Temperature Increase of the Foil Stripping Material in the AGS-RHIC Beam Transfer Line

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1. Introduction

As part of the injection scenario of RHIC,\(^1\) heavy ions with \(Z \geq 79\) will be accelerated in the AGS accelerator with all but their two K-shell electrons stripped. These electrons must be removed before the beam bunch is injected into RHIC for further acceleration. An efficient method of removing the K-shell electrons from an ion, is to allow the beam to pass through a foil material of appropriate thickness. Such a foil can be positioned at a chosen location along the AGS-RHIC beam transfer line. As the beam passes through the foil, the beam suffers Energy loss, Energy spread, (Energy Straggling), Angular Spread, (Angular Straggling) and the foil itself may undergo some sublimation. It is the purpose of this tech note to calculate the effect of various foil materials (C, Al, Cu, and Au) on two different beam bunches, i.e. \(^{197}\text{Au}^{+77}\) and \(^{238}\text{U}^{+90}\) as well as the mechanical effect of the beam bunches on the foils. In this way we recommend the optimal foil thickness needed to remove the K-shell electrons. All the calculations presented here, are based on the formalism appearing in ref. 2, and the calculated physical quantities are tabulated in Table I for Au and U beams.

The physical quantities which are calculated for this tech note appear in Table I and are based on the following assumptions:

a) Beam Energy = 10.4 GeV/u with energy spread \(\Delta E/E = 10^{-4}\).

b) Number of ions per bunch = \(10^9\).

c) The 2.5 \(\sigma\) beam emittance is \(0.8 \times 10^{-6}\) m-rad.
d) At the point of interaction the beam forms waists in both horizontal and vertical planes with beam transport parameters $\beta_x = \beta_y = 6 \text{ m}$.

2. Effects of Stripping Foil on the Beam

This section is a summary of ref. 2 which is included for completeness. The formulae used to calculate the physical quantities of Table I are given next:

a) Foil thickness of a material to yield 98% bare beam ions is given by the formula

$$ T = \frac{1.66 A_t \ell n \xi}{\sigma_k} \left( \text{gr/cm}^2 \right) $$

(1)

where $A_t = \text{mass number of target}$, $\xi = \text{survival probability (for 98% efficient foil \( \xi = 0.2 \))}$, $\sigma_k$ is the single electron K-shell ionization cross section in barns given by the formula.\(^3\)

$$ \sigma_k = S(Z_t, Z_p) \left( \frac{Z_t}{Z_p} \right)^2 + (Z_t, Z_p) \left( \frac{Z_t}{Z_p} \right)^2 \left( \frac{\ell n \gamma^2 - \beta^2}{\beta^2} \right) $$

(2)

where $Z_t$ and $Z_p$ are the Atomic numbers for the target and projectile respectively. $S(Z_t, Z_p)$ and $t(Z_t, Z_p)$ are obtained from tables of ref. 4 and $\beta = v/c$ and $\gamma = (1 - \beta^{-2})^{1/2}$.

b) The multiple scattering (Angular spread) of the beam when the beam passes through the foil.

$$ < \sigma_{rms} > = 14.98 \text{(MeV/c)} \times \frac{Z_p}{p \beta} \left( \frac{T}{X_0} \right)^{1/2} $$

(3)

where $p$ is the beam momentum in MeV/c, $\beta = v/c$ and $X_0$ is the unit radiation length.\(^5\) The rest of the symbols are defined above. Substituting in (3) the target thickness given by (1) and expressing $p$ and $\beta$ in terms of $\gamma$, the formula (3) can be expressed as:

$$ < \sigma_{rms} >^2 = -3 \times 10^{-6} \frac{Z_p^2 A_t}{A_p X_0} \ell n \xi \sigma_k \left( \text{rad}^2 \right) $$

