(s) \frac{\Delta V_{rf}}{V_{rf}} \quad \text{with} \quad B(s) = -\frac{\alpha \left(2\omega_s\right)^2}{s^2 + \left(2\omega_s\right)^2}$$

The $\alpha$ factor depends on the bunch length:

$$\alpha = \left[\frac{2}{\theta} - \frac{J_0(\theta)}{J_1(\theta)}\right] \theta^{-1} \quad \text{(eq. a)},$$

where $\theta$ is the half bunch length in radians ($=\omega_s l$), and $J_0$ and $J_1$ are Bessel functions [2]. For short bunches, $\alpha = \frac{1}{4}$.

3. OSCILLATION DETECTION

The detection of the oscillations usually relies on the measurement of the peak line density of the bunches. A peak detector following a broad band pick up is often used. We make the assumption we have parabolic bunches.

3.1 Relation between $V_m$ and $l$

The main variables are represented in fig. 2. $V_m$ is the measured voltage. The difference between $V_p$ and $V_m$ is $h$. $T$ is the space between bunches ($T = 2\pi \omega_s$).
If $A$ is the area under one bunch ($A = a_1 + a_2$), for a parabolic shape:

$$A = \frac{2V_p l}{3}.$$  

But $a_1 = a_3$ because the signal is ac-coupled, so:

$$A = a_3 + a_2 = hT$$

Then

$$\frac{2V_p l}{3} = hT.$$  

Now $V_p = V_m + h$ so $h = \frac{2V_m l}{3T - 2l}$.

Moreover the area $A$ is a constant and so $V_p l$ or $(V_m + h)l$ and $h$ are constant. As a result, $l \partial V_m + (h + V_m)\partial l = 0$ and finally:

$$\frac{\Delta V_m}{V_m} = -\frac{3T}{3T - 2l} \frac{\Delta l}{l}$$

For a given bunch length $l$, its relative variations $\frac{\Delta l}{l}$ are proportional to the relative variations of the detected peak voltage.

### 3.2 Practical solution

The voltage $V_m$ was found to be very noisy, due to non smooth and unequal intensity bunches. To get a signal proportional to $\frac{\Delta l}{l}$, the third harmonic of the beam is calculated (for the time being with a spectrum analyzer). A dedicated circuit has to be built.
4. **BLOCK DIAGRAM OF THE SYSTEM**

F(s) is the transfer function of the loop amplifier to be designed. It is fed by the difference between the measured signal and the reference signal. After being amplified by F(s), the signal is injected into the high level RF system (HLRF) to modulate the total RF voltage. The cavities' response which can be considered as a low pass filter with a cut off frequency of 10 kHz \( C(s) = \frac{s_0}{s+s_0} \) with \( s_0 = 2\pi \times 10^4 \text{ rad s}^{-1} \). The block diagram is drawn in Fig.3.

![Block diagram](image)

**Fig. 3 Block diagram**

5. **STUDY OF THE LOOP**

5.1 **Characteristic equation, derivative action**

In a first approximation, the high level RF system is supposed to be equal to one. In this case, the open loop transfer function is:

\[
\frac{-2\omega_s^2}{s^2 + (2\omega_s)^2} \left( \frac{3T}{3T-2l} \right) F(s) K,
\]

K being a proportional constant taking into account the pick up system and the RF system.

To get rid of the DC offset of the peak detector, a derivative action is required:

\[
F(s) = K_d s
\]

This action also provides a 90° phase shift at \( 2\omega_s \) which is required to damp the oscillations.

The characteristic equation is then:

\[
s^2 + (2\omega_s)^2 \alpha K K_d \left( \frac{3T}{3T-2l} \right) s + (2\omega_s) = 0
\]
5.2 **Study of the damper**

According to the Routh criterion, the system is stable for $K(3T-2l)>0$ which is true and it is critically damped for $K_d = K_{dl} = \frac{1}{K_\alpha \omega_s} \frac{3T-2l}{3l}$.

Moreover, $K_{dl}$ is maximum when both $l$ and $\omega_s$ are minimum, according to eq. a. $K_{dl}$ is plot as a function of $\theta$ in fig. 4.

![Fig. 4 K_{dl} vs bunch length](image)

Theoretically, the system can always be critically damped (or over damped) if $K_{dl}$ is chosen for $l$ and $\omega_s$ are minimum.

5.2 **Practical implementation**

In practice, one pole due to the high level RF system and one to the peak detector have to be taken into account. These poles will reduce the phase margin and the system will be damped and stable just around a selected synchrotron frequency.

The final block diagram of the damper is drawn in fig. 5.
6. **PRACTICAL RESULTS**

The beam signal has been measured with and without feedback system. Measurements are given in fig. 6a (without) and in fig. 7b (with).

The quadrupole oscillations are damped, before and after transition (corresponding to the peak in fig. 7a).
CONCLUSION

The AGS beam now has a better behavior. Coherent quadrupole oscillations are well damped. Some modifications of the system are foreseen. A dedicated pick up (based on the third harmonic detection) will be added to have a clean input for the loop amplifier.

The loop amplifier could also be based on an integral action if the DC offset of the system could be rejected before closing the loop. This solution, which has to be studied, would provide a higher gain at low frequencies, which could be interesting according to the usual low values of the synchrotron frequency.

REFERENCES