(4)

c) The Energy loss of the beam passing through the foil has been calculated from ref. 6 which shows in various graphs the Energy loss of beam ions per mg/cm\(^2\) of target material for beam energies up to 1 GeV/u. To calculate the Energy loss for ions of 10.4 GeV/u we extrapolated the graphs up to 10.4 GeV/u assuming that the percent change of the energy loss between 0.1 GeV/u and 1 GeV/u for a given beam ion and target material is the same as for a beam with energy range 1 GeV/u and 10 GeV/u.
d) The percent change of the beam emittance after the beam passes through the foil is calculated using the formula

$$\frac{\Delta \epsilon}{\epsilon} (%) = (\beta < \theta_{rms} >^2 / \epsilon) * 100$$  \hspace{1cm} (5)

where \( \epsilon \) is the beam emittance and \( \beta \) is the beam transport parameter at the foil location. To prove (5) we assume that the beam forms a waist at the location of the foil. For a relatively thin foil it is a good approximation to assume that the beam exiting the foil forms also a waist, and has the same \( \beta \) value as at the entrance of the foil.

3. Effects of the Beam on the Foil

Here we focus on the temperature increase of the section of the foil which interacts with the beam. In these calculations the foil is considered as a disk of 2.54 cm in radius, (Fig. 1) and of uniform thickness that is taken to be equivalent to 98% stripping efficiency.\(^2\) The beam spot on the foil has a radius of 2.2 mm and includes 96% of the beam intensity (2.5\( \sigma \)). The temperature increase of the foil has been calculated under three different scenarios, which are described below. The results of the calculations appear in columns 6, 7 and 8 of Table I. These three scenarios are summarized below.

1. In the first way of calculating the temperature increase of the foil we assumed that the energy lost by the beam is totally converted into heat which increases the temperature of the section of the foil which intercepts the beam. The formula to calculate the temperature increase is

$$\Delta T = \frac{Q}{m \cdot c}$$  \hspace{1cm} (6)

where \( Q \) is the Energy loss of two beam pulses deposited on the foil in the form of heat, \( c \) is the specific heat capacity and \( m \) is the mass of the foil which interacts with the beam. The reason we calculate the temperature increase after two pulses hit the foil is because the time difference between the two pulses is 20 msec and the time constant of cooling of the foil because of its finite thermal conductivity (see next two cases) is \(~ 60\) msec.
2. In the second way of calculating the temperature increase, we assume that the beam dissipates on the foil a constant energy per unit time which equals the energy loss of the beam per unit time (three bunches per second). We also assume that the heat is transferred by means of conduction from the center region of the foil to the outer edge of the foil, where the rest of the foil mass absorbs the heat. The formula to calculate the temperature increase in this case is given by (Appendix B)

$$\Delta T = \frac{A}{(m_s + m)c} t + \frac{A \ln (r_2/r_1)}{(1 + m_s/m)^2} \frac{k \ln (r_2/r_1)}{2\pi d} \left[ 1 - \exp \left( -\frac{k2\pi d(1/m + 1/m_s)}{c \ln (r_2/r_1)} t \right) \right]$$

where $m_s$ is the mass of foil in the beam-foil interaction area (gr)
m is the mass of the rest of the foil (gr)
c is the specific heat capacity of the foil (J/gr°C)
k is the thermal heat conductivity of the foil [J/(sec·cm°C)]
d is the foil thickness (cm)
r$_1$ is the radius of the beam spot (cm)
r$_2$ is the outer radius of the foil (cm)
$A$ is the heat per unit time delivered on the foil by the beam (J/sec)

3. In the third way of calculating the temperature increase we assume that the outer edge of the foil is kept at a constant temperature acting as a heat sink. We also assume that the part of the foil which does not interact with the beam is acting as a conductor and does not absorb any heat. In this case the beam-foil interaction region will achieve asymptotically a final temperature difference which is given by (Appendix C)

$$\Delta T = \frac{A \ln (r_2/r_1)}{k2\pi d} \left[ 1 - \exp \left( -\frac{k2\pi d}{m_s c \ln (r_2/r_1)} t \right) \right]$$

CONCLUSIONS

Utilizing the results of ref. 2 and the derived formulas above we have constructed Table I. From these results we can conclude that all the foils considered here can withstand the temperature increase due to the beam energy loss. In addition the Au foil has the minimum effect on the beam because it requires smallest thickness in order to strip the beam with
98% efficiency. Since Au foil of thickness of 40 mg/cm² can be easily made, it appears to be the most advantageous stripping foil and we recommend its utility for the AGS-RHIC beam line.

References

1. M.J. Rhoades-Brown, "An Alternative Injection Scheme for Heavy Ions into RHIC", AD/RHIC-44.
2. M.J. Rhoades-Brown, "The Heavy Ion Stripping Foil Requirements Between AGS and RHIC", AD/RHIC-68.
5. Y.S. Tsai, Rev. of Mod. Phys. 46 815 (1974).

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<th>Table I.</th>
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<td>Au^{+77}</td>
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case 1: Temperature increase due to two beam bunches
case 2: Temperature increase after 120 s (3 bunches/sec)
case 3: Final temperature at t = ∞
Appendix A. Calculations of Heat Flow

In this Appendix we calculate the heat flow between the inner part of the film where the beam deposits its energy (a disk of radius \( r_1 \)) (see Fig. 1) and the outer part of the foil at radius \( r = r_2 \). The result of this Appendix will be used in Appendices B and C.

In a steady state condition the heat flow per unit time \( \frac{dQ}{dt} \) across a cylindrical cell \( dr \) and thickness \( d \) (normal to the paper, see Fig. 1) is given by

\[
\frac{dQ}{dt} = k2\pi rd \frac{dT}{dt} \Rightarrow dT \frac{dQ}{dt} = \frac{k2\pi d}{k2\pi d} dr/r
\]

(A.1)

Since \( \frac{dQ}{dt} = \text{constant across the cylinder} \), integration of the equation A.1 yields

\[
\frac{dQ}{dt} = \frac{k2\pi d}{\ell n (r_2/r_1)} (T_2 - T_1)
\]

(A.2)

The formula above expresses the heat transferred radially per unit time in the cylinder shown in Fig. 1 in terms of known quantities.

Appendix B.

In this Appendix we will derive formula 7 which describes the temperature increase for case 2 above.

Conservation of Energy for the central spot is expressed as

\[
A - \frac{\Delta Q}{\Delta t} = mgcc \Delta T \Rightarrow \frac{A}{mgcc} - \frac{k2\pi d}{\ell n (r_2/r_1)} (T_2 - T_1) = \frac{dT_1}{dt}
\]

(B.1)

Similarly, Conservation of Energy for the mass residing on the ring can be written as

\[
\frac{\Delta Q}{\Delta t} = mc \Delta T_2 \Rightarrow \frac{k2\pi d}{mcc\ell n (r_2/r_1)} (T_2 - T_1) = \frac{dT_2}{dt}
\]

(B.2)

Subtracting Eq. (B.2) from (B.1) we obtain

\[
\frac{A}{mgcc} - \frac{k2\pi d}{\ell n (r_2/r_1)} \left( \frac{1}{mc} + \frac{1}{mcc} \right) (T_1 - T_2) = \frac{d(T_1 - T_2)}{dt}
\]

(B.3)

Integrating eq. (B.3) above we obtain

\[
(T_1 - T_2) = \frac{\frac{A}{mcc}}{\frac{k2\pi d}{\ell n (r_2/r_1)} \left( \frac{1}{mcc} + \frac{1}{mc} \right)} \left[ 1 - \exp \left( -\frac{k2\pi d}{\ell n (r_2/r_1)} \left( \frac{1}{mcc} + \frac{1}{mc} \right) t \right) \right]
\]

(B.4)

Substituting Eq. (B.4) above into Eq. (B.1) and integrating, we obtain Eq. (7).
Appendix C.

In this Appendix we will derive formula 8 which describes the temperature increase for case 3 above.

We apply conservation of energy for the central spot where the beam delivers $A$ units of energy per unit time.

$$A - \frac{dQ}{dt} (\text{Heat Transfer by Cond.}) = m_sc \frac{dT}{dt} \rightarrow A - \frac{k2\pi d}{\ell_n (r_2/r_1)} (T - T_0) = m_sc \frac{dT}{dt} \quad (C.1)$$

Integration of equation C.1 above yields equation (8).
Fig. 1 Schematic diagram of the beam spot in reference to the stripping foil. The beam broadening due to the dispersion is not included since the calculation of the emittance does not require it.